PUZZLER

For thousands of years the spinning Earth provided a natural standard for our measurements of time. However, since 1972 we have added more than 20 "leap seconds" to our clocks to keep them synchronized to the Earth. Why are such adjustments needed? What does it take to be a good standard? *(Don Mason/The Stock Market and NASA)*

chapter

1 Physics and Measurement

Chapter Outline

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of bodies at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's is a more general theory of motion.

Classical physics, which means all of the physics developed before 1900, includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, and electromagnetism.

Important contributions to classical physics were provided by Newton, who developed classical mechanics as a systematic theory and was one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electricity and magnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments was either too crude or unavailable.

A new era in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in modern physics were the theories of relativity and quantum mechanics. Einstein's theory of relativity revolutionized the traditional concepts of space, time, and energy; quantum mechanics, which applies to both the microscopic and macroscopic worlds, was originally formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists constantly work at improving our understanding of phenomena and fundamental laws, and new discoveries are made every day. In many research areas, a great deal of overlap exists between physics, chemistry, geology, and biology, as well as engineering. Some of the most notable developments are (1) numerous space missions and the landing of astronauts on the Moon, (2) microcircuitry and high-speed computers, and (3) sophisticated imaging techniques used in scientific research and medicine. The impact such developments and discoveries have had on our society has indeed been great, and it is very likely that future discoveries and developments will be just as exciting and challenging and of great benefit to humanity.

1.1 STANDARDS OF LENGTH, MASS, AND TIME

The laws of physics are expressed in terms of basic quantities that require a clear definition. In mechanics, the three basic quantities are length (L), mass (M), and time (T). All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 "glitches" if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.[1] Whatever is chosen as a standard must be readily accessible and possess some property that can be measured reliably—measurements taken by different people in different places must yield the same result.

In 1960, an international committee established a set of standards for length, mass, and other basic quantities. The system established is an adaptation of the metric system, and it is called the **SI system** of units. (The abbreviation SI comes from the system's French name "Système International.") In this system, the units of length, mass, and time are the meter, kilogram, and second, respectively. Other SI standards established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*). In our study of mechanics we shall be concerned only with the units of length, mass, and time.

Length

In A.D. 1120 the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the *meter,* defined as one ten-millionth the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris.

Many other systems for measuring length have been developed over the years, but the advantages of the French system have caused it to prevail in almost all countries and in scientific circles everywhere. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines on the bar can be determined does not meet the current requirements of science and technology. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. However, in October 1983, the **meter (m) was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second.** In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 m per second.

Table 1.1 lists approximate values of some measured lengths.

[1] The need for assigning numerical values to various measured physical quantities was expressed by Lord Kelvin (William Thomson) as follows: "I often say that when you can measure what you are speaking about, and express it in numbers, you should know something about it, but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. It may be the beginning of knowledge but you have scarcely in your thoughts advanced to the state of science."

TABLE 1.1 Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to most remote known quasar	1.4×10^{26}
Distance from the Earth to most remote known normal galaxies	9×10^{25}
Distance from the Earth to nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

Mass

The basic SI unit of mass, **the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy (Fig. 1.1a). A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland.

Table 1.2 lists approximate values of the masses of various objects.

web
Visit the Bureau at **www.bipm.fr** or the National Institute of Standards at **www.NIST.gov**

TABLE 1.2 Masses of Various Bodies (Approximate Values)

Body	Mass (kg)
Visible Universe	$\sim 10^{52}$
Milky Way galaxy	7×10^{41}
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Horse	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

Time

Before 1960, the standard of time was defined in terms of the *mean solar day* for the year 1900.[2] The *mean solar second* was originally defined as $(\frac{1}{60})(\frac{1}{60})(\frac{1}{24})$ of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a standard.

In 1967, consequently, the second was redefined to take advantage of the high precision obtainable in a device known as an *atomic clock* (Fig. 1.1b). In this device, the frequencies associated with certain atomic transitions can be measured to a precision of one part in 10^{12}. This is equivalent to an uncertainty of less than one second every 30 000 years. Thus, in 1967 the SI unit of time, the *second,* was redefined using the characteristic frequency of a particular kind of cesium atom as the "reference clock." The basic SI unit of time, **the second (s), is defined as 9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.**[3] To keep these atomic clocks—and therefore all common clocks and

[2] One solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.

[3] *Period* is defined as the time interval needed for one complete vibration.

Figure 1.1 *(Top)* The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology (NIST). *(Bottom)* The primary frequency standard (an atomic clock) at the NIST. This device keeps time with an accuracy of about 3 millionths of a second per year. *(Courtesy of National Institute of Standards and Technology, U.S. Department of Commerce)*

watches that are set to them—synchronized, it has sometimes been necessary to add leap seconds to our clocks. This is not a new idea. In 46 B.C. Julius Caesar began the practice of adding extra days to the calendar during leap years so that the seasons occurred at about the same date each year.

Since Einstein's discovery of the linkage between space and time, precise measurement of time intervals requires that we know both the state of motion of the clock used to measure the interval and, in some cases, the location of the clock as well. Otherwise, for example, global positioning system satellites might be unable to pinpoint your location with sufficient accuracy, should you need rescuing.

Approximate values of time intervals are presented in Table 1.3.

In addition to SI, another system of units, the *British engineering system* (sometimes called the *conventional system*), is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this text we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of British engineering units in the study of classical mechanics.

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote various powers of ten. Some of the most frequently used prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For

TABLE 1.3 Approximate Values of Some Time Intervals

	Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.16×10^{7}
One day (time for one rotation of the Earth about its axis)	8.64×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time for light to cross a proton	$\sim 10^{-24}$

TABLE 1.4 Prefixes for SI Units

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deka	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m corresponds to 1 kilometer (km). Likewise, 1 kg is 10^3 grams (g), and 1 megavolt (MV) is 10^6 volts (V).

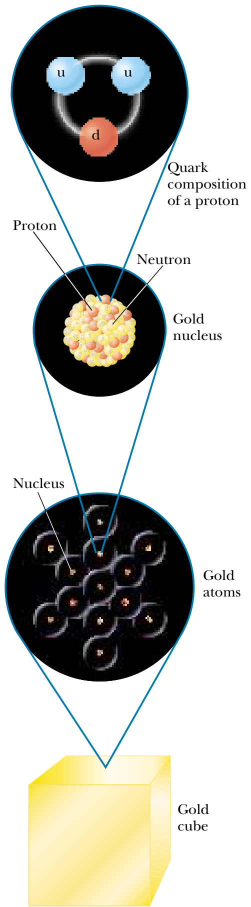


Figure 1.2 Levels of organization in matter. Ordinary matter consists of atoms, and at the center of each atom is a compact nucleus consisting of protons and neutrons. Protons and neutrons are composed of quarks. The quark composition of a proton is shown.

1.2 THE BUILDING BLOCKS OF MATTER

A 1-kg cube of solid gold has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. But what if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Questions such as these can be traced back to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They speculated that the process ultimately must end when it produces a particle that can no longer be cut. In Greek, *atomos* means "not sliceable." From this comes our English word *atom.*

Let us review briefly what is known about the structure of matter. All ordinary matter consists of atoms, and each atom is made up of electrons surrounding a central nucleus. Following the discovery of the nucleus in 1911, the question arose: Does it have structure? That is, is the nucleus a single particle or a collection of particles? The exact composition of the nucleus is not known completely even today, but by the early 1930s a model evolved that helped us understand how the nucleus behaves. Specifically, scientists determined that occupying the nucleus are two basic entities, protons and neutrons. The *proton* carries a positive charge, and a specific element is identified by the number of protons in its nucleus. This number is called the **atomic number** of the element. For instance, the nucleus of a hydrogen atom contains one proton (and so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, there is a second number characterizing atoms—**mass number,** defined as the number of protons plus neutrons in a nucleus. As we shall see, the atomic number of an element never varies (i.e., the number of protons does not vary) but the mass number can vary (i.e., the number of neutrons varies). Two or more atoms of the same element having different mass numbers are **isotopes** of one another.

The existence of neutrons was verified conclusively in 1932. A *neutron* has no charge and a mass that is about equal to that of a proton. One of its primary purposes is to act as a "glue" that holds the nucleus together. If neutrons were not present in the nucleus, the repulsive force between the positively charged particles would cause the nucleus to come apart.

But is this where the breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks,** which have been given the names of *up, down, strange, charm, bottom,* and *top.* The up, charm, and top quarks have charges of $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks have charges of $-\frac{1}{3}$ that of the proton. The proton consists of two up quarks and one down quark (Fig. 1.2), which you can easily show leads to the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.

1.3 DENSITY

A property of any substance is its **density** ρ (Greek letter rho), defined as the amount of mass contained in a unit volume, which we usually express as *mass per unit volume:*

A table of the letters in the Greek alphabet is provided on the back endsheet of this textbook.

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

For example, aluminum has a density of 2.70 g/cm^3, and lead has a density of 11.3 g/cm^3. Therefore, a piece of aluminum of volume 10.0 cm^3 has a mass of 27.0 g, whereas an equivalent volume of lead has a mass of 113 g. A list of densities for various substances is given Table 1.5.

The difference in density between aluminum and lead is due, in part, to their different *atomic masses.* The **atomic mass** of an element is the average mass of one atom in a sample of the element that contains all the element's isotopes, where the relative amounts of isotopes are the same as the relative amounts found in nature. The unit for atomic mass is the *atomic mass unit* (u), where 1 u = 1.660 540 2 $\times$ 10^{-27} kg. The atomic mass of lead is 207 u, and that of aluminum is 27.0 u. However, the ratio of atomic masses, 207 u/27.0 u = 7.67, does not correspond to the ratio of densities, (11.3 g/cm^3)/(2.70 g/cm^3) = 4.19. The discrepancy is due to the difference in atomic separations and atomic arrangements in the crystal structure of these two substances.

The mass of a nucleus is measured relative to the mass of the nucleus of the carbon-12 isotope, often written as ^{12}C. (This isotope of carbon has six protons and six neutrons. Other carbon isotopes have six protons but different numbers of neutrons.) Practically all of the mass of an atom is contained within the nucleus. Because the atomic mass of ^{12}C is defined to be exactly 12 u, the proton and neutron each have a mass of about 1 u.

One mole (mol) of a substance is that amount of the substance that contains as many particles (atoms, molecules, or other particles) as there are atoms in 12 g of the carbon-12 isotope. One mole of substance A contains the same number of particles as there are in 1 mol of any other substance B. For example, 1 mol of aluminum contains the same number of atoms as 1 mol of lead.

TABLE 1.5 Densities of Various Substances

Substance	Density ρ (10^3 kg/m^3)
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air	0.0012

Experiments have shown that this number, known as Avogadro's number, N_A, is

$$N_A = 6.022\ 137 \times 10^{23} \text{ particles/mol}$$

Avogadro's number is defined so that 1 mol of carbon-12 atoms has a mass of exactly 12 g. In general, the mass in 1 mol of any element is the element's atomic mass expressed in grams. For example, 1 mol of iron (atomic mass = 55.85 u) has a mass of 55.85 g (we say its *molar mass* is 55.85 g/mol), and 1 mol of lead (atomic mass = 207 u) has a mass of 207 g (its molar mass is 207 g/mol). Because there are 6.02×10^{23} particles in 1 mol of *any* element, the mass per atom for a given element is

$$m_{\text{atom}} = \frac{\text{molar mass}}{N_A} \tag{1.2}$$

For example, the mass of an iron atom is

$$m_{\text{Fe}} = \frac{55.85 \text{ g/mol}}{6.02 \times 10^{23} \text{ atoms/mol}} = 9.28 \times 10^{-23} \text{ g/atom}$$

EXAMPLE 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.7 g/cm^3) has a volume of 0.20 cm^3. How many aluminum atoms are contained in the cube?

Solution Since density equals mass per unit volume, the mass m of the cube is

$$m = \rho V = (2.7 \text{ g/cm}^3)(0.20 \text{ cm}^3) = 0.54 \text{ g}$$

To find the number of atoms N in this mass of aluminum, we can set up a proportion using the fact that one mole of aluminum (27 g) contains 6.02×10^{23} atoms:

$$\frac{N_A}{27 \text{ g}} = \frac{N}{0.54 \text{ g}}$$

$$\frac{6.02 \times 10^{23} \text{ atoms}}{27 \text{ g}} = \frac{N}{0.54 \text{ g}}$$

$$N = \frac{(0.54 \text{ g})(6.02 \times 10^{23} \text{ atoms})}{27 \text{ g}} = 1.2 \times 10^{22} \text{ atoms}$$

1.4 DIMENSIONAL ANALYSIS

The word *dimension* has a special meaning in physics. It usually denotes the physical nature of a quantity. Whether a distance is measured in the length unit feet or the length unit meters, it is still a distance. We say the dimension—the physical nature—of distance is *length*.

The symbols we use in this book to specify length, mass, and time are L, M, and T, respectively. We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation the dimensions of speed are written $[v] = \text{L/T}$. As another example, the dimensions of area, for which we use the symbol A, are $[A] = \text{L}^2$. The dimensions of area, volume, speed, and acceleration are listed in Table 1.6.

In solving problems in physics, there is a useful and powerful procedure called *dimensional analysis*. This procedure, which should always be used, will help minimize the need for rote memorization of equations. Dimensional analysis makes use of the fact that **dimensions can be treated as algebraic quantities.** That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

TABLE 1.6 Dimensions and Common Units of Area, Volume, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
British engineering	ft^2	ft^3	ft/s	ft/s^2

By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions are the same on both sides of the equation.

To illustrate this procedure, suppose you wish to derive a formula for the distance x traveled by a car in a time t if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2, and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$\text{L} = \frac{\text{L}}{\text{T}^2} \cdot \text{T}^2 = \text{L}$$

The units of time squared cancel as shown, leaving the unit of length.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = \text{L} = \text{LT}^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$\left(\frac{\text{L}}{\text{T}^2}\right)^n \text{T}^m = \text{L}^1$$

$$\text{L}^n \text{T}^{m-2n} = \text{L}^1$$

Because the exponents of L and T must be the same on both sides, the dimensional equation is balanced under the conditions $m - 2n = 0$, $n = 1$, and $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$. This result differs by a factor of 2 from the correct expression, which is $x = \frac{1}{2}at^2$. Because the factor $\frac{1}{2}$ is dimensionless, there is no way of determining it using dimensional analysis.

Quick Quiz 1.1

True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

EXAMPLE 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t a time interval.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{\mathrm{L}}{\mathrm{T}}$$

The same table gives us $\mathrm{L/T^2}$ for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \left(\frac{\mathrm{L}}{\mathrm{T}^{\cancel{2}}}\right)(\cancel{\mathrm{T}}) = \frac{\mathrm{L}}{\mathrm{T}}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$, it would be dimensionally *incorrect.* Try it and see!)

EXAMPLE 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. How can we determine the values of n and m?

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\mathrm{L/T^2} = \mathrm{L}^n(\mathrm{L/T})^m = \mathrm{L}^{n+m}/\mathrm{T}^m$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \quad \text{and} \quad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in $\mathrm{m/s^2}$.

QuickLab

Estimate the weight (in pounds) of two large bottles of soda pop. Note that 1 L of water has a mass of about 1 kg. Use the fact that an object weighing 2.2 lb has a mass of 1 kg. Find some bathroom scales and check your estimate.

1.5 CONVERSION OF UNITS

Sometimes it is necessary to convert units from one system to another. Conversion factors between the SI units and conventional units of length are as follows:

$$1 \text{ mi} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} \equiv 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0\ \cancel{\text{in.}})(2.54 \text{ cm}/\cancel{\text{in.}}) = 38.1 \text{ cm}$$

This works because multiplying by $\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)$ is the same as multiplying by 1, because the numerator and denominator describe identical things.

(Left) This road sign near Raleigh, North Carolina, shows distances in miles and kilometers. How accurate are the conversions? *(Billy E. Barnes/Stock Boston).*

(Right) This vehicle's speedometer gives speed readings in miles per hour and in kilometers per hour. Try confirming the conversion between the two sets of units for a few readings of the dial. *(Paul Silverman/Fundamental Photographs)*

EXAMPLE 1.4 The Density of a Cube

The mass of a solid cube is 856 g, and each edge has a length of 5.35 cm. Determine the density ρ of the cube in basic SI units.

Solution Because $1 \text{ g} = 10^{-3}$ kg and $1 \text{ cm} = 10^{-2}$ m, the mass m and volume V in basic SI units are

$$m = 856\ \cancel{\text{g}} \times 10^{-3} \text{ kg}/\cancel{\text{g}} = 0.856 \text{ kg}$$

$$V = L^3 = (5.35\ \cancel{\text{cm}} \times 10^{-2} \text{ m}/\cancel{\text{cm}})^3$$
$$= (5.35)^3 \times 10^{-6} \text{ m}^3 = 1.53 \times 10^{-4} \text{ m}^3$$

Therefore,

$$\rho = \frac{m}{V} = \frac{0.856 \text{ kg}}{1.53 \times 10^{-4} \text{ m}^3} = 5.59 \times 10^3 \text{ kg/m}^3$$

1.6 ESTIMATES AND ORDER-OF-MAGNITUDE CALCULATIONS

It is often useful to compute an approximate answer to a physical problem even where little information is available. Such an approximate answer can then be used to determine whether a more accurate calculation is necessary. Approximations are usually based on certain assumptions, which must be modified if greater accuracy is needed. Thus, we shall sometimes refer to the order of magnitude of a certain quantity as the power of ten of the number that describes that quantity. If, for example, we say that a quantity increases in value by three orders of magnitude, this means that its value is increased by a factor of $10^3 = 1000$. Also, if a quantity is given as 3×10^3, we say that the order of magnitude of that quantity is 10^3 (or in symbolic form, $3 \times 10^3 \sim 10^3$). Likewise, the quantity $8 \times 10^7 \sim 10^8$.

The spirit of order-of-magnitude calculations, sometimes referred to as "guesstimates" or "ball-park figures," is given in the following quotation: "Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle. Courage: no one else needs to know what the guess is."[4] Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates get better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head.

EXAMPLE 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We shall start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is approximately

$$1\ \cancel{\text{yr}} \times 400\ \frac{\cancel{\text{days}}}{\cancel{\text{yr}}} \times 25\ \frac{\cancel{\text{h}}}{\cancel{\text{day}}} \times 60\ \frac{\text{min}}{\cancel{\text{h}}} = 6 \times 10^5\ \text{min}$$

Notice how much simpler it is to multiply 400×25 than it is to work with the more accurate 365×24. These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70\ \text{yr})(6 \times 10^5\ \text{min/yr}) = 4 \times 10^7$ min. At a rate of 10 breaths/min, an individual would take

4×10^8 breaths in a lifetime.

EXAMPLE 1.6 It's a Long Way to San Jose

Estimate the number of steps a person would take walking from New York to Los Angeles.

Solution Without looking up the distance between these two cities, you might remember from a geography class that they are about 3 000 mi apart. The next approximation we must make is the length of one step. Of course, this length depends on the person doing the walking, but we can estimate that each step covers about 2 ft. With our estimated step size, we can determine the number of steps in 1 mi. Because this is a rough calculation, we round 5 280 ft/mi to 5 000 ft/mi. (What percentage error does this introduce?) This conversion factor gives us

$$\frac{5\ 000\ \cancel{\text{ft}}/\text{mi}}{2\ \cancel{\text{ft}}/\text{step}} = 2\ 500\ \text{steps/mi}$$

Now we switch to scientific notation so that we can do the calculation mentally:

$$(3 \times 10^3\ \cancel{\text{mi}})(2.5 \times 10^3\ \text{steps}/\cancel{\text{mi}}) = 7.5 \times 10^6\ \text{steps}$$

$$\sim 10^7\ \text{steps}$$

So if we intend to walk across the United States, it will take us on the order of ten million steps. This estimate is almost certainly too small because we have not accounted for curving roads and going up and down hills and mountains. Nonetheless, it is probably within an order of magnitude of the correct answer.

EXAMPLE 1.7 How Much Gas Do We Use?

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solution There are about 270 million people in the United States, and so we estimate that the number of cars in the country is 100 million (guessing that there are between two and three people per car). We also estimate that the average distance each car travels per year is 10 000 mi. If we assume a gasoline consumption of 20 mi/gal or 0.05 gal/mi, then each car uses about 500 gal/yr. Multiplying this by the total number of cars in the United States gives an estimated total consumption of 5×10^{10} gal $\sim$ 10^{11} gal.

[4] E. Taylor and J. A. Wheeler, *Spacetime Physics,* San Francisco, W. H. Freeman & Company, Publishers, 1966, p. 60.

1.7 SIGNIFICANT FIGURES

When physical quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of this uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed.

Suppose that we are asked to measure the area of a computer disk label using a meter stick as a measuring instrument. Let us assume that the accuracy to which we can measure with this stick is ± 0.1 cm. If the length of the label is measured to be 5.5 cm, we can claim only that its length lies somewhere between 5.4 cm and 5.6 cm. In this case, we say that the measured value has two significant figures. Likewise, if the label's width is measured to be 6.4 cm, the actual value lies between 6.3 cm and 6.5 cm. Note that the significant figures include the first estimated digit. Thus we could write the measured values as (5.5 ± 0.1) cm and (6.4 ± 0.1) cm.

Now suppose we want to find the area of the label by multiplying the two measured values. If we were to claim the area is $(5.5\ \text{cm})(6.4\ \text{cm}) = 35.2\ \text{cm}^2$, our answer would be unjustifiable because it contains three significant figures, which is greater than the number of significant figures in either of the measured lengths. A good rule of thumb to use in determining the number of significant figures that can be claimed is as follows:

When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the *least* accurate of the quantities being multiplied, where "least accurate" means "having the lowest number of significant figures." The same rule applies to division.

Applying this rule to the multiplication example above, we see that the answer for the area can have only two significant figures because our measured lengths have only two significant figures. Thus, all we can claim is that the area is $35\ \text{cm}^2$, realizing that the value can range between $(5.4\ \text{cm})(6.3\ \text{cm}) = 34\ \text{cm}^2$ and $(5.6\ \text{cm})(6.5\ \text{cm}) = 36\ \text{cm}^2$.

QuickLab

Determine the thickness of a page from this book. (Note that numbers that have no measurement errors—like the count of a number of pages—do not affect the significant figures in a calculation.) In terms of significant figures, why is it better to measure the thickness of as many pages as possible and then divide by the number of sheets?

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant. Thus, there are one and two significant figures, respectively, in these two values. When the zeros come after other digits, however, there is the possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. To remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. The same rule holds when the number is less than 1, so that 2.3×10^{-4} has two significant figures (and so could be written 0.000 23) and 2.30×10^{-4} has three significant figures (also written 0.000 230). In general, **a significant figure is a reliably known digit** (other than a zero used to locate the decimal point).

For addition and subtraction, you must consider the number of decimal places when you are determining how many significant figures to report.

When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

For example, if we wish to compute 123 + 5.35, the answer given to the correct number of significant figures is 128 and not 128.35. If we compute the sum 1.000 1 + 0.000 3 = 1.000 4, the result has five significant figures, even though one of the terms in the sum, 0.000 3, has only one significant figure. Likewise, if we perform the subtraction 1.002 − 0.998 = 0.004, the result has only one significant figure even though one term has four significant figures and the other has three. In this book, **most of the numerical examples and end-of-chapter problems will yield answers having three significant figures.** When carrying out estimates we shall typically work with a single significant figure.

Quick Quiz 1.2

Suppose you measure the position of a chair with a meter stick and record that the center of the seat is 1.043 860 564 2 m from a wall. What would a reader conclude from this recorded measurement?

EXAMPLE 1.8 The Area of a Rectangle

A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.80 ± 0.1) cm. Find the area of the plate and the uncertainty in the calculated area.

Solution

$$\text{Area} = \ell w = (21.3 \pm 0.2 \text{ cm}) \times (9.80 \pm 0.1 \text{ cm})$$

$$\approx (21.3 \times 9.80 \pm 21.3 \times 0.1 \pm 0.2 \times 9.80) \text{ cm}^2$$

$$\approx (209 \pm 4) \text{ cm}^2$$

Because the input data were given to only three significant figures, we cannot claim any more in our result. Do you see why we did not need to multiply the uncertainties 0.2 cm and 0.1 cm?

EXAMPLE 1.9 Installing a Carpet

A carpet is to be installed in a room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

Solution If you multiply 12.71 m by 3.46 m on your calculator, you will get an answer of 43.976 6 m^2. How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in the least accurate of the quantities being measured. In this example, we have only three significant figures in our least accurate measurement, so we should express our final answer as 44.0 m^2.

Note that in reducing 43.976 6 to three significant figures for our answer, we used a general rule for rounding off numbers that states that the last digit retained (the 9 in this example) is increased by 1 if the first digit dropped (here, the 7) is 5 or greater. (A technique for avoiding error accumulation is to delay rounding of numbers in a long calculation until you have the final result. Wait until you are ready to copy the answer from your calculator before rounding to the correct number of significant figures.)

SUMMARY

The three fundamental physical quantities of mechanics are length, mass, and time, which in the SI system have the units meters (m), kilograms (kg), and seconds (s), respectively. Prefixes indicating various powers of ten are used with these three basic units. The **density** of a substance is defined as its *mass per unit volume.* Different substances have different densities mainly because of differences in their atomic masses and atomic arrangements.

The number of particles in one mole of any element or compound, called **Avogadro's number,** N_A, is 6.02×10^{23}.

The method of *dimensional analysis* is very powerful in solving physics problems. Dimensions can be treated as algebraic quantities. By making estimates and making order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to completely specify an exact solution.

When you compute a result from several measured numbers, each of which has a certain accuracy, you should give the result with the correct number of significant figures.

QUESTIONS

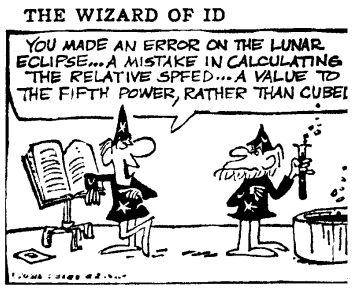
1. In this chapter we described how the Earth's daily rotation on its axis was once used to define the standard unit of time. What other types of natural phenomena could serve as alternative time standards?
2. Suppose that the three fundamental standards of the metric system were length, density, and time rather than length, mass, and time. The standard of density in this system is to be defined as that of water. What considerations about water would you need to address to make sure that the standard of density is as accurate as possible?
3. A hand is defined as 4 in.; a foot is defined as 12 in. Why should the hand be any less acceptable as a unit than the foot, which we use all the time?
4. Express the following quantities using the prefixes given in Table 1.4: (a) 3×10^{-4} m (b) 5×10^{-5} s (c) 72×10^2 g.
5. Suppose that two quantities A and B have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful: (a) $A + B$ (b) A/B (c) $B - A$ (d) AB.
6. What level of accuracy is implied in an order-of-magnitude calculation?
7. Do an order-of-magnitude calculation for an everyday situation you might encounter. For example, how far do you walk or drive each day?
8. Estimate your age in seconds.
9. Estimate the mass of this textbook in kilograms. If a scale is available, check your estimate.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 1.3 Density

1. The standard kilogram is a platinum–iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?
2. The mass of the planet Saturn (Fig. P1.2) is 5.64×10^{26} kg, and its radius is 6.00×10^7 m. Calculate its density.
3. How many grams of copper are required to make a hollow spherical shell having an inner radius of 5.70 cm and an outer radius of 5.75 cm? The density of copper is 8.92 g/cm^3.
4. What mass of a material with density ρ is required to make a hollow spherical shell having inner radius r_1 and outer radius r_2?
5. Iron has molar mass 55.8 g/mol. (a) Find the volume of 1 mol of iron. (b) Use the value found in (a) to determine the volume of one iron atom. (c) Calculate the cube root of the atomic volume, to have an estimate for the distance between atoms in the solid. (d) Repeat the calculations for uranium, finding its molar mass in the periodic table of the elements in Appendix C.



By permission of John Hart and Field Enterprises, Inc.

Figure P1.2 A view of Saturn from *Voyager 2*. *(Courtesy of NASA)*

6. Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the other is five times greater. Find its radius.

WEB **7.** Calculate the mass of an atom of (a) helium, (b) iron, and (c) lead. Give your answers in atomic mass units and in grams. The molar masses are 4.00, 55.9, and 207 g/mol, respectively, for the atoms given.

8. On your wedding day your lover gives you a gold ring of mass 3.80 g. Fifty years later its mass is 3.35 g. As an average, how many atoms were abraded from the ring during each second of your marriage? The molar mass of gold is 197 g/mol.

9. A small cube of iron is observed under a microscope. The edge of the cube is 5.00×10^{-6} cm long. Find (a) the mass of the cube and (b) the number of iron atoms in the cube. The molar mass of iron is 55.9 g/mol, and its density is 7.86 g/cm^3.

10. A structural I-beam is made of steel. A view of its cross-section and its dimensions are shown in Figure P1.10.

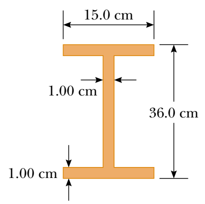


Figure P1.10

(a) What is the mass of a section 1.50 m long? (b) How many atoms are there in this section? The density of steel is 7.56×10^3 kg/m^3.

11. A child at the beach digs a hole in the sand and, using a pail, fills it with water having a mass of 1.20 kg. The molar mass of water is 18.0 g/mol. (a) Find the number of water molecules in this pail of water. (b) Suppose the quantity of water on the Earth is 1.32×10^{21} kg and remains constant. How many of the water molecules in this pail of water were likely to have been in an equal quantity of water that once filled a particular claw print left by a dinosaur?

Section 1.4 Dimensional Analysis

12. The radius r of a circle inscribed in any triangle whose sides are a, b, and c is given by

$$r = [(s - a)(s - b)(s - c)/s]^{1/2}$$

where s is an abbreviation for $(a + b + c)/2$. Check this formula for dimensional consistency.

13. The displacement of a particle moving under uniform acceleration is some function of the elapsed time and the acceleration. Suppose we write this displacement $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

14. The period T of a simple pendulum is measured in time units and is described by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and g is the free-fall acceleration in units of length divided by the square of time. Show that this equation is dimensionally correct.

15. Which of the equations below are dimensionally correct?
(a) $v = v_0 + ax$
(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$

16. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the gravitational force, M and m are masses, and r is a length. Force has the SI units kg·m/s^2. What are the SI units of the proportionality constant G?

WEB **17.** The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where V is the volume in millions of cubic feet and t the time in months. Express this equation in units of cubic feet and seconds. Put the proper units on the coefficients. Assume a month is 30.0 days.

Section 1.5 Conversion of Units

18. Suppose your hair grows at the rate 1/32 in. per day. Find the rate at which it grows in nanometers per second. Since the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly layers of atoms are assembled in this protein synthesis.

19. A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in m^2.

20. An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m^3. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

21. Assume that it takes 7.00 min to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time, in hours, required to fill a 1-cubic-meter volume at the same rate. (1 U.S. gal = 231 in.^{3})

22. A creature moves at a speed of 5.00 furlongs per fortnight (not a very common unit of speed). Given that 1 furlong = 220 yards and 1 fortnight = 14 days, determine the speed of the creature in meters per second. What kind of creature do you think it might be?

23. A section of land has an area of 1 mi^2 and contains 640 acres. Determine the number of square meters in 1 acre.

24. A quart container of ice cream is to be made in the form of a cube. What should be the length of each edge in centimeters? (Use the conversion 1 gal = 3.786 L.)

25. A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kg/m^3).

26. An astronomical unit (AU) is defined as the average distance between the Earth and the Sun. (a) How many astronomical units are there in one lightyear? (b) Determine the distance from the Earth to the Andromeda galaxy in astronomical units.

27. The mass of the Sun is 1.99×10^{30} kg, and the mass of an atom of hydrogen, of which the Sun is mostly composed, is 1.67×10^{-27} kg. How many atoms are there in the Sun?

28. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) In the past, a federal law mandated that highway speed limits would be 55 mi/h. Use the conversion factor of part (a) to find this speed in kilometers per hour. (c) The maximum highway speed is now 65 mi/h in some places. In kilometers per hour, how much of an increase is this over the 55-mi/h limit?

29. At the time of this book's printing, the U. S. national debt is about \$6 trillion. (a) If payments were made at the rate of \$1 000/s, how many years would it take to pay off a \$6-trillion debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If six trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the Earth? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

30. (a) How many seconds are there in a year? (b) If one micrometeorite (a sphere with a diameter of 1.00×10^{-6} m) strikes each square meter of the Moon each second, how many years will it take to cover the Moon to a depth of 1.00 m? (*Hint:* Consider a cubic box on the Moon 1.00 m on a side, and find how long it will take to fill the box.)

WEB **31.** One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the paint on the wall?

32. A pyramid has a height of 481 ft, and its base covers an area of 13.0 acres (Fig. P1.32). If the volume of a pyramid is given by the expression $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height, find the volume of this pyramid in cubic meters. (1 acre = 43 560 ft^2)

Figure P1.32 Problems 32 and 33.

33. The pyramid described in Problem 32 contains approximately two million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

34. Assuming that 70% of the Earth's surface is covered with water at an average depth of 2.3 mi, estimate the mass of the water on the Earth in kilograms.

35. The amount of water in reservoirs is often measured in acre-feet. One acre-foot is a volume that covers an area of 1 acre to a depth of 1 ft. An acre is an area of 43 560 ft^2. Find the volume in SI units of a reservoir containing 25.0 acre-ft of water.

36. A hydrogen atom has a diameter of approximately 1.06×10^{-10} m, as defined by the diameter of the spherical electron cloud around the nucleus. The hydrogen nucleus has a diameter of approximately 2.40×10^{-15} m. (a) For a scale model, represent the diameter of the hydrogen atom by the length of an American football field (100 yards = 300 ft), and determine the diameter of the nucleus in millimeters. (b) The atom is how many times larger in volume than its nucleus?

37. The diameter of our disk-shaped galaxy, the Milky Way, is about 1.0×10^5 lightyears. The distance to Messier 31—which is Andromeda, the spiral galaxy nearest to the Milky Way—is about 2.0 million lightyears. If a scale model represents the Milky Way and Andromeda galaxies as dinner plates 25 cm in diameter, determine the distance between the two plates.

38. The mean radius of the Earth is 6.37×10^6 m, and that of the Moon is 1.74×10^8 cm. From these data calculate (a) the ratio of the Earth's surface area to that of the Moon and (b) the ratio of the Earth's volume to that of the Moon. Recall that the surface area of a sphere is $4\pi r^2$ and that the volume of a sphere is $\frac{4}{3}\pi r^3$.

WEB **39.** One cubic meter (1.00 m^3) of aluminum has a mass of 2.70×10^3 kg, and 1.00 m^3 of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius 2.00 cm on an equal-arm balance.

40. Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.

Section 1.6 Estimates and Order-of-Magnitude Calculations

WEB **41.** Estimate the number of Ping-Pong balls that would fit into an average-size room (without being crushed). In your solution state the quantities you measure or estimate and the values you take for them.

42. McDonald's sells about 250 million packages of French fries per year. If these fries were placed end to end, estimate how far they would reach.

43. An automobile tire is rated to last for 50 000 miles. Estimate the number of revolutions the tire will make in its lifetime.

44. Approximately how many raindrops fall on a 1.0-acre lot during a 1.0-in. rainfall?

45. Grass grows densely everywhere on a quarter-acre plot of land. What is the order of magnitude of the number of blades of grass on this plot of land? Explain your reasoning. (1 acre = 43 560 ft^2.)

46. Suppose that someone offers to give you $1 billion if you can finish counting it out using only one-dollar bills. Should you accept this offer? Assume you can count one bill every second, and be sure to note that you need about 8 hours a day for sleeping and eating and that right now you are probably at least 18 years old.

47. Compute the order of magnitude of the mass of a bathtub half full of water and of the mass of a bathtub half full of pennies. In your solution, list the quantities you take as data and the value you measure or estimate for each.

48. Soft drinks are commonly sold in aluminum containers. Estimate the number of such containers thrown away or recycled each year by U.S. consumers. Approximately how many tons of aluminum does this represent?

49. To an order of magnitude, how many piano tuners are there in New York City? The physicist Enrico Fermi was famous for asking questions like this on oral Ph.D. qualifying examinations and for his own facility in making order-of-magnitude calculations.

Section 1.7 Significant Figures

50. Determine the number of significant figures in the following measured values: (a) 23 cm (b) 3.589 s (c) 4.67×10^3 m/s (d) 0.003 2 m.

51. The radius of a circle is measured to be 10.5 ± 0.2 m. Calculate the (a) area and (b) circumference of the circle and give the uncertainty in each value.

52. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product $0.003\,2 \times 356.3$; (c) the product $5.620 \times \pi$.

53. The radius of a solid sphere is measured to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

54. How many significant figures are in the following numbers: (a) 78.9 ± 0.2, (b) 3.788×10^9, (c) 2.46×10^{-6}, and (d) 0.005 3?

55. A farmer measures the distance around a rectangular field. The length of the long sides of the rectangle is found to be 38.44 m, and the length of the short sides is found to be 19.5 m. What is the total distance around the field?

56. A sidewalk is to be constructed around a swimming pool that measures (10.0 ± 0.1) m by (17.0 ± 0.1) m. If the sidewalk is to measure (1.00 ± 0.01) m wide by (9.0 ± 0.1) cm thick, what volume of concrete is needed, and what is the approximate uncertainty of this volume?

ADDITIONAL PROBLEMS

57. In a situation where data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," since we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which we consider factors rather than increments. We write 500 m $\sim 10^3$ m because 500 differs from 100 by a factor of 5 whereas it differs from 1000 by only a factor of 2. We write 437 m $\sim 10^3$ m and 305 m $\sim 10^2$ m. What distance differs from 100 m and from 1000 m by equal factors, so that we could equally well choose to represent its order of magnitude either as $\sim 10^2$ m or as $\sim 10^3$ m?

58. When a droplet of oil spreads out on a smooth water surface, the resulting "oil slick" is approximately one molecule thick. An oil droplet of mass 9.00×10^{-7} kg and density 918 kg/m^3 spreads out into a circle of radius 41.8 cm on the water surface. What is the diameter of an oil molecule?

59. The basic function of the carburetor of an automobile is to "atomize" the gasoline and mix it with air to promote rapid combustion. As an example, assume that 30.0 cm^3 of gasoline is atomized into N spherical droplets, each with a radius of 2.00×10^{-5} m. What is the total surface area of these N spherical droplets?

60. In physics it is important to use mathematical approximations. Demonstrate for yourself that for small angles ($< 20°$)

$$\tan \alpha \approx \sin \alpha \approx \alpha = \pi\alpha'/180°$$

where α is in radians and α' is in degrees. Use a calculator to find the largest angle for which $\tan \alpha$ may be approximated by $\sin \alpha$ if the error is to be less than 10.0%.

61. A high fountain of water is located at the center of a circular pool as in Figure P1.61. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be 55.0°. How high is the fountain?

Figure P1.61

62. Assume that an object covers an area A and has a uniform height h. If its cross-sectional area is uniform over its height, then its volume is given by $V = Ah$. (a) Show that $V = Ah$ is dimensionally correct. (b) Show that the volumes of a cylinder and of a rectangular box can be written in the form $V = Ah$, identifying A in each case. (Note that A, sometimes called the "footprint" of the object, can have any shape and that the height can be replaced by average thickness in general.)

63. A useful fact is that there are about $\pi \times 10^7$ s in one year. Find the percentage error in this approximation, where "percentage error" is defined as

$$\frac{|\text{Assumed value} - \text{true value}|}{\text{True value}} \times 100\%$$

64. A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.64a. The atoms reside at the corners of cubes of side $L = 0.200$ nm. One piece of evidence for the regular arrangement of atoms comes from the flat surfaces along which a crystal separates, or "cleaves," when it is broken. Suppose this crystal cleaves along a face diagonal, as shown in Figure P1.64b. Calculate the spacing d between two adjacent atomic planes that separate when the crystal cleaves.

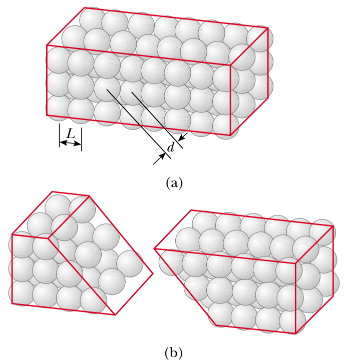


Figure P1.64

65. A child loves to watch as you fill a transparent plastic bottle with shampoo. Every horizontal cross-section of the bottle is a circle, but the diameters of the circles all have different values, so that the bottle is much wider in some places than in others. You pour in bright green shampoo with constant volume flow rate 16.5 cm^3/s. At what rate is its level in the bottle rising (a) at a point where the diameter of the bottle is 6.30 cm and (b) at a point where the diameter is 1.35 cm?

66. As a child, the educator and national leader Booker T. Washington was given a spoonful (about 12.0 cm^3) of molasses as a treat. He pretended that the quantity increased when he spread it out to cover uniformly all of a tin plate (with a diameter of about 23.0 cm). How thick a layer did it make?

67. Assume there are 100 million passenger cars in the United States and that the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?

68. One cubic centimeter of water has a mass of 1.00×10^{-3} kg. (a) Determine the mass of 1.00 m^3 of water. (b) Assuming biological substances are 98% water, estimate the mass of a cell that has a diameter of 1.0 μm, a human kidney, and a fly. Assume that a kidney is roughly a sphere with a radius of 4.0 cm and that a fly is roughly a cylinder 4.0 mm long and 2.0 mm in diameter.

69. The distance from the Sun to the nearest star is 4×10^{16} m. The Milky Way galaxy is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the 4×10^{16}-m distance between the Sun and the nearest star is typical.

70. The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. Use these data to calculate the densities of these substances. Compare your results for aluminum, copper, and iron with those given in Table 1.5.

Substance	Mass (g)	Diameter (cm)	Length (cm)
Aluminum	51.5	2.52	3.75
Copper	56.3	1.23	5.06
Brass	94.4	1.54	5.69
Tin	69.1	1.75	3.74
Iron	216.1	1.89	9.77

Answers to Quick Quizzes

1.1 False. Dimensional analysis gives the units of the proportionality constant but provides no information about its numerical value. For example, experiments show that doubling the radius of a solid sphere increases its mass 8-fold, and tripling the radius increases the mass 27-fold. Therefore, its mass is proportional to the cube of its radius. Because $m \propto r^3$ we can write $m = kr^3$. Dimensional analysis shows that the proportionality constant k must have units kg/m^3, but to determine its numerical value requires either experimental data or geometrical reasoning.

1.2 Reporting all these digits implies you have determined the location of the center of the chair's seat to the nearest $\pm$ 0.000 000 000 1 m. This roughly corresponds to being able to count the atoms in your meter stick because each of them is about that size! It would probably be better to record the measurement as 1.044 m: this indicates that you know the position to the nearest millimeter, assuming the meter stick has millimeter markings on its scale.

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P U Z Z L E R

In a moment the arresting cable will be pulled taut, and the 140-mi/h landing of this F/A-18 Hornet on the aircraft carrier USS *Nimitz* will be brought to a sudden conclusion. The pilot cuts power to the engine, and the plane is stopped in less than 2 s. If the cable had not been successfully engaged, the pilot would have had to take off quickly before reaching the end of the flight deck. Can the motion of the plane be described quantitatively in a way that is useful to ship and aircraft designers and to pilots learning to land on a "postage stamp?" *(Courtesy of the USS* Nimitz*/U.S. Navy)*

c h a p t e r

2

Motion in One Dimension

Chapter Outline

As a first step in studying classical mechanics, we describe motion in terms of space and time while ignoring the agents that caused that motion. This portion of classical mechanics is called *kinematics.* (The word *kinematics* has the same root as *cinema.* Can you see why?) In this chapter we consider only motion in one dimension. We first define displacement, velocity, and acceleration. Then, using these concepts, we study the motion of objects traveling in one dimension with a constant acceleration.

From everyday experience we recognize that motion represents a continuous change in the position of an object. In physics we are concerned with three types of motion: translational, rotational, and vibrational. A car moving down a highway is an example of translational motion, the Earth's spin on its axis is an example of rotational motion, and the back-and-forth movement of a pendulum is an example of vibrational motion. In this and the next few chapters, we are concerned only with translational motion. (Later in the book we shall discuss rotational and vibrational motions.)

In our study of translational motion, we describe the moving object as a *particle* regardless of its size. In general, **a particle is a point-like mass having infinitesimal size.** For example, if we wish to describe the motion of the Earth around the Sun, we can treat the Earth as a particle and obtain reasonably accurate data about its orbit. This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the Sun. As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles.

2.1 DISPLACEMENT, VELOCITY, AND SPEED

TABLE 2.1 Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

The motion of a particle is completely known if the particle's position in space is known at all times. Consider a car moving back and forth along the x axis, as shown in Figure 2.1a. When we begin collecting position data, the car is 30 m to the right of a road sign. (Let us assume that all data in this example are known to two significant figures. To convey this information, we should report the initial position as 3.0×10^1 m. We have written this value in this simpler form to make the discussion easier to follow.) We start our clock and once every 10 s note the car's location relative to the sign. As you can see from Table 2.1, the car is moving to the right (which we have defined as the positive direction) during the first 10 s of motion, from position Ⓐ to position Ⓑ. The position values now begin to decrease, however, because the car is backing up from position Ⓑ through position Ⓕ. In fact, at Ⓓ, 30 s after we start measuring, the car is alongside the sign we are using as our origin of coordinates. It continues moving to the left and is more than 50 m to the left of the sign when we stop recording information after our sixth data point. A graph of this information is presented in Figure 2.1b. Such a plot is called a *position–time graph.*

If a particle is moving, we can easily determine its change in position. **The displacement of a particle is defined as its change in position.** As it moves from an initial position x_i to a final position x_f, its displacement is given by $x_f - x_i$. We use the Greek letter delta (Δ) to denote the *change* in a quantity. Therefore, we write the displacement, or change in position, of the particle as

$$\Delta x \equiv x_f - x_i \qquad (2.1)$$

From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

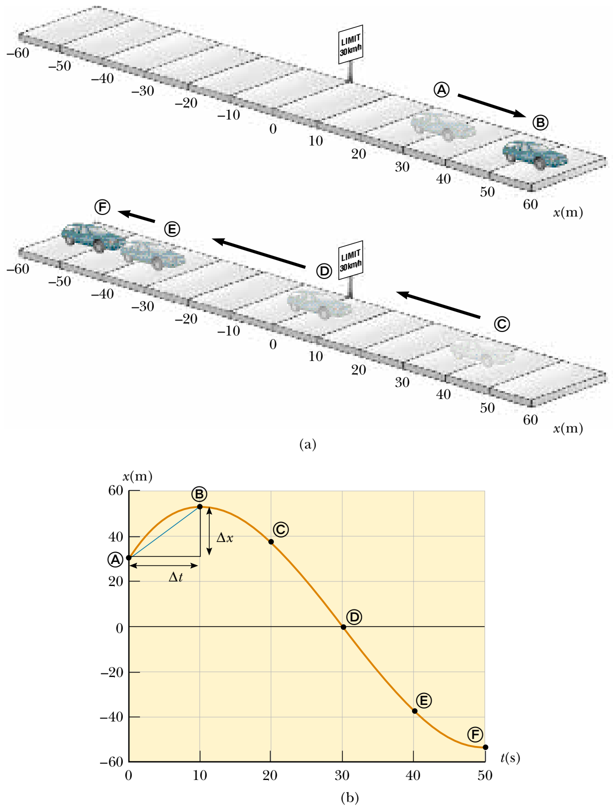


Figure 2.1 (a) A car moves back and forth along a straight line taken to be the *x* axis. Because we are interested only in the car's translational motion, we can treat it as a particle. (b) Position–time graph for the motion of the "particle."

A very easy mistake to make is not to recognize the difference between displacement and distance traveled (Fig. 2.2). A baseball player hitting a home run travels a distance of 360 ft in the trip around the bases. However, the player's displacement is zero because his final and initial positions are identical.

Displacement is an example of a vector quantity. Many other physical quantities, including velocity and acceleration, also are vectors. In general, **a vector is a physical quantity that requires the specification of both direction and magnitude.** By contrast, **a scalar is a quantity that has magnitude and no direction.** In this chapter, we use plus and minus signs to indicate vector direction. We can do this because the chapter deals with one-dimensional motion only; this means that any object we study can be moving only along a straight line. For example, for horizontal motion, let us arbitrarily specify to the right as being the positive direction. It follows that any object always moving to the right undergoes a

Figure 2.2 Bird's-eye view of a baseball diamond. A batter who hits a home run travels 360 ft as he rounds the bases, but his displacement for the round trip is zero. *(Mark C. Burnett/Photo Researchers, Inc.)*

positive displacement $+\Delta x$, and any object moving to the left undergoes a negative displacement $-\Delta x$. We shall treat vectors in greater detail in Chapter 3.

There is one very important point that has not yet been mentioned. Note that the graph in Figure 2.1b does not consist of just six data points but is actually a smooth curve. The graph contains information about the entire 50-s interval during which we watched the car move. It is much easier to see changes in position from the graph than from a verbal description or even a table of numbers. For example, it is clear that the car was covering more ground during the middle of the 50-s interval than at the end. Between positions Ⓒ and Ⓓ, the car traveled almost 40 m, but during the last 10 s, between positions Ⓔ and Ⓕ, it moved less than half that far. A common way of comparing these different motions is to divide the displacement Δx that occurs between two clock readings by the length of that particular time interval Δt. This turns out to be a very useful ratio, one that we shall use many times. For convenience, the ratio has been given a special name—*average velocity.* **The average velocity $\bar{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurred:**

Average velocity

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t} \qquad \textbf{(2.2)}$$

3.2

where the subscript *x* indicates motion along the *x* axis. From this definition we see that average velocity has dimensions of length divided by time (L/T)—meters per second in SI units.

Although the distance traveled for any motion is always positive, the average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), then Δx is positive and $\bar{v}_x = \Delta x/\Delta t$ is positive. This case corresponds to motion in the positive *x* direction. If the coordinate decreases in time (that is, if $x_f < x_i$), then Δx is negative and hence $\bar{v}_x$ is negative. This case corresponds to motion in the negative *x* direction.

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph in Figure 2.1b. This line forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$. For example, the line between positions Ⓐ and Ⓑ has a slope equal to the average velocity of the car between those two times, (52 m − 30 m)/(10 s − 0) = 2.2 m/s.

In everyday usage, the terms *speed* and *velocity* are interchangeable. In physics, however, there is a clear distinction between these two quantities. Consider a marathon runner who runs more than 40 km, yet ends up at his starting point. His average velocity is zero! Nonetheless, we need to be able to quantify how fast he was running. A slightly different ratio accomplishes this for us. **The average speed of a particle, a scalar quantity, is defined as the total distance traveled divided by the total time it takes to travel that distance:**

Average speed

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

The SI unit of average speed is the same as the unit of average velocity: meters per second. However, unlike average velocity, average speed has no direction and hence carries no algebraic sign.

Knowledge of the average speed of a particle tells us nothing about the details of the trip. For example, suppose it takes you 8.0 h to travel 280 km in your car. The average speed for your trip is 35 km/h. However, you most likely traveled at various speeds during the trip, and the average speed of 35 km/h could result from an infinite number of possible speed values.

EXAMPLE 2.1 Calculating the Variables of Motion

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

Solution The units of displacement must be meters, and the numerical result should be of the same order of magnitude as the given position data (which means probably not 10 or 100 times bigger or smaller). From the position–time graph given in Figure 2.1b, note that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s. Using these values along with the definition of displacement, Equation 2.1, we find that

$$\Delta x = x_F - x_A = -53\text{ m} - 30\text{ m} = \boxed{-83\text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that this is the correct answer.

It is difficult to estimate the average velocity without completing the calculation, but we expect the units to be meters per second. Because the car ends up to the left of where we started taking data, we know the average velocity must be negative. From Equation 2.2,

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}} = \boxed{-1.7\text{ m/s}}$$

We find the car's average speed for this trip by adding the distances traveled and dividing by the total time:

$$\text{Average speed} = \frac{22\text{ m} + 52\text{ m} + 53\text{ m}}{50\text{ s}} = \boxed{2.5\text{ m/s}}$$

2.2 INSTANTANEOUS VELOCITY AND SPEED

Often we need to know the velocity of a particle at a particular instant in time, rather than over a finite time interval. For example, even though you might want to calculate your average velocity during a long automobile trip, you would be especially interested in knowing your velocity at the *instant* you noticed the police

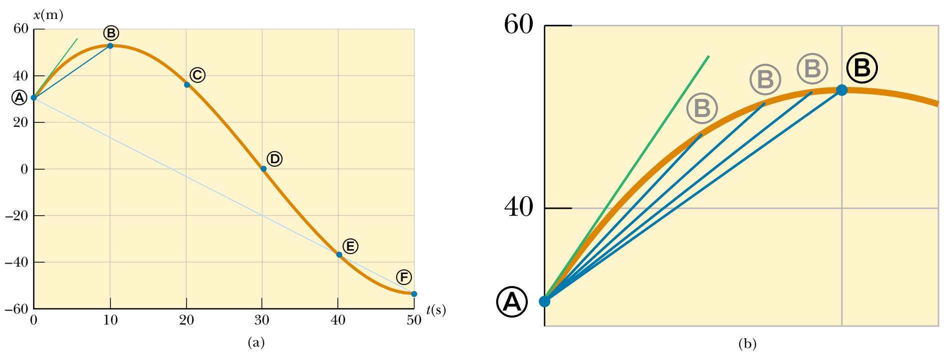


Figure 2.3 (a) Graph representing the motion of the car in Figure 2.1. (b) An enlargement of the upper left-hand corner of the graph shows how the blue line between positions Ⓐ and Ⓑ approaches the green tangent line as point Ⓑ gets closer to point Ⓐ.

car parked alongside the road in front of you. In other words, you would like to be able to specify your velocity just as precisely as you can specify your position by noting what is happening at a specific clock reading—that is, at some specific instant. It may not be immediately obvious how to do this. What does it mean to talk about how fast something is moving if we "freeze time" and talk only about an individual instant? This is a subtle point not thoroughly understood until the late 1600s. At that time, with the invention of calculus, scientists began to understand how to describe an object's motion at any moment in time.

To see how this is done, consider Figure 2.3a. We have already discussed the average velocity for the interval during which the car moved from position Ⓐ to position Ⓑ (given by the slope of the dark blue line) and for the interval during which it moved from Ⓐ to Ⓕ (represented by the slope of the light blue line). Which of these two lines do you think is a closer approximation of the initial velocity of the car? The car starts out by moving to the right, which we defined to be the positive direction. Therefore, being positive, the value of the average velocity during the Ⓐ to Ⓑ interval is probably closer to the initial value than is the value of the average velocity during the Ⓐ to Ⓕ interval, which we determined to be negative in Example 2.1. Now imagine that we start with the dark blue line and slide point Ⓑ to the left along the curve, toward point Ⓐ, as in Figure 2.3b. The line between the points becomes steeper and steeper, and as the two points get extremely close together, the line becomes a tangent line to the curve, indicated by the green line on the graph. The slope of this tangent line represents the velocity of the car at the moment we started taking data, at point Ⓐ. What we have done is determine the *instantaneous velocity* at that moment. In other words, the **instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:**[1]

Definition of instantaneous velocity

3.3

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \textbf{(2.3)}$$

[1] Note that the displacement Δx also approaches zero as Δt approaches zero. As Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the slope of the line tangent to the x-versus-t curve.

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \textbf{(2.4)}$$

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at any time during the first 10 s in Figure 2.3, v_x is positive. After point Ⓑ, v_x is negative because the slope is negative. At the peak, the slope and the instantaneous velocity are zero.

From here on, we use the word *velocity* to designate instantaneous velocity. When it is *average velocity* we are interested in, we always use the adjective *average*.

The instantaneous speed of a particle is defined as the magnitude of its velocity. As with average speed, instantaneous speed has no direction associated with it and hence carries no algebraic sign. For example, if one particle has a velocity of $+25$ m/s along a given line and another particle has a velocity of -25 m/s along the same line, both have a speed[2] of 25 m/s.

EXAMPLE 2.2 Average and Instantaneous Velocity

A particle moves along the x axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, is at rest at the moment $t = 1$ s, and moves in the positive x direction for $t > 1$ s. (a) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution During the first time interval, we have a negative slope and hence a negative velocity. Thus, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, we set $t_i = t_A = 0$ and $t_f = t_B = 1$ s. Using Equation 2.1, with $x = -4t + 2t^2$, we obtain for the first displacement

$$\Delta x_{A\to B} = x_f - x_i = x_B - x_A$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]$$
$$= \boxed{-2\text{ m}}$$

To calculate the displacement during the second time interval, we set $t_i = t_B = 1$ s and $t_f = t_D = 3$ s:

$$\Delta x_{B\to D} = x_f - x_i = x_D - x_B$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]$$
$$= \boxed{+8\text{ m}}$$

These displacements can also be read directly from the position–time graph.

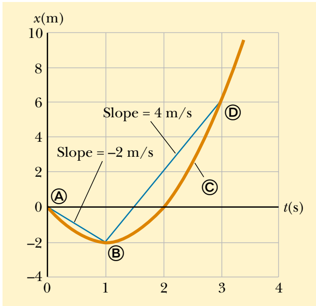


Figure 2.4 Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$.

[2] As with velocity, we drop the adjective for instantaneous speed: "Speed" means instantaneous speed.

[3] Simply to make it easier to read, we write the empirical equation as $x = -4t + 2t^2$ rather than as $x = (-4.00\text{ m/s})t + (2.00\text{ m/s}^2)t^{2.00}$. When an equation summarizes measurements, consider its coefficients to have as many significant digits as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$ s, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

(b) Calculate the average velocity during these two time intervals.

Solution In the first time interval, $\Delta t = t_f - t_i = t_B - t_A = 1$ s. Therefore, using Equation 2.2 and the displacement calculated in (a), we find that

$$\bar{v}_{x(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

In the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{v}_{x(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

These values agree with the slopes of the lines joining these points in Figure 2.4.

(c) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution Certainly we can guess that this instantaneous velocity must be of the same order of magnitude as our previous results, that is, around 4 m/s. Examining the graph, we see that the slope of the tangent at position Ⓒ is greater than the slope of the blue line connecting points Ⓑ and Ⓓ. Thus, we expect the answer to be greater than 4 m/s. By measuring the slope of the position–time graph at $t = 2.5$ s, we find that

$$v_x = \boxed{+6\text{ m/s}}$$

2.3 ACCELERATION

In the last example, we worked with a situation in which the velocity of a particle changed while the particle was moving. This is an extremely common occurrence. (How constant is your velocity as you ride a city bus?) It is easy to quantify changes in velocity as a function of time in exactly the same way we quantify changes in position as a function of time. When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the velocity of a car increases when you step on the gas and decreases when you apply the brakes. However, we need a better definition of acceleration than this.

Suppose a particle moving along the x axis has a velocity v_{xi} at time t_i and a velocity v_{xf} at time t_f, as in Figure 2.5a.

The average acceleration of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurred:

Average acceleration

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad \textbf{(2.5)}$$

As with velocity, when the motion being analyzed is one-dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, accelera-

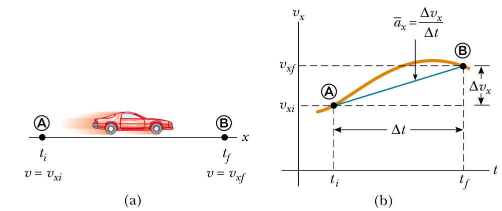


Figure 2.5 (a) A "particle" moving along the x axis from Ⓐ to Ⓑ has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph for the particle moving in a straight line. The slope of the blue straight line connecting Ⓐ and Ⓑ is the average acceleration in the time interval $\Delta t = t_f - t_i$.

tion has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/s^2). It might be easier to interpret these units if you think of them as meters per second per second. For example, suppose an object has an acceleration of 2 m/s^2. You should form a mental image of the object having a velocity that is along a straight line and is increasing by 2 m/s during every 1-s interval. If the object starts from rest, you should be able to picture it moving at a velocity of +2 m/s after 1 s, at +4 m/s after 2 s, and so on.

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the *instantaneous acceleration* as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in the previous section. If we imagine that point Ⓑ is brought closer and closer to point Ⓐ in Figure 2.5a and take the limit of $\Delta v_x/\Delta t$ as Δt approaches zero, we obtain the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad \textbf{(2.6)}$$

Instantaneous acceleration

That is, **the instantaneous acceleration equals the derivative of the velocity with respect to time,** which by definition is the slope of the velocity–time graph (Fig. 2.5b). Thus, we see that just as the velocity of a moving particle is the slope of the particle's x-t graph, the acceleration of a particle is the slope of the particle's v_x-t graph. One can interpret the derivative of the velocity with respect to time as the time rate of change of velocity. If a_x is positive, then the acceleration is in the positive x direction; if a_x is negative, then the acceleration is in the negative x direction.

From now on we shall use the term *acceleration* to mean instantaneous acceleration. When we mean average acceleration, we shall always use the adjective *average*.

Because $v_x = dx/dt$, the acceleration can also be written

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad \textbf{(2.7)}$$

That is, in one-dimensional motion, the acceleration equals the *second derivative* of x with respect to time.

Figure 2.6 illustrates how an acceleration–time graph is related to a velocity–time graph. The acceleration at any time is the slope of the velocity–time graph at that time. Positive values of acceleration correspond to those points in Figure 2.6a where the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time t_A, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time t_B, when the velocity is a maximum (that is, when the slope of the v_x-t graph is zero). The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time t_C.

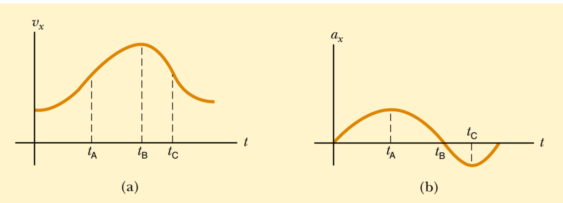


Figure 2.6 Instantaneous acceleration can be obtained from the v_x-t graph. (a) The velocity–time graph for some motion. (b) The acceleration–time graph for the same motion. The acceleration given by the a_x-t graph for any value of t equals the slope of the line tangent to the v_x-t graph at the same value of t.

CONCEPTUAL EXAMPLE 2.3 Graphical Relationships Between x, v_x, and a_x

The position of an object moving along the x axis varies with time as in Figure 2.7a. Graph the velocity versus time and the acceleration versus time for the object.

Solution The velocity at any instant is the slope of the tangent to the x-t graph at that instant. Between $t = 0$ and $t = t_A$, the slope of the x-t graph increases uniformly, and so the velocity increases linearly, as shown in Figure 2.7b. Between t_A and t_B, the slope of the x-t graph is constant, and so the velocity remains constant. At t_D, the slope of the x-t graph is zero, so the velocity is zero at that instant. Between t_D and t_E, the slope of the x-t graph and thus the velocity are negative and decrease uniformly in this interval. In the interval t_E to t_F, the slope of the x-t graph is still negative, and at t_F it goes to zero. Finally, after t_F, the slope of the x-t graph is zero, meaning that the object is at rest for $t > t_F$.

The acceleration at any instant is the slope of the tangent to the v_x-t graph at that instant. The graph of acceleration versus time for this object is shown in Figure 2.7c. The acceleration is constant and positive between 0 and t_A, where the slope of the v_x-t graph is positive. It is zero between t_A and t_B and for $t > t_F$ because the slope of the v_x-t graph is zero at these times. It is negative between t_B and t_E because the slope of the v_x-t graph is negative during this interval.

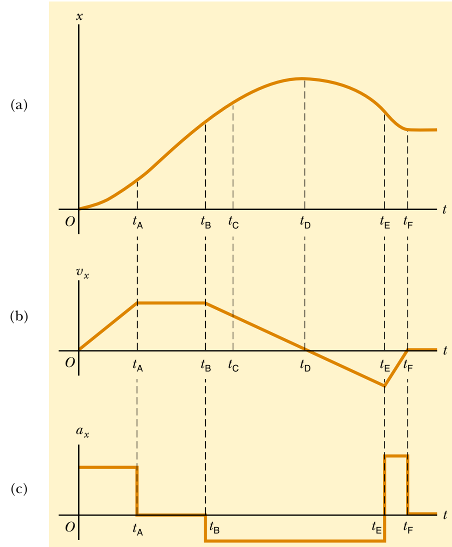


Figure 2.7 (a) Position–time graph for an object moving along the x axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

Quick Quiz 2.1

Make a velocity–time graph for the car in Figure 2.1a and use your graph to determine whether the car ever exceeds the speed limit posted on the road sign (30 km/h).

EXAMPLE 2.4 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds. (a) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

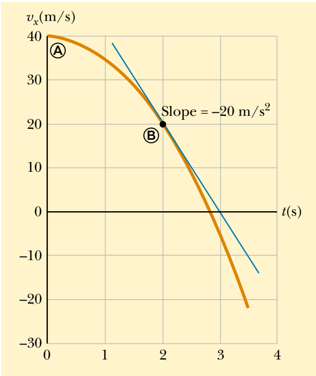


Figure 2.8 The velocity–time graph for a particle moving along the *x* axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the blue tangent line at that time.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of *t* into the expression for the velocity:

$$v_{xA} = (40 - 5t_A{}^2)\text{ m/s} = [40 - 5(0)^2]\text{ m/s} = +40\text{ m/s}$$

$$v_{xB} = (40 - 5t_B{}^2)\text{ m/s} = [40 - 5(2.0)^2]\text{ m/s} = +20\text{ m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\text{ m/s}}{(2.0 - 0)\text{ s}}$$

$$= \boxed{-10\text{ m/s}^2}$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line (not shown) joining the initial and final points on the velocity–time graph, is negative.

(b) Determine the acceleration at $t = 2.0$ s.

Solution The velocity at any time *t* is $v_{xi} = (40 - 5t^2)$ m/s, and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2]\text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time *t*:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0}(-10t - 5\Delta t) = -10t\text{ m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0)\text{ m/s}^2 = \boxed{-20\text{ m/s}^2}$$

What we have done by comparing the average acceleration during the interval between Ⓐ and Ⓑ (-10 m/s²) with the instantaneous value at Ⓑ (-20 m/s²) is compare the slope of the line (not shown) joining Ⓐ and Ⓑ with the slope of the tangent at Ⓑ.

Note that the acceleration is not constant in this example. Situations involving constant acceleration are treated in Section 2.5.

So far we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. Those of you familiar with calculus should recognize that there are specific rules for taking derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose *x* is proportional to some power of *t*, such as in the expression

$$x = At^n$$

where *A* and *n* are constants. (This is a very common functional form.) The derivative of *x* with respect to *t* is

$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 2.4, in which $v_x = 40 - 5t^2$, we find that $a_x = dv_x/dt = -10t$.

2.4 MOTION DIAGRAMS

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. It is instructive to use motion diagrams to describe the velocity and acceleration while an object is in motion. In order not to confuse these two vector quantities, for which both magnitude and direction are important, we use red for velocity vectors and violet for acceleration vectors, as shown in Figure 2.9. The vectors are sketched at several instants during the motion of the object, and the time intervals between adjacent positions are assumed to be equal. This illustration represents three sets of strobe photographs of a car moving from left to right along a straight roadway. The time intervals between flashes are equal in each diagram.

In Figure 2.9a, the images of the car are equally spaced, showing us that the car moves the same distance in each time interval. Thus, the car moves with *constant positive velocity* and has *zero acceleration.*

In Figure 2.9b, the images become farther apart as time progresses. In this case, the velocity vector increases in time because the car's displacement between adjacent positions increases in time. The car is moving with a *positive velocity* and a *positive acceleration.*

In Figure 2.9c, we can tell that the car slows as it moves to the right because its displacement between adjacent images decreases with time. In this case, the car moves to the right with a constant negative acceleration. The velocity vector decreases in time and eventually reaches zero. From this diagram we see that the acceleration and velocity vectors are *not* in the same direction. The car is moving with a *positive velocity* but with a *negative acceleration.*

You should be able to construct motion diagrams for a car that moves initially to the left with a constant positive or negative acceleration.

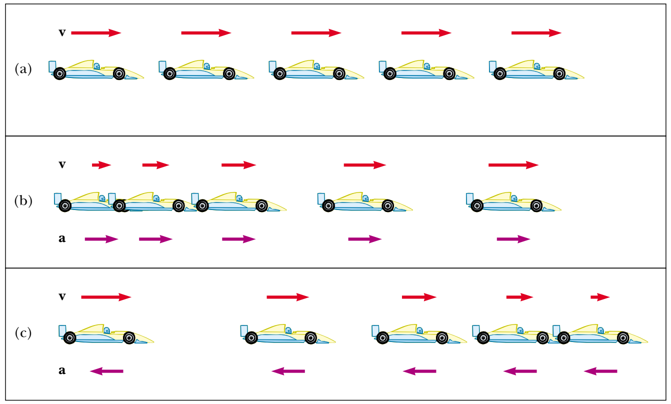


Figure 2.9 (a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration by a violet arrow. (c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

Quick Quiz 2.2

(a) If a car is traveling eastward, can its acceleration be westward? (b) If a car is slowing down, can its acceleration be positive?

2.5 ONE-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant. When this is the case, the average acceleration over any time interval equals the instantaneous acceleration at any instant within the interval, and the velocity changes at the same rate throughout the motion.

If we replace $\bar{a}_x$ by a_x in Equation 2.5 and take $t_i = 0$ and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

or

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.8)}$$

Velocity as a function of time

This powerful expression enables us to determine an object's velocity at *any* time t if we know the object's initial velocity and its (constant) acceleration. A velocity–time graph for this constant-acceleration motion is shown in Figure 2.10a. The graph is a straight line, the (constant) slope of which is the acceleration a_x; this is consistent with the fact that $a_x = dv_x/dt$ is a constant. Note that the slope is positive; this indicates a positive acceleration. If the acceleration were negative, then the slope of the line in Figure 2.10a would be negative.

When the acceleration is constant, the graph of acceleration versus time (Fig. 2.10b) is a straight line having a slope of zero.

Quick Quiz 2.3

Describe the meaning of each term in Equation 2.8.

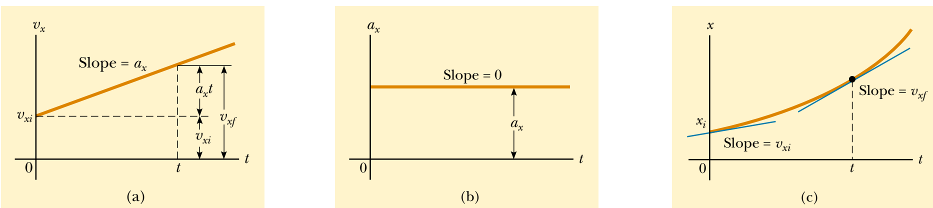


Figure 2.10 An object moving along the x axis with constant acceleration a_x. (a) The velocity–time graph. (b) The acceleration–time graph. (c) The position–time graph.

Because velocity at constant acceleration varies linearly in time according to Equation 2.8, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}:

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.9)}$$

Note that this expression for average velocity applies *only* in situations in which the acceleration is constant.

We can now use Equations 2.1, 2.2, and 2.9 to obtain the displacement of any object as a function of time. Recalling that Δx in Equation 2.2 represents $x_f - x_i$, and now using t in place of Δt (because we take $t_i = 0$), we can say

Displacement as a function of velocity and time

$$x_f - x_i = \bar{v}_x t = \tfrac{1}{2}(v_{xi} + v_{xf})\,t \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.10)}$$

We can obtain another useful expression for displacement at constant acceleration by substituting Equation 2.8 into Equation 2.10:

$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xi} + a_x t)\,t$$

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \textbf{(2.11)}$$

The position–time graph for motion at constant (positive) acceleration shown in Figure 2.10c is obtained from Equation 2.11. Note that the curve is a parabola. The slope of the tangent line to this curve at $t = t_i = 0$ equals the initial velocity v_{xi}, and the slope of the tangent line at any later time t equals the velocity at that time, v_{xf}.

We can check the validity of Equation 2.11 by moving the x_i term to the right-hand side of the equation and differentiating the equation with respect to time:

$$v_{xf} = \frac{dx_f}{dt} = \frac{d}{dt}\left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right) = v_{xi} + a_x t$$

Finally, we can obtain an expression for the final velocity that does not contain a time interval by substituting the value of t from Equation 2.8 into Equation 2.10:

$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.12)}$$

For motion at *zero* acceleration, we see from Equations 2.8 and 2.11 that

$$\left.\begin{aligned} v_{xf} = v_{xi} = v_x \\ x_f - x_i = v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when acceleration is zero, velocity is constant and displacement changes linearly with time.

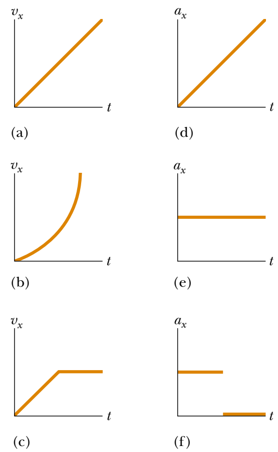


Figure 2.11 Parts (a), (b), and (c) are v_x-t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

Quick Quiz 2.4

In Figure 2.11, match each v_x-t graph with the a_x-t graph that best describes the motion.

Equations 2.8 through 2.12 are **kinematic expressions that may be used to solve any problem involving one-dimensional motion at constant accelera-**

TABLE 2.2 Kinematic Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t$	Displacement as a function of velocity and time
$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$	Displacement as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of displacement

Note: Motion is along the *x* axis.

tion. Keep in mind that these relationships were derived from the definitions of velocity and acceleration, together with some simple algebraic manipulations and the requirement that the acceleration be constant.

The four kinematic equations used most often are listed in Table 2.2 for convenience. The choice of which equation you use in a given situation depends on what you know beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns. For example, suppose initial velocity v_{xi} and acceleration a_x are given. You can then find (1) the velocity after an interval t has elapsed, using $v_{xf} = v_{xi} + a_x t$, and (2) the displacement after an interval t has elapsed, using $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$. You should recognize that the quantities that vary during the motion are velocity, displacement, and time.

You will get a great deal of practice in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than one method can be used to obtain a solution. Remember that these equations of kinematics *cannot* be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

CONCEPTUAL EXAMPLE 2.5 The Velocity of Different Objects

Consider the following one-dimensional motions: (a) A ball thrown directly upward rises to a highest point and falls back into the thrower's hand. (b) A race car starts from rest and speeds up to 100 m/s. (c) A spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity is the same as the average velocity over the entire motion? If so, identify the point(s).

Solution (a) The average velocity for the thrown ball is zero because the ball returns to the starting point; thus its displacement is zero. (Remember that average velocity is defined as $\Delta x/\Delta t$.) There is one point at which the instantaneous velocity is zero—at the top of the motion.

(b) The car's average velocity cannot be evaluated unambiguously with the information given, but it must be some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity.

(c) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at *any* time and its average velocity over *any* time interval are the same.

EXAMPLE 2.6 Entering the Traffic Flow

(a) Estimate your average acceleration as you drive up the entrance ramp to an interstate highway.

Solution This problem involves more than our usual amount of estimating! We are trying to come up with a value of a_x, but that value is hard to guess directly. The other three variables involved in kinematics are position, velocity, and time. Velocity is probably the easiest one to approximate. Let us assume a final velocity of 100 km/h, so that you can merge with traffic. We multiply this value by 1 000 to convert kilome-

ters to meters and then divide by 3 600 to convert hours to seconds. These two calculations together are roughly equivalent to dividing by 3. In fact, let us just say that the final velocity is $v_{xf} \approx 30$ m/s. (Remember, you can get away with this type of approximation and with dropping digits when performing mental calculations. If you were starting with British units, you could approximate 1 mi/h as roughly 0.5 m/s and continue from there.)

Now we assume that you started up the ramp at about one-third your final velocity, so that $v_{xi} \approx 10$ m/s. Finally, we assume that it takes about 10 s to get from v_{xi} to v_{xf}, basing this guess on our previous experience in automobiles. We can then find the acceleration, using Equation 2.8:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{30\text{ m/s} - 10\text{ m/s}}{10\text{ s}} = \boxed{2\text{ m/s}^2}$$

Granted, we made many approximations along the way, but this type of mental effort can be surprisingly useful and often yields results that are not too different from those derived from careful measurements.

(b) How far did you go during the first half of the time interval during which you accelerated?

Solution We can calculate the distance traveled during the first 5 s from Equation 2.11:

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2 \approx (10\text{ m/s})(5\text{ s}) + \tfrac{1}{2}(2\text{ m/s}^2)(5\text{ s})^2$$
$$= 50\text{ m} + 25\text{ m} = \boxed{75\text{ m}}$$

This result indicates that if you had not accelerated, your initial velocity of 10 m/s would have resulted in a 50-m movement up the ramp during the first 5 s. The additional 25 m is the result of your increasing velocity during that interval.

Do not be afraid to attempt making educated guesses and doing some fairly drastic number rounding to simplify mental calculations. Physicists engage in this type of thought analysis all the time.

EXAMPLE 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$63 m/s). (a) What is its acceleration if it stops in 2.0 s?

Solution We define our *x* axis as the direction of motion of the jet. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We also note that we are not given the displacement of the jet while it is slowing down. Equation 2.8 is the only equation in Table 2.2 that does not involve displacement, and so we use it to find the acceleration:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\text{ m/s}}{2.0\text{ s}} = \boxed{-31\text{ m/s}^2}$$

(b) What is the displacement of the plane while it is stopping?

Solution We can now use any of the other three equations in Table 2.2 to solve for the displacement. Let us choose Equation 2.10:

$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xf})t = \tfrac{1}{2}(63\text{ m/s} + 0)(2.0\text{ s}) = \boxed{63\text{ m}}$$

If the plane travels much farther than this, it might fall into the ocean. Although the idea of using arresting cables to enable planes to land safely on ships originated at about the time of the First World War, the cables are still a vital part of the operation of modern aircraft carriers.

EXAMPLE 2.8 Watch Out for the Speed Limit!

A car traveling at a constant speed of 45.0 m/s passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

Solution A careful reading lets us categorize this as a constant-acceleration problem. We know that after the 1-s delay in starting, it will take the trooper 15 additional seconds to accelerate up to 45.0 m/s. Of course, she then has to continue to pick up speed (at a rate of 3.00 m/s per second) to catch up to the car. While all this is going on, the car continues to move. We should therefore expect our result to be well over 15 s. A sketch (Fig. 2.12) helps clarify the sequence of events.

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_B \equiv 0$ as the time the trooper begins moving. At that instant, the car has already traveled a distance of 45.0 m because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Thus, the initial position of the speeding car is $x_B = 45.0$ m.

Because the car moves with constant speed, its accelera-

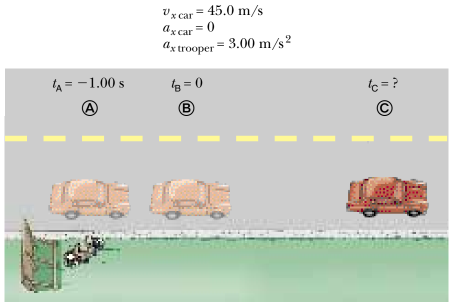


Figure 2.12 A speeding car passes a hidden police officer.

tion is zero, and applying Equation 2.11 (with $a_x = 0$) gives for the car's position at any time t:

$$x_{car} = x_B + v_{x\,car}t = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

A quick check shows that at $t = 0$, this expression gives the car's correct initial position when the trooper begins to move: $x_{car} = x_B = 45.0$ m. Looking at limiting cases to see whether they yield expected values is a very useful way to make sure that you are obtaining reasonable results.

The trooper starts from rest at $t = 0$ and accelerates at 3.00 m/s^2 away from the origin. Hence, her position after any time interval t can be found from Equation 2.11:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_xt^2$$

$$x_{trooper} = 0 + 0t + \tfrac{1}{2}a_xt^2 = \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2$$

The trooper overtakes the car at the instant her position matches that of the car, which is position Ⓒ:

$$x_{trooper} = x_{car}$$

$$\tfrac{1}{2}(3.00 \text{ m/s}^2)t^2 = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

This gives the quadratic equation

$$1.50t^2 - 45.0t - 45.0 = 0$$

The positive solution of this equation is $t =$ 31.0 s.

(For help in solving quadratic equations, see Appendix B.2.) Note that in this 31.0-s time interval, the trooper travels a distance of about 1440 m. [This distance can be calculated from the car's constant speed: (45.0 m/s)(31 + 1) s = 1 440 m.]

Exercise This problem can be solved graphically. On the same graph, plot position versus time for each vehicle, and from the intersection of the two curves determine the time at which the trooper overtakes the car.

2.6 FREELY FALLING OBJECTS

It is now well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. It was not until about 1600 that this conclusion was accepted. Before that time, the teachings of the great philosopher Aristotle (384–322 B.C.) had held that heavier objects fall faster than lighter ones.

It was the Italian Galileo Galilei (1564–1642) who originated our present-day ideas concerning falling objects. There is a legend that he demonstrated the law of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. In his experiments he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration; with the acceleration reduced, Galileo was able to make accurate measurements of the time intervals. By gradually increasing the slope of the incline, he was finally able to draw conclusions about freely falling objects because a freely falling ball is equivalent to a ball moving down a vertical incline.

Astronaut David Scott released a hammer and a feather simultaneously, and they fell in unison to the lunar surface. *(Courtesy of NASA)*

QuickLab

Use a pencil to poke a hole in the bottom of a paper or polystyrene cup. Cover the hole with your finger and fill the cup with water. Hold the cup up in front of you and release it. Does water come out of the hole while the cup is falling? Why or why not?

You might want to try the following experiment. Simultaneously drop a coin and a crumpled-up piece of paper from the same height. If the effects of air resistance are negligible, both will have the same motion and will hit the floor at the same time. In the idealized case, in which air resistance is absent, such motion is referred to as *free fall.* If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, such a demonstration was conducted on the Moon by astronaut David Scott. He simultaneously released a hammer and a feather, and in unison they fell to the lunar surface. This demonstration surely would have pleased Galileo!

Definition of free fall

When we use the expression *freely falling object,* we do not necessarily refer to an object dropped from rest. **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed *downward*, regardless of its initial motion.**

We shall denote the magnitude of the *free-fall acceleration* by the symbol g. The value of g near the Earth's surface decreases with increasing altitude. Furthermore, slight variations in g occur with changes in latitude. It is common to define "up" as the $+y$ direction and to use y as the position variable in the kinematic equations. At the Earth's surface, the value of g is approximately 9.80 m/s^2. Unless stated otherwise, we shall use this value for g when performing calculations. For making quick estimates, use $g = 10$ m/s^2.

Free-fall acceleration $g = 9.80$ m/s^2

If we neglect air resistance and assume that the free-fall acceleration does not vary with altitude over short vertical distances, then the motion of a freely falling object moving vertically is equivalent to motion in one dimension under constant acceleration. Therefore, the equations developed in Section 2.5 for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal (x) direction and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Thus, we always take $a_y = -g = -9.80$ m/s^2, where the minus sign means that the acceleration of a freely falling object is downward. In Chapter 14 we shall study how to deal with variations in g with altitude.

CONCEPTUAL EXAMPLE 2.9 The Daring Sky Divers

A sky diver jumps out of a hovering helicopter. A few seconds later, another sky diver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both sky divers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall? If the two divers were connected by a long bungee cord, would the tension in the cord increase, lessen, or stay the same during the fall?

Solution At any given instant, the speeds of the divers are different because one had a head start. In any time interval Δt after this instant, however, the two divers increase their speeds by the same amount because they have the same acceleration. Thus, the difference in their speeds remains the same throughout the fall.

The first jumper always has a greater speed than the second. Thus, in a given time interval, the first diver covers a greater distance than the second. Thus, the separation distance between them increases.

Once the distance between the divers reaches the length of the bungee cord, the tension in the cord begins to increase. As the tension increases, the distance between the divers becomes greater and greater.

EXAMPLE 2.10 Describing the Motion of a Tossed Ball

A ball is tossed straight up at 25 m/s. Estimate its velocity at 1-s intervals.

Solution Let us choose the upward direction to be positive. Regardless of whether the ball is moving upward or downward, its vertical velocity changes by approximately −10 m/s for every second it remains in the air. It starts out at 25 m/s. After 1 s has elapsed, it is still moving upward but at 15 m/s because its acceleration is downward (downward acceleration causes its velocity to decrease). After another second, its upward velocity has dropped to 5 m/s. Now comes the tricky part—after another half second, its velocity is zero. The ball has gone as high as it will go. After the last half of this 1-s interval, the ball is moving at −5 m/s. (The minus sign tells us that the ball is now moving in the negative direction, that is, *downward*. Its velocity has changed from +5 m/s to −5 m/s during that 1-s interval. The change in velocity is still −5 − [+5] = −10 m/s in that second.) It continues downward, and after another 1 s has elapsed, it is falling at a velocity of −15 m/s. Finally, after another 1 s, it has reached its original starting point and is moving downward at −25 m/s. If the ball had been tossed vertically off a cliff so that it could continue downward, its velocity would continue to change by about −10 m/s every second.

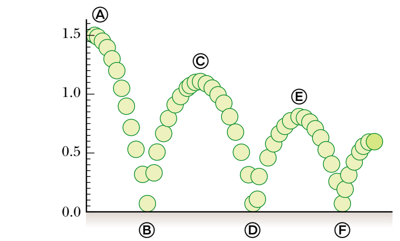
CONCEPTUAL EXAMPLE 2.11 Follow the Bouncing Ball

A tennis ball is dropped from shoulder height (about 1.5 m) and bounces three times before it is caught. Sketch graphs of its position, velocity, and acceleration as functions of time, with the $+y$ direction defined as upward.

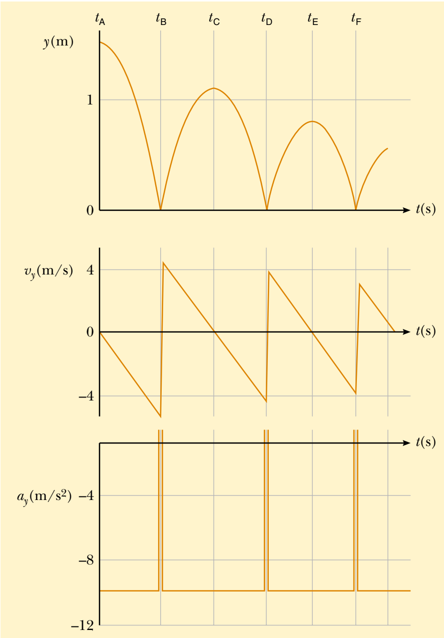
Solution For our sketch let us stretch things out horizontally so that we can see what is going on. (Even if the ball were moving horizontally, this motion would not affect its vertical motion.)

From Figure 2.13 we see that the ball is in contact with the floor at points Ⓑ, Ⓓ, and Ⓕ. Because the velocity of the ball changes from negative to positive three times during these bounces, the slope of the position–time graph must change in the same way. Note that the time interval between bounces decreases. Why is that?

During the rest of the ball's motion, the slope of the velocity–time graph should be $-9.80\ \text{m/s}^2$. The acceleration–time graph is a horizontal line at these times because the acceleration does not change when the ball is in free fall. When the ball is in contact with the floor, the velocity changes substantially during a very short time interval, and so the acceleration must be quite great. This corresponds to the very steep upward lines on the velocity–time graph and to the spikes on the acceleration–time graph.



(a)



(b)

Figure 2.13 (a) A ball is dropped from a height of 1.5 m and bounces from the floor. (The horizontal motion is not considered here because it does not affect the vertical motion.) (b) Graphs of position, velocity, and acceleration versus time.

Quick Quiz 2.5

Which values represent the ball's velocity and acceleration at points Ⓐ, Ⓒ, and Ⓔ in Figure 2.13?

(a) $v_y = 0$, $a_y = 0$
(b) $v_y = 0$, $a_y = 9.80\ \text{m/s}^2$
(c) $v_y = 0$, $a_y = -9.80\ \text{m/s}^2$
(d) $v_y = -9.80\ \text{m/s}$, $a_y = 0$

EXAMPLE 2.12 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine (a) the time at which the stone reaches its maximum height, (b) the maximum height, (c) the time at which the stone returns to the height from which it was thrown, (d) the velocity of the stone at this instant, and (e) the velocity and position of the stone at $t = 5.00$ s.

Solution (a) As the stone travels from Ⓐ to Ⓑ, its velocity must change by 20 m/s because it stops at Ⓑ. Because gravity causes vertical velocities to change by about 10 m/s for every second of free fall, it should take the stone about 2 s to go from Ⓐ to Ⓑ in our drawing. (In a problem like this, a sketch definitely helps you organize your thoughts.) To calculate the time t_B at which the stone reaches maximum height, we use Equation 2.8, $v_{yB} = v_{yA} + a_y t$, noting that $v_{yB} = 0$ and setting the start of our clock readings at $t_A \equiv 0$:

$$20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)t = 0$$

$$t = t_B = \frac{20.0\ \text{m/s}}{9.80\ \text{m/s}^2} = \boxed{2.04\ \text{s}}$$

Our estimate was pretty close.

(b) Because the average velocity for this first interval is 10 m/s (the average of 20 m/s and 0 m/s) and because it travels for about 2 s, we expect the stone to travel about 20 m. By substituting our time interval into Equation 2.11, we can find the maximum height as measured from the position of the thrower, where we set $y_i = y_A = 0$:

$$y_{max} = y_B = v_{yA}\,t + \tfrac{1}{2}a_y t^2$$

$$y_B = (20.0\ \text{m/s})(2.04\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.04\ \text{s})^2$$

$$= \boxed{20.4\ \text{m}}$$

Our free-fall estimates are very accurate.

(c) There is no reason to believe that the stone's motion from Ⓑ to Ⓒ is anything other than the reverse of its motion

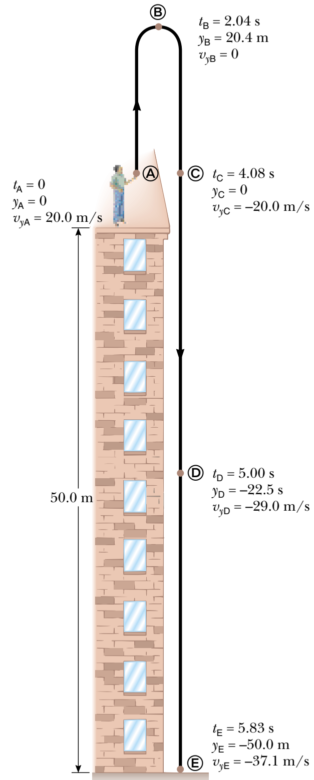


Figure 2.14 Position and velocity versus time for a freely falling stone thrown initially upward with a velocity v_{yi} = 20.0 m/s.

from Ⓐ to Ⓑ. Thus, the time needed for it to go from Ⓐ to Ⓒ should be twice the time needed for it to go from Ⓐ to Ⓑ. When the stone is back at the height from which it was thrown (position Ⓒ), the y coordinate is again zero. Using Equation 2.11, with $y_f = y_C = 0$ and $y_i = y_A = 0$, we obtain

$$y_C - y_A = v_{yA}\, t + \tfrac{1}{2}a_y t^2$$
$$0 = 20.0t - 4.90t^2$$

This is a quadratic equation and so has two solutions for $t = t_C$. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution is $t = 4.08$ s, which is the solution we are after. Notice that it is double the value we calculated for t_B.

(d) Again, we expect everything at Ⓒ to be the same as it is at Ⓐ, except that the velocity is now in the opposite direction. The value for t found in (c) can be inserted into Equation 2.8 to give

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$
$$= -20.0 \text{ m/s}$$

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction. This indicates that the motion is symmetric.

(e) For this part we consider what happens as the stone falls from position Ⓑ, where it had zero vertical velocity, to position Ⓓ. Because the elapsed time for this part of the motion is about 3 s, we estimate that the acceleration due to gravity will have changed the speed by about 30 m/s. We can calculate this from Equation 2.8, where we take $t = t_D - t_B$:

$$v_{yD} = v_{yB} + a_y t = 0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s} - 2.04 \text{ s})$$
$$= -29.0 \text{ m/s}$$

We could just as easily have made our calculation between positions Ⓐ and Ⓓ by making sure we use the correct time interval, $t = t_D - t_A = 5.00$ s:

$$v_{yD} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s})$$
$$= -29.0 \text{ m/s}$$

To demonstrate the power of our kinematic equations, we can use Equation 2.11 to find the position of the stone at $t_D = 5.00$ s by considering the change in position between a different pair of positions, Ⓒ and Ⓓ. In this case, the time is $t_D - t_C$:

$$y_D = y_C + v_{yC}t + \tfrac{1}{2}a_y t^2$$
$$= 0 \text{ m} + (-20.0 \text{ m/s})(5.00 \text{ s} - 4.08 \text{ s})$$
$$+ \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s} - 4.08 \text{ s})^2$$
$$= -22.5 \text{ m}$$

Exercise Find (a) the velocity of the stone just before it hits the ground at Ⓔ and (b) the total time the stone is in the air.

Answer (a) -37.1 m/s (b) 5.83 s

Optional Section

2.7 KINEMATIC EQUATIONS DERIVED FROM CALCULUS

This is an optional section that assumes the reader is familiar with the techniques of integral calculus. If you have not yet studied integration in your calculus course, you should skip this section or cover it after you become familiar with integration.

The velocity of a particle moving in a straight line can be obtained if its position as a function of time is known. Mathematically, the velocity equals the derivative of the position coordinate with respect to time. It is also possible to find the displacement of a particle if its velocity is known as a function of time. In calculus, the procedure used to perform this task is referred to either as *integration* or as finding the *antiderivative.* Graphically, it is equivalent to finding the area under a curve.

Suppose the v_x-t graph for a particle moving along the x axis is as shown in Figure 2.15. Let us divide the time interval $t_f - t_i$ into many small intervals, each of duration Δt_n. From the definition of average velocity we see that the displacement during any small interval, such as the one shaded in Figure 2.15, is given by $\Delta x_n = \bar{v}_{xn}\,\Delta t_n$, where $\bar{v}_{xn}$ is the average velocity in that interval. Therefore, the displacement during this small interval is simply the area of the shaded rectangle.

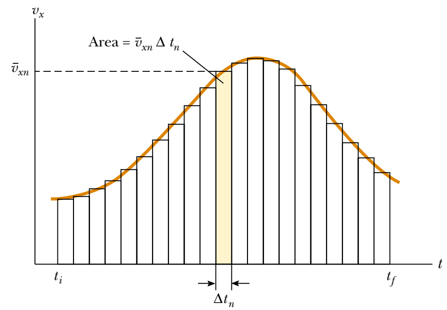


Figure 2.15 Velocity versus time for a particle moving along the x axis. The area of the shaded rectangle is equal to the displacement Δx in the time interval Δt_n, while the total area under the curve is the total displacement of the particle.

The total displacement for the interval $t_f - t_i$ is the sum of the areas of all the rectangles:

$$\Delta x = \sum_n \bar{v}_{xn}\,\Delta t_n$$

where the symbol Σ (upper case Greek sigma) signifies a sum over all terms. In this case, the sum is taken over all the rectangles from t_i to t_f. Now, as the intervals are made smaller and smaller, the number of terms in the sum increases and the sum approaches a value equal to the area under the velocity–time graph. Therefore, in the limit $n \rightarrow \infty$, or $\Delta t_n \rightarrow 0$, the displacement is

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn}\,\Delta t_n \qquad \textbf{(2.13)}$$

or

Displacement = area under the v_x-t graph

Note that we have replaced the average velocity $\bar{v}_{xn}$ with the instantaneous velocity v_{xn} in the sum. As you can see from Figure 2.15, this approximation is clearly valid in the limit of very small intervals. We conclude that if we know the v_x-t graph for motion along a straight line, we can obtain the displacement during any time interval by measuring the area under the curve corresponding to that time interval.

Definite integral

The limit of the sum shown in Equation 2.13 is called a **definite integral** and is written

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn}\Delta t_n = \int_{t_i}^{t_f} v_x(t)\; dt \qquad \textbf{(2.14)}$$

where $v_x(t)$ denotes the velocity at any time t. If the explicit functional form of $v_x(t)$ is known and the limits are given, then the integral can be evaluated.

Sometimes the v_x-t graph for a moving particle has a shape much simpler than that shown in Figure 2.15. For example, suppose a particle moves at a constant ve-

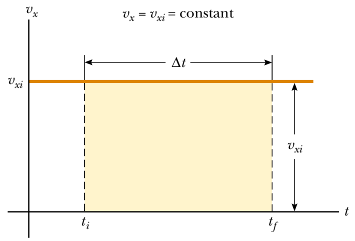


Figure 2.16 The velocity–time curve for a particle moving with constant velocity v_{xi}. The displacement of the particle during the time interval $t_f - t_i$ is equal to the area of the shaded rectangle.

locity v_{xi}. In this case, the v_x-t graph is a horizontal line, as shown in Figure 2.16, and its displacement during the time interval Δt is simply the area of the shaded rectangle:

$$\Delta x = v_{xi}\Delta t \qquad (\text{when } v_{xf} = v_{xi} = \text{constant})$$

As another example, consider a particle moving with a velocity that is proportional to t, as shown in Figure 2.17. Taking $v_x = a_x t$, where a_x is the constant of proportionality (the acceleration), we find that the displacement of the particle during the time interval $t = 0$ to $t = t_{\text{A}}$ is equal to the area of the shaded triangle in Figure 2.17:

$$\Delta x = \tfrac{1}{2}(t_{\text{A}})(a_x t_{\text{A}}) = \tfrac{1}{2}a_x {t_{\text{A}}}^2$$

Kinematic Equations

We now use the defining equations for acceleration and velocity to derive two of our kinematic equations, Equations 2.8 and 2.11.

The defining equation for acceleration (Eq. 2.6),

$$a_x = \frac{dv_x}{dt}$$

may be written as $dv_x = a_x dt$ or, in terms of an integral (or antiderivative), as

$$v_x = \int a_x\, dt + C_1$$

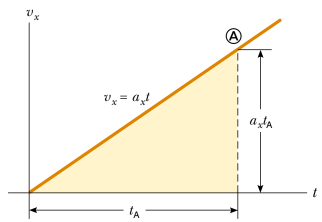


Figure 2.17 The velocity–time curve for a particle moving with a velocity that is proportional to the time.

where C_1 is a constant of integration. For the special case in which the acceleration is constant, the a_x can be removed from the integral to give

$$v_x = a_x \int dt + C_1 = a_x t + C_1 \qquad \textbf{(2.15)}$$

The value of C_1 depends on the initial conditions of the motion. If we take $v_x = v_{xi}$ when $t = 0$ and substitute these values into the last equation, we have

$$v_{xi} = a_x(0) + C_1$$

$$C_1 = v_{xi}$$

Calling $v_x = v_{xf}$ the velocity after the time interval t has passed and substituting this and the value just found for C_1 into Equation 2.15, we obtain kinematic Equation 2.8:

$$v_{xf} = v_{xi} + a_x t \qquad (\text{for constant } a_x)$$

Now let us consider the defining equation for velocity (Eq. 2.4):

$$v_x = \frac{dx}{dt}$$

We can write this as $dx = v_x dt$ or in integral form as

$$x = \int v_x\, dt + C_2$$

where C_2 is another constant of integration. Because $v_x = v_{xf} = v_{xi} + a_x t$, this expression becomes

$$x = \int (v_{xi} + a_x t)\, dt + C_2$$

$$x = \int v_{xi}\, dt + a_x \int t\, dt + C_2$$

$$x = v_{xi}t + \tfrac{1}{2}a_x t^2 + C_2$$

To find C_2, we make use of the initial condition that $x = x_i$ when $t = 0$. This gives $C_2 = x_i$. Therefore, after substituting x_f for x, we have

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad (\text{for constant } a_x)$$

Once we move x_i to the left side of the equation, we have kinematic Equation 2.11. Recall that $x_f - x_i$ is equal to the displacement of the object, where x_i is its initial position.

Besides what you might expect to learn about physics concepts, a very valuable skill you should hope to take away from your physics course is the ability to solve complicated problems. The way physicists approach complex situations and break them down into manageable pieces is extremely useful. We have developed a memory aid to help you easily recall the steps required for successful problem solving. When working on problems, the secret is to keep your GOAL in mind!

GOAL Problem-Solving Steps

Gather information

The first thing to do when approaching a problem is to understand the situation. Carefully read the problem statement, looking for key phrases like "at rest" or "freely falls." What information is given? Exactly what is the question asking? Don't forget to gather information from your own experiences and common sense. What should a reasonable answer look like? You wouldn't expect to calculate the speed of an automobile to be 5×10^6 m/s. Do you know what units to expect? Are there any limiting cases you can consider? What happens when an angle approaches 0° or 90° or when a mass becomes huge or goes to zero? Also make sure you carefully study any drawings that accompany the problem.

Organize your approach

Once you have a really good idea of what the problem is about, you need to think about what to do next. Have you seen this type of question before? Being able to classify a problem can make it much easier to lay out a plan to solve it. You should almost always make a quick drawing of the situation. Label important events with circled letters. Indicate any known values, perhaps in a table or directly on your sketch.

Analyze the problem

Because you have already categorized the problem, it should not be too difficult to select relevant equations that apply to this type of situation. Use algebra (and calculus, if necessary) to solve for the unknown variable in terms of what is given. Substitute in the appropriate numbers, calculate the result, and round it to the proper number of significant figures.

Learn from your efforts

This is the most important part. Examine your numerical answer. Does it meet your expectations from the first step? What about the algebraic form of the result—before you plugged in numbers? Does it make sense? (Try looking at the variables in it to see whether the answer would change in a physically meaningful way if they were drastically increased or decreased or even became zero.) Think about how this problem compares with others you have done. How was it similar? In what critical ways did it differ? Why was this problem assigned? You should have learned something by doing it. Can you figure out what?

When solving complex problems, you may need to identify a series of subproblems and apply the GOAL process to each. For very simple problems, you probably don't need GOAL at all. But when you are looking at a problem and you don't know what to do next, remember what the letters in GOAL stand for and use that as a guide.

Summary

After a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

The **average velocity** of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurred:

$$\overline{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time it takes to travel that distance.

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero. By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.4}$$

The **instantaneous speed** of a particle is equal to the magnitude of its velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurred:

$$\overline{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.5}$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches 0. By definition, this limit equals the derivative of v_x with respect to t, or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.6}$$

The **equations of kinematics** for a particle moving along the x axis with uniform acceleration a_x (constant in magnitude and direction) are

$$v_{xf} = v_{xi} + a_x t \tag{2.8}$$

$$x_f - x_i = \overline{v}_x t = \tfrac{1}{2}(v_{xi} + v_{xf})t \tag{2.10}$$

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.11}$$

$$v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i) \tag{2.12}$$

You should be able to use these equations and the definitions in this chapter to analyze the motion of any object moving with constant acceleration.

An object falling freely in the presence of the Earth's gravity experiences a free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, then the free-fall acceleration g is constant over the range of motion, where g is equal to 9.80 m/s^2.

Complicated problems are best approached in an organized manner. You should be able to recall and apply the steps of the GOAL strategy when you need them.

QUESTIONS

1. Average velocity and instantaneous velocity are generally different quantities. Can they ever be equal for a specific type of motion? Explain.
2. If the average velocity is nonzero for some time interval, does this mean that the instantaneous velocity is never zero during this interval? Explain.
3. If the average velocity equals zero for some time interval Δt and if $v_x(t)$ is a continuous function, show that the instantaneous velocity must go to zero at some time in this interval. (A sketch of x versus t might be useful in your proof.)
4. Is it possible to have a situation in which the velocity and acceleration have opposite signs? If so, sketch a velocity–time graph to prove your point.
5. If the velocity of a particle is nonzero, can its acceleration be zero? Explain.
6. If the velocity of a particle is zero, can its acceleration be nonzero? Explain.
7. Can an object having constant acceleration ever stop and stay stopped?
8. A stone is thrown vertically upward from the top of a building. Does the stone's displacement depend on the location of the origin of the coordinate system? Does the stone's velocity depend on the origin? (Assume that the coordinate system is stationary with respect to the building.) Explain.
9. A student at the top of a building of height h throws one ball upward with an initial speed v_{yi} and then throws a second ball downward with the same initial speed. How do the final speeds of the balls compare when they reach the ground?
10. Can the magnitude of the instantaneous velocity of an object ever be greater than the magnitude of its average velocity? Can it ever be less?
11. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?
12. A rapidly growing plant doubles in height each week. At the end of the 25th day, the plant reaches the height of a building. At what time was the plant one-fourth the height of the building?
13. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does this mean that the acceleration of car A is greater than that of car B? Explain.
14. An apple is dropped from some height above the Earth's surface. Neglecting air resistance, how much does the apple's speed increase each second during its descent?
15. Consider the following combinations of signs and values for velocity and acceleration of a particle with respect to a one-dimensional x axis:

Velocity	Acceleration
a. Positive	Positive
b. Positive	Negative
c. Positive	Zero
d. Negative	Positive
e. Negative	Negative
f. Negative	Zero
g. Zero	Positive
h. Zero	Negative

Describe what the particle is doing in each case, and give a real-life example for an automobile on an east-west one-dimensional axis, with east considered to be the positive direction.

16. A pebble is dropped into a water well, and the splash is heard 16 s later, as illustrated in Figure Q2.16. Estimate the distance from the rim of the well to the water's surface.
17. Average velocity is an entirely contrived quantity, and other combinations of data may prove useful in other contexts. For example, the ratio $\Delta t/\Delta x$, called the "slowness" of a moving object, is used by geophysicists when discussing the motion of continental plates. Explain what this quantity means.

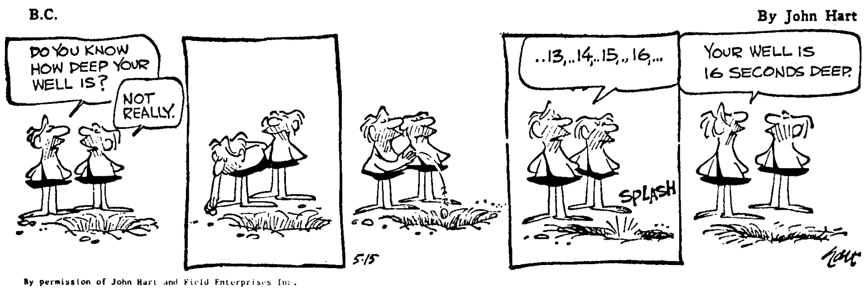


Figure Q2.16

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
□ = paired numerical/symbolic problems

Section 2.1 Displacement, Velocity, and Speed

1. The position of a pinewood derby car was observed at various times; the results are summarized in the table below. Find the average velocity of the car for (a) the first second, (b) the last 3 s, and (c) the entire period of observation.

x (m)	0	2.3	9.2	20.7	36.8	57.5
t (s)	0	1.0	2.0	3.0	4.0	5.0

2. A motorist drives north for 35.0 min at 85.0 km/h and then stops for 15.0 min. He then continues north, traveling 130 km in 2.00 h. (a) What is his total displacement? (b) What is his average velocity?
3. The displacement versus time for a certain particle moving along the x axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.

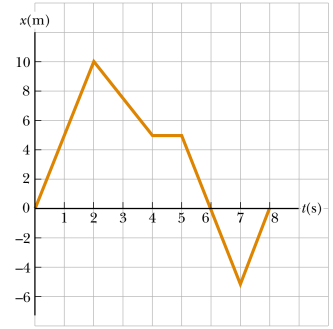


Figure P2.3 Problems 3 and 11.

4. A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.0 s to 3.0 s. (b) Find the average velocity for the time interval from 2.0 s to 2.1 s.
5. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. What are (a) her average speed over the entire trip and (b) her average velocity over the entire trip?
6. A person first walks at a constant speed v_1 along a straight line from A to B and then back along the line from B to A at a constant speed v_2. What are (a) her average speed over the entire trip and (b) her average velocity over the entire trip?

Section 2.2 Instantaneous Velocity and Speed

7. At $t = 1.00$ s, a particle moving with constant velocity is located at $x = -3.00$ m, and at $t = 6.00$ s the particle is located at $x = 5.00$ m. (a) From this information, plot the position as a function of time. (b) Determine the velocity of the particle from the slope of this graph.
8. The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds. Evaluate its position (a) at $t = 3.00$ s and (b) at 3.00 s + Δt. (c) Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero to find the velocity at $t = 3.00$ s.
9. WEB A position–time graph for a particle moving along the x axis is shown in Figure P2.9. (a) Find the average velocity in the time interval $t = 1.5$ s to $t = 4.0$ s. (b) Determine the instantaneous velocity at $t = 2.0$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

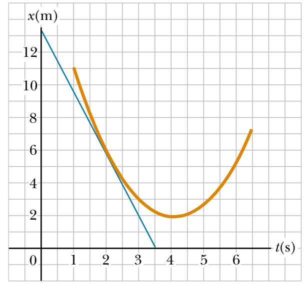


Figure P2.9

10. (a) Use the data in Problem 1 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from this, determine the average acceleration of the car. (d) What was the initial velocity of the car?

11. Find the instantaneous velocity of the particle described in Figure P2.3 at the following times: (a) $t = 1.0$ s, (b) $t = 3.0$ s, (c) $t = 4.5$ s, and (d) $t = 7.5$ s.

Section 2.3 Acceleration

12. A particle is moving with a velocity of 60.0 m/s in the positive x direction at $t = 0$. Between $t = 0$ and $t = 15.0$ s, the velocity decreases uniformly to zero. What was the acceleration during this 15.0-s interval? What is the significance of the sign of your answer?

13. A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? (*Note:* 1 ms $= 10^{-3}$ s.)

14. A particle starts from rest and accelerates as shown in Figure P2.14. Determine: (a) the particle's speed at $t = 10$ s and at $t = 20$ s, and (b) the distance traveled in the first 20 s.

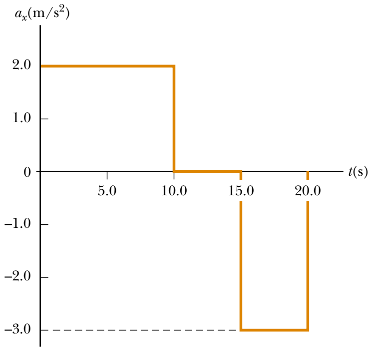


Figure P2.14

15. A velocity–time graph for an object moving along the x axis is shown in Figure P2.15. (a) Plot a graph of the acceleration versus time. (b) Determine the average acceleration of the object in the time intervals $t = 5.00$ s to $t = 15.0$ s and $t = 0$ to $t = 20.0$ s.

16. A student drives a moped along a straight road as described by the velocity–time graph in Figure P2.16. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x-t graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?

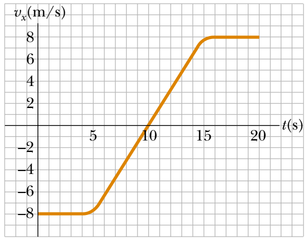


Figure P2.15

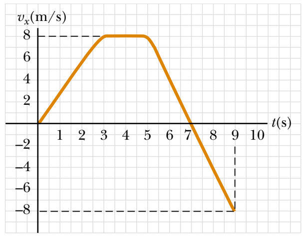


Figure P2.16

WEB **17.** A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

18. An object moves along the x axis according to the equation $x = (3.00t^2 - 2.00t + 3.00)$ m. Determine (a) the average speed between $t = 2.00$ s and $t = 3.00$ s, (b) the instantaneous speed at $t = 2.00$ s and at $t = 3.00$ s, (c) the average acceleration between $t = 2.00$ s and $t = 3.00$ s, and (d) the instantaneous acceleration at $t = 2.00$ s and $t = 3.00$ s.

19. Figure P2.19 shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

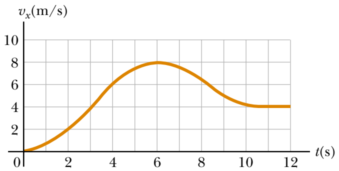


Figure P2.19

Section 2.4 Motion Diagrams

20. Draw motion diagrams for (a) an object moving to the right at constant speed, (b) an object moving to the right and speeding up at a constant rate, (c) an object moving to the right and slowing down at a constant rate, (d) an object moving to the left and speeding up at a constant rate, and (e) an object moving to the left and slowing down at a constant rate. (f) How would your drawings change if the changes in speed were not uniform; that is, if the speed were not changing at a constant rate?

Section 2.5 One-Dimensional Motion with Constant Acceleration

21. Jules Verne in 1865 proposed sending people to the Moon by firing a space capsule from a 220-m-long cannon with a final velocity of 10.97 km/s. What would have been the unrealistically large acceleration experienced by the space travelers during launch? Compare your answer with the free-fall acceleration, 9.80 m/s^2.

22. A certain automobile manufacturer claims that its superdeluxe sports car will accelerate from rest to a speed of 42.0 m/s in 8.00 s. Under the (improbable) assumption that the acceleration is constant, (a) determine the acceleration of the car. (b) Find the distance the car travels in the first 8.00 s. (c) What is the speed of the car 10.0 s after it begins its motion, assuming it continues to move with the same acceleration?

23. A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

24. The minimum distance required to stop a car moving at 35.0 mi/h is 40.0 ft. What is the minimum stopping distance for the same car moving at 70.0 mi/h, assuming the same rate of acceleration?

WEB **25.** A body moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is the magnitude of its acceleration?

26. Figure P2.26 represents part of the performance data of a car owned by a proud physics student. (a) Calculate from the graph the total distance traveled. (b) What distance does the car travel between the times $t = 10$ s and $t = 40$ s? (c) Draw a graph of its acceleration versus time between $t = 0$ and $t = 50$ s. (d) Write an equation for x as a function of time for each phase of the motion, represented by (i) 0a, (ii) ab, (iii) bc. (e) What is the average velocity of the car between $t = 0$ and $t = 50$ s?

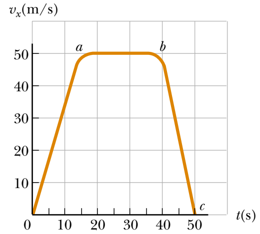


Figure P2.26

27. A particle moves along the x axis. Its position is given by the equation $x = 2.00 + 3.00t - 4.00t^2$ with x in meters and t in seconds. Determine (a) its position at the instant it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

28. The initial velocity of a body is 5.20 m/s. What is its velocity after 2.50 s (a) if it accelerates uniformly at 3.00 m/s^2 and (b) if it accelerates uniformly at -3.00 m/s^2?

29. A drag racer starts her car from rest and accelerates at 10.0 m/s^2 for the entire distance of 400 m ($\frac{1}{4}$ mi). (a) How long did it take the race car to travel this distance? (b) What is the speed of the race car at the end of the run?

30. A car is approaching a hill at 30.0 m/s when its engine suddenly fails, just at the bottom of the hill. The car moves with a constant acceleration of -2.00 m/s^2 while coasting up the hill. (a) Write equations for the position along the slope and for the velocity as functions of time, taking $x = 0$ at the bottom of the hill, where $v_i = 30.0$ m/s. (b) Determine the maximum distance the car travels up the hill.

31. A jet plane lands with a speed of 100 m/s and can accelerate at a maximum rate of -5.00 m/s^2 as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time it needs before it can come to rest? (b) Can this plane land at a small tropical island airport where the runway is 0.800 km long?

32. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of -5.60 m/s^2 for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

33. *Help! One of our equations is missing!* We describe constant-acceleration motion with the variables and parameters v_{xi}, v_{xf}, a_x, t, and $x_f - x_i$. Of the equations in Table 2.2, the first does not involve $x_f - x_i$. The second does not contain a_x, the third omits v_{xf}, and the last

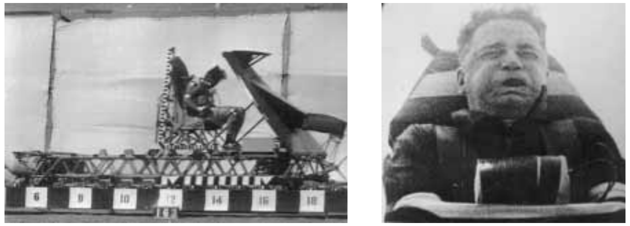


Figure P2.37 *(Left)* Col. John Stapp on rocket sled. *(Courtesy of the U.S. Air Force)* *(Right)* Col. Stapp's face is contorted by the stress of rapid negative acceleration. *(Photri, Inc.)*

leaves out t. So to complete the set there should be an equation *not* involving v_{xi}. Derive it from the others. Use it to solve Problem 32 in one step.

34. An indestructible bullet 2.00 cm long is fired straight through a board that is 10.0 cm thick. The bullet strikes the board with a speed of 420 m/s and emerges with a speed of 280 m/s. (a) What is the average acceleration of the bullet as it passes through the board? (b) What is the total time that the bullet is in contact with the board? (c) What thickness of board (calculated to 0.1 cm) would it take to stop the bullet, assuming the bullet's acceleration through all parts of the board is the same?

35. A truck on a straight road starts from rest, accelerating at 2.00 m/s^2 until it reaches a speed of 20.0 m/s. Then the truck travels for 20.0 s at constant speed until the brakes are applied, stopping the truck in a uniform manner in an additional 5.00 s. (a) How long is the truck in motion? (b) What is the average velocity of the truck for the motion described?

36. A train is traveling down a straight track at 20.0 m/s when the engineer applies the brakes. This results in an acceleration of -1.00 m/s^2 as long as the train is in motion. How far does the train move during a 40.0-s time interval starting at the instant the brakes are applied?

37. For many years the world's land speed record was held by Colonel John P. Stapp, USAF (Fig. P2.37). On March 19, 1954, he rode a rocket-propelled sled that moved down the track at 632 mi/h. He and the sled were safely brought to rest in 1.40 s. Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.

38. An electron in a cathode-ray tube (CRT) accelerates uniformly from 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) How long does the electron take to travel this 1.50 cm? (b) What is its acceleration?

39. A ball starts from rest and accelerates at 0.500 m/s^2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed 8.00 m along the second plane?

40. Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? If so, determine how far into the tunnel and at what time the collision occurs. If not, determine the distance of closest approach between Sue's car and the van.

Section 2.6 Freely Falling Objects

Note: In all problems in this section, ignore the effects of air resistance.

41. A golf ball is released from rest from the top of a very tall building. Calculate (a) the position and (b) the velocity of the ball after 1.00 s, 2.00 s, and 3.00 s.

42. *Every morning at seven o'clock*
There's twenty terriers drilling on the rock.
The boss comes around and he says, "Keep still
And bear down heavy on the cast-iron drill
And drill, ye terriers, drill." And drill, ye terriers, drill.
It's work all day for sugar in your tea . . .
And drill, ye terriers, drill.

One day a premature blast went off
And a mile in the air went big Jim Goff. And drill . . .

Then when next payday came around
Jim Goff a dollar short was found.
When he asked what for, came this reply:
"You were docked for the time you were up in the sky." And drill . . .

—American folksong

What was Goff's hourly wage? State the assumptions you make in computing it.

WEB **43.** A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

44. A ball is thrown directly downward with an initial speed of 8.00 m/s from a height of 30.0 m. How many seconds later does the ball strike the ground?

45. Emily challenges her friend David to catch a dollar bill as follows: She holds the bill vertically, as in Figure P2.45, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

Figure P2.45 *(George Semple)*

46. A ball is dropped from rest from a height h above the ground. Another ball is thrown vertically upward from the ground at the instant the first ball is released. Determine the speed of the second ball if the two balls are to meet at a height $h/2$ above the ground.

47. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the maximum height it reaches.

48. A woman is reported to have fallen 144 ft from the 17th floor of a building, landing on a metal ventilator box, which she crushed to a depth of 18.0 in. She suffered only minor injuries. Calculate (a) the speed of the woman just before she collided with the ventilator box, (b) her average acceleration while in contact with the box, and (c) the time it took to crush the box.

WEB **49.** A daring ranch hand sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The speed of the horse is 10.0 m/s, and the distance from the limb to the saddle is 3.00 m. (a) What must be the horizontal distance between the saddle and limb when the ranch hand makes his move? (b) How long is he in the air?

50. A ball thrown vertically upward is caught by the thrower after 20.0 s. Find (a) the initial velocity of the ball and (b) the maximum height it reaches.

51. A ball is thrown vertically upward from the ground with an initial speed of 15.0 m/s. (a) How long does it take the ball to reach its maximum altitude? (b) What is its maximum altitude? (c) Determine the velocity and acceleration of the ball at $t = 2.00$ s.

52. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

(Optional)
2.7 Kinematic Equations Derived from Calculus

53. Automotive engineers refer to the time rate of change of acceleration as the "jerk." If an object moves in one dimension such that its jerk J is constant, (a) determine expressions for its acceleration a_x, velocity v_x, and position x, given that its initial acceleration, speed, and position are a_{xi}, v_{xi}, and x_i, respectively. (b) Show that $a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})$.

54. The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by the expression $v = (-5.0 \times 10^7)t^2 + (3.0 \times 10^5)t$, where v is in meters per second and t is in seconds. The acceleration of the bullet just as it leaves the barrel is zero. (a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (b) Determine the length of time the bullet is accelerated. (c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

55. The acceleration of a marble in a certain fluid is proportional to the speed of the marble squared and is given (in SI units) by $a = -3.00v^2$ for $v > 0$. If the marble enters this fluid with a speed of 1.50 m/s, how long will it take before the marble's speed is reduced to half of its initial value?

ADDITIONAL PROBLEMS

56. A motorist is traveling at 18.0 m/s when he sees a deer in the road 38.0 m ahead. (a) If the maximum negative acceleration of the vehicle is -4.50 m/s^2, what is the maximum reaction time Δt of the motorist that will allow him to avoid hitting the deer? (b) If his reaction time is actually 0.300 s, how fast will he be traveling when he hits the deer?

57. Another scheme to catch the roadrunner has failed. A safe falls from rest from the top of a 25.0-m-high cliff toward Wile E. Coyote, who is standing at the base. Wile first notices the safe after it has fallen 15.0 m. How long does he have to get out of the way?

58. A dog's hair has been cut and is now getting longer by 1.04 mm each day. With winter coming on, this rate of hair growth is steadily increasing by 0.132 mm/day every week. By how much will the dog's hair grow during five weeks?

59. A test rocket is fired vertically upward from a well. A catapult gives it an initial velocity of 80.0 m/s at ground level. Subsequently, its engines fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1000 m. At that point its engines fail, and the rocket goes into free fall, with an acceleration of -9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (*Hint:* Consider the motion while the engine is operating separate from the free-fall motion.)

60. A motorist drives along a straight road at a constant speed of 15.0 m/s. Just as she passes a parked motorcycle police officer, the officer starts to accelerate at 2.00 m/s^2 to overtake her. Assuming the officer maintains this acceleration, (a) determine the time it takes the police officer to reach the motorist. Also find (b) the speed and (c) the total displacement of the officer as he overtakes the motorist.

61. In Figure 2.10a, the area under the velocity–time curve between the vertical axis and time t (vertical dashed line) represents the displacement. As shown, this area consists of a rectangle and a triangle. Compute their areas and compare the sum of the two areas with the expression on the righthand side of Equation 2.11.

62. A commuter train travels between two downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. The engineer minimizes the time t between the two stations by accelerating at a rate $a_1 = 0.100$ m/s^2 for a time t_1 and then by braking with acceleration $a_2 = -0.500$ m/s^2 for a time t_2. Find the minimum time of travel t and the time t_1.

63. In a 100-m race, Maggie and Judy cross the finish line in a dead heat, both taking 10.2 s. Accelerating uniformly, Maggie took 2.00 s and Judy 3.00 s to attain maximum speed, which they maintained for the rest of the race. (a) What was the acceleration of each sprinter? (b) What were their respective maximum speeds? (c) Which sprinter was ahead at the 6.00-s mark, and by how much?

64. A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened. Suppose the maximum depth of the dent is on the order of 1 cm. Compute an order-of-magnitude estimate for the maximum acceleration of the ball while it is in contact with the pavement. State your assumptions, the quantities you estimate, and the values you estimate for them.

65. A teenager has a car that speeds up at 3.00 m/s^2 and slows down at -4.50 m/s^2. On a trip to the store, he accelerates from rest to 12.0 m/s, drives at a constant speed for 5.00 s, and then comes to a momentary stop at an intersection. He then accelerates to 18.0 m/s, drives at a constant speed for 20.0 s, slows down for 2.67 s, continues for 4.00 s at this speed, and then comes to a stop. (a) How long does the trip take? (b) How far has he traveled? (c) What is his average speed for the trip? (d) How long would it take to walk to the store and back if he walks at 1.50 m/s?

66. A rock is dropped from rest into a well. (a) If the sound of the splash is heard 2.40 s later, how far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

67. An inquisitive physics student and mountain climber climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s. (a) How long after release of the first stone do the two stones hit the water? (b) What was the initial velocity of the second stone? (c) What is the velocity of each stone at the instant the two hit the water?

68. A car and train move together along parallel paths at 25.0 m/s, with the car adjacent to the rear of the train. Then, because of a red light, the car undergoes a uniform acceleration of -2.50 m/s^2 and comes to rest. It remains at rest for 45.0 s and then accelerates back to a speed of 25.0 m/s at a rate of 2.50 m/s^2. How far behind the rear of the train is the car when it reaches the speed of 25.0 m/s, assuming that the speed of the train has remained 25.0 m/s?

69. Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by racing with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. If Stan moves with a constant acceleration of 3.50 m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2, find (a) the time it takes Kathy to overtake Stan, (b) the distance she travels before she catches up with him, and (c) the speeds of both cars at the instant she overtakes him.

70. To protect his food from hungry bears, a boy scout raises his food pack with a rope that is thrown over a tree limb at height h above his hands. He walks away from the vertical rope with constant velocity v_{boy}, holding the free end of the rope in his hands (Fig. P2.70).

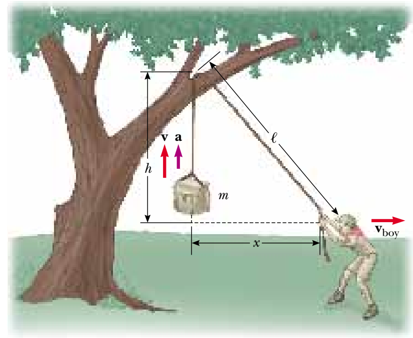


Figure P2.70

(a) Show that the speed v of the food pack is $x(x^2 + h^2)^{-1/2}\, v_{boy}$, where x is the distance he has walked away from the vertical rope. (b) Show that the acceleration a of the food pack is $h^2(x^2 + h^2)^{-3/2}\, v_{boy}{}^2$. (c) What values do the acceleration and velocity have shortly after he leaves the point under the pack ($x = 0$)? (d) What values do the pack's velocity and acceleration approach as the distance x continues to increase?

71. In Problem 70, let the height h equal 6.00 m and the speed v_{boy} equal 2.00 m/s. Assume that the food pack starts from rest. (a) Tabulate and graph the speed–time graph. (b) Tabulate and graph the acceleration–time graph. (Let the range of time be from 0 to 5.00 s and the time intervals be 0.500 s.)

72. Astronauts on a distant planet toss a rock into the air. With the aid of a camera that takes pictures at a steady rate, they record the height of the rock as a function of time as given in Table P2.72. (a) Find the average velocity of the rock in the time interval between each measurement and the next. (b) Using these average velocities to approximate instantaneous velocities at the midpoints of the time intervals, make a graph of velocity as a function of time. Does the rock move with constant acceleration? If so, plot a straight line of best fit on the graph and calculate its slope to find the acceleration.

TABLE P2.72 Height of a Rock versus Time

Time (s)	Height (m)	Time (s)	Height (m)
0.00	5.00	2.75	7.62
0.25	5.75	3.00	7.25
0.50	6.40	3.25	6.77
0.75	6.94	3.50	6.20
1.00	7.38	3.75	5.52
1.25	7.72	4.00	4.73
1.50	7.96	4.25	3.85
1.75	8.10	4.50	2.86
2.00	8.13	4.75	1.77
2.25	8.07	5.00	0.58
2.50	7.90		

73. Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.73. If A slides to the left with a constant speed v, find the speed of B when $\alpha = 60.0°$.

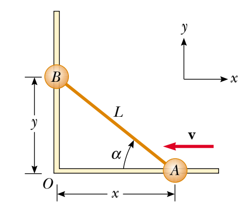


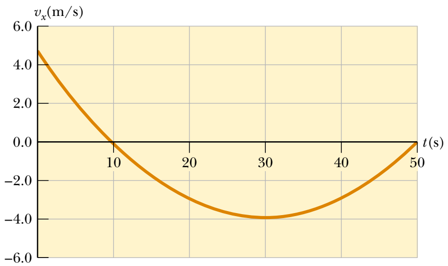
Figure P2.73

Answers to Quick Quizzes

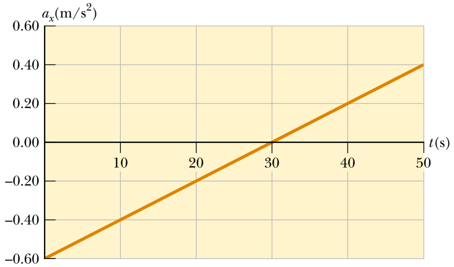
2.1 Your graph should look something like the one in (a). This v_x-t graph shows that the maximum speed is about 5.0 m/s, which is 18 km/h (= 11 mi/h), and so the driver was not speeding. Can you derive the acceleration–time graph from the velocity–time graph? It should look something like the one in (b).

2.2 (a) Yes. This occurs when the car is slowing down, so that the direction of its acceleration is opposite the direction of its motion. (b) Yes. If the motion is in the direction chosen as negative, a positive acceleration causes a decrease in speed.

2.3 The left side represents the final velocity of an object. The first term on the right side is the velocity that the object had initially when we started watching it. The second term is the change in that initial velocity that is caused by the acceleration. If this second term is positive, then the initial velocity has increased ($v_{xf} > v_{xi}$). If this term is negative, then the initial velocity has decreased ($v_{xf} < v_{xi}$).



(a)



(b)

2.4 Graph (a) has a constant slope, indicating a constant acceleration; this is represented by graph (e).

Graph (b) represents a speed that is increasing constantly but not at a uniform rate. Thus, the acceleration must be increasing, and the graph that best indicates this is (d).

Graph (c) depicts a velocity that first increases at a constant rate, indicating constant acceleration. Then the velocity stops increasing and becomes constant, indicating zero acceleration. The best match to this situation is graph (f).

2.5 (c). As can be seen from Figure 2.13b, the ball is at rest for an infinitesimally short time at these three points. Nonetheless, gravity continues to act even though the ball is instantaneously not moving.

PUZZLER

When this honeybee gets back to its hive, it will tell the other bees how to return to the food it has found. By moving in a special, very precisely defined pattern, the bee conveys to other workers the information they need to find a flower bed. Bees communicate by "speaking in vectors." What does the bee have to tell the other bees in order to specify where the flower bed is located relative to the hive? *(E. Webber/Visuals Unlimited)*

chapter

3 Vectors

Chapter Outline

We often need to work with physical quantities that have both numerical and directional properties. As noted in Section 2.1, quantities of this nature are represented by vectors. This chapter is primarily concerned with vector algebra and with some general properties of vector quantities. We discuss the addition and subtraction of vector quantities, together with some common applications to physical situations.

Vector quantities are used throughout this text, and it is therefore imperative that you master both their graphical and their algebraic properties.

3.1 COORDINATE SYSTEMS

Many aspects of physics deal in some form or other with locations in space. In Chapter 2, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. This description is accomplished with the use of coordinates, and in Chapter 2 we used the cartesian coordinate system, in which horizontal and vertical axes intersect at a point taken to be the origin (Fig. 3.1). Cartesian coordinates are also called *rectangular coordinates.*

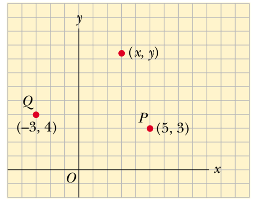


Figure 3.1 Designation of points in a cartesian coordinate system. Every point is labeled with coordinates (x, y).

2.2 Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates* (r, θ), as shown in Figure 3.2a. In this *polar coordinate system,* r is the distance from the origin to the point having cartesian coordinates (x, y), and θ is the angle between r and a fixed axis. This fixed axis is usually the positive x axis, and θ is usually measured counterclockwise from it. From the right triangle in Figure 3.2b, we find that $\sin\theta = y/r$ and that $\cos\theta = x/r$. (A review of trigonometric functions is given in Appendix B.4.) Therefore, starting with the plane polar coordinates of any point, we can obtain the cartesian coordinates, using the equations

$$x = r\cos\theta \tag{3.1}$$

$$y = r\sin\theta \tag{3.2}$$

Furthermore, the definitions of trigonometry tell us that

$$\tan\theta = \frac{y}{x} \tag{3.3}$$

$$r = \sqrt{x^2 + y^2} \tag{3.4}$$

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined, as shown in Figure 3.2a—in other words, when positive θ is an angle measured *counterclockwise* from the positive x axis. (Some scientific calculators perform conversions between cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.

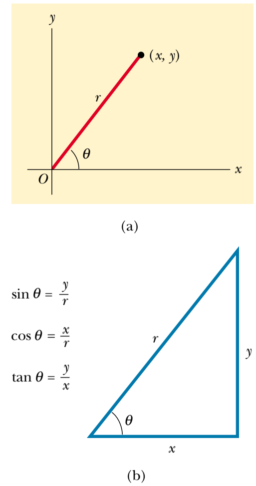


Figure 3.2 (a) The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. (b) The right triangle used to relate (x, y) to (r, θ).

Quick Quiz 3.1

Would the honeybee at the beginning of the chapter use cartesian or polar coordinates when specifying the location of the flower? Why? What is the honeybee using as an origin of coordinates?

You may want to read *Talking Apes and Dancing Bees* (1997) by Betsy Wyckoff.

EXAMPLE 3.1 Polar Coordinates

The cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.

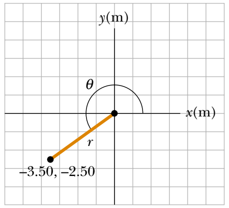


Figure 3.3 Finding polar coordinates when cartesian coordinates are given.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\ \text{m})^2 + (-2.50\ \text{m})^2} = \boxed{4.30\ \text{m}}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.50\ \text{m}}{-3.50\ \text{m}} = 0.714$$

$$\theta = \boxed{216°}$$

Note that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$ and not 35.5°.

3.2 VECTOR AND SCALAR QUANTITIES

2.3 As noted in Chapter 2, some physical quantities are scalar quantities whereas others are vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit "degrees C" or "degrees F." Temperature is therefore an example of a **scalar quantity,** which is defined as a quantity that is completely specified by a number and appropriate units. That is,

> A scalar quantity is specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are getting ready to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is part of the information it gives, velocity is a **vector quantity,** which is defined as a physical quantity that is completely specified by a number and appropriate units plus a direction. That is,

> A vector quantity has both magnitude and direction.

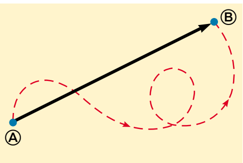


Figure 3.4 As a particle moves from Ⓐ to Ⓑ along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from Ⓐ to Ⓑ.

Another example of a vector quantity is displacement, as you know from Chapter 2. Suppose a particle moves from some point Ⓐ to some point Ⓑ along a straight path, as shown in Figure 3.4. We represent this displacement by drawing an arrow from Ⓐ to Ⓑ, with the tip of the arrow pointing away from the starting point. The direction of the arrowhead represents the direction of the displacement, and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from Ⓐ to Ⓑ, such as the broken line in Figure 3.4, its displacement is still the arrow drawn from Ⓐ to Ⓑ.

(a) (b) (c)

(a) The number of apples in the basket is one example of a scalar quantity. Can you think of other examples? *(Superstock)* (b) Jennifer pointing to the right. A vector quantity is one that must be specified by both magnitude and direction. *(Photo by Ray Serway)* (c) An anemometer is a device meteorologists use in weather forecasting. The cups spin around and reveal the magnitude of the wind velocity. The pointer indicates the direction. *(Courtesy of Peet Bros.Company, 1308 Doris Avenue, Ocean, NJ 07712)*

In this text, we use a boldface letter, such as **A**, to represent a vector quantity. Another common method for vector notation that you should be aware of is the use of an arrow over a letter, such as $\vec{A}$. The magnitude of the vector **A** is written either A or $|\mathbf{A}|$. The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity.

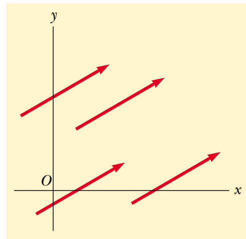


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

3.3 SOME PROPERTIES OF VECTORS

Equality of Two Vectors

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is, **A** = **B** only if $A = B$ *and* if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

Adding Vectors

2.4 The rules for adding vectors are conveniently described by geometric methods. To add vector **B** to vector **A**, first draw vector **A**, with its magnitude represented by a convenient scale, on graph paper and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure 3.6. The **resultant vector R** = **A** + **B** is the vector drawn from the tail of **A** to the tip of **B**. This procedure is known as the **triangle method of addition.**

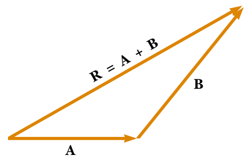


Figure 3.6 When vector **B** is added to vector **A**, the resultant **R** is the vector that runs from the tail of **A** to the tip of **B**.

For example, if you walked 3.0 m toward the east and then 4.0 m toward the north, as shown in Figure 3.7, you would find yourself 5.0 m from where you

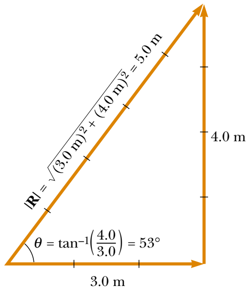


Figure 3.7 Vector addition. Walking first 3.0 m due east and then 4.0 m due north leaves you $|\mathbf{R}|$ = 5.0 m from your starting point.

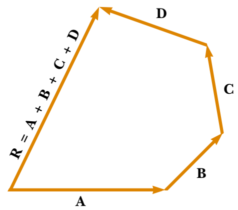


Figure 3.8 Geometric construction for summing four vectors. The resultant vector **R** is by definition the one that completes the polygon.

started, measured at an angle of 53° north of east. Your total displacement is the vector sum of the individual displacements.

A geometric construction can also be used to add more than two vectors. This is shown in Figure 3.8 for the case of four vectors. The resultant vector **R** = **A** + **B** + **C** + **D** is the vector that completes the polygon. In other words, **R is the vector drawn from the tail of the first vector to the tip of the last vector.**

An alternative graphical procedure for adding two vectors, known as the **parallelogram rule of addition,** is shown in Figure 3.9a. In this construction, the tails of the two vectors **A** and **B** are joined together and the resultant vector **R** is the diagonal of a parallelogram formed with **A** and **B** as two of its four sides.

When two vectors are added, the sum is independent of the order of the addition. (This fact may seem trivial, but as you will see in Chapter 11, the order is important when vectors are multiplied). This can be seen from the geometric construction in Figure 3.9b and is known as the **commutative law of addition:**

Commutative law

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \tag{3.5}$$

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule

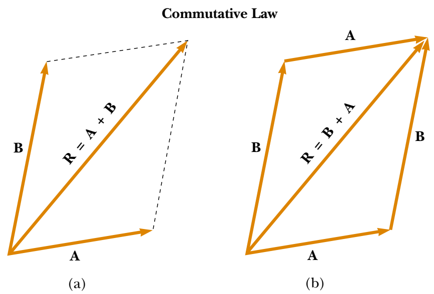


Figure 3.9 (a) In this construction, the resultant **R** is the diagonal of a parallelogram having sides **A** and **B**. (b) This construction shows that **A** + **B** = **B** + **A**—in other words, that vector addition is commutative.

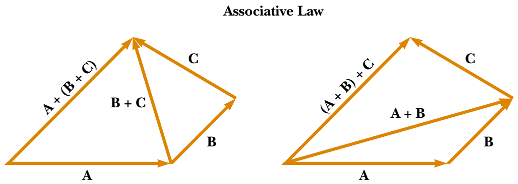


Figure 3.10 Geometric constructions for verifying the associative law of addition.

for three vectors is given in Figure 3.10. This is called the **associative law of addition:**

Associative law

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \tag{3.6}$$

In summary, **a vector quantity has both magnitude and direction and also obeys the laws of vector addition** as described in Figures 3.6 to 3.10. When two or more vectors are added together, *all* of them *must* have the same units. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because they represent different physical quantities. The same rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

Negative of a Vector

The negative of the vector **A** is defined as the vector that when added to **A** gives zero for the vector sum. That is, **A** + (−**A**) = 0. The vectors **A** and −**A** have the same magnitude but point in opposite directions.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation **A** − **B** as vector −**B** added to vector **A**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \tag{3.7}$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 3.11a.

Another way of looking at vector subtraction is to note that the difference **A** − **B** between two vectors **A** and **B** is what you have to add to the second vector to obtain the first. In this case, the vector **A** − **B** points from the tip of the second vector to the tip of the first, as Figure 3.11b shows.

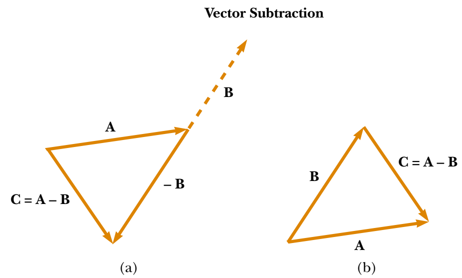


Figure 3.11 (a) This construction shows how to subtract vector **B** from vector **A**. The vector −**B** is equal in magnitude to vector **B** and points in the opposite direction. To subtract **B** from **A**, apply the rule of vector addition to the combination of **A** and −**B**: Draw **A** along some convenient axis, place the tail of −**B** at the tip of **A**, and **C** is the difference **A** − **B**. (b) A second way of looking at vector subtraction. The difference vector **C** = **A** − **B** is the vector that we must add to **B** to obtain **A**.

EXAMPLE 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as shown in Figure 3.12. Find the magnitude and direction of the car's resultant displacement.

Solution In this example, we show two ways to find the resultant of two vectors. We can solve the problem geometrically, using graph paper and a protractor, as shown in Figure 3.12. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) The displacement **R** is the resultant when the two individual displacements **A** and **B** are added.

To solve the problem algebraically, we note that the magnitude of **R** can be obtained from the law of cosines as applied to the triangle (see Appendix B.4). With $\theta = 180° - 60° = 120°$ and $R^2 = A^2 + B^2 - 2AB\cos\theta$, we find that

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$= \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$

$$= 48.2\text{ km}$$

The direction of **R** measured from the northerly direction can be obtained from the law of sines (Appendix B.4):

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = 38.9°$$

The resultant displacement of the car is 48.2 km in a direction 38.9° west of north. This result matches what we found graphically.

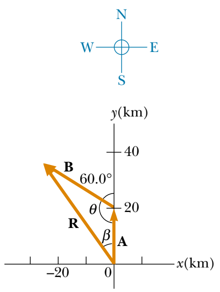


Figure 3.12 Graphical method for finding the resultant displacement vector **R** = **A** + **B**.

Multiplying a Vector by a Scalar

If vector **A** is multiplied by a positive scalar quantity m, then the product m**A** is a vector that has the same direction as **A** and magnitude mA. If vector **A** is multiplied by a negative scalar quantity $-m$, then the product $-m$**A** is directed opposite **A**. For example, the vector 5**A** is five times as long as **A** and points in the same direction as **A**; the vector $-\frac{1}{3}$**A** is one-third the length of **A** and points in the direction opposite **A**.

Quick Quiz 3.2

If vector **B** is added to vector **A**, under what condition does the resultant vector **A** + **B** have magnitude $A + B$? Under what conditions is the resultant vector equal to zero?

3.4 COMPONENTS OF A VECTOR AND UNIT VECTORS

2.5

The geometric method of adding vectors is not recommended whenever great accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the *projections* of vectors along coordinate axes. These projections are called the **components** of the vector. Any vector can be completely described by its components.

Consider a vector **A** lying in the xy plane and making an arbitrary angle θ with the positive x axis, as shown in Figure 3.13. This vector can be expressed as the sum of two other vectors $\mathbf{A}_x$ and $\mathbf{A}_y$. From Figure 3.13, we see that the three vectors form a right triangle and that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$. (If you cannot see why this equality holds, go back to Figure 3.9 and review the parallelogram rule.) We shall often refer to the "components of a vector **A**," written A_x and A_y (without the boldface notation). The component A_x represents the projection of **A** along the x axis, and the component A_y represents the projection of **A** along the y axis. These components can be positive or negative. The component A_x is positive if $\mathbf{A}_x$ points in the positive x direction and is negative if $\mathbf{A}_x$ points in the negative x direction. The same is true for the component A_y.

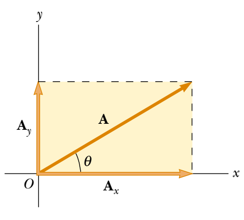


Figure 3.13 Any vector **A** lying in the xy plane can be represented by a vector $\mathbf{A}_x$ lying along the x axis and by a vector $\mathbf{A}_y$ lying along the y axis, where $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$.

From Figure 3.13 and the definition of sine and cosine, we see that $\cos\theta = A_x/A$ and that $\sin\theta = A_y/A$. Hence, the components of **A** are

$$A_x = A\cos\theta \quad (3.8)$$

$$A_y = A\sin\theta \quad (3.9)$$

Components of the vector **A**

These components form two sides of a right triangle with a hypotenuse of length A. Thus, it follows that the magnitude and direction of **A** are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \quad (3.10)$$

Magnitude of **A**

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad (3.11)$$

Direction of **A**

Note that **the signs of the components A_x and A_y depend on the angle θ.** For example, if $\theta = 120°$, then A_x is negative and A_y is positive. If $\theta = 225°$, then both A_x and A_y are negative. Figure 3.14 summarizes the signs of the components when **A** lies in the various quadrants.

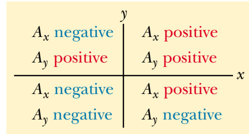


Figure 3.14 The signs of the components of a vector **A** depend on the quadrant in which the vector is located.

When solving problems, you can specify a vector **A** either with its components A_x and A_y or with its magnitude and direction A and θ.

Quick Quiz 3.3

Can the component of a vector ever be greater than the magnitude of the vector?

Suppose you are working a physics problem that requires resolving a vector into its components. In many applications it is convenient to express the components in a coordinate system having axes that are not horizontal and vertical but are still perpendicular to each other. If you choose reference axes or an angle other than the axes and angle shown in Figure 3.13, the components must be modified accordingly. Suppose a vector **B** makes an angle θ' with the x' axis defined in Figure 3.15. The components of **B** along the x' and y' axes are $B_{x'} = B\cos\theta'$ and $B_{y'} = B\sin\theta'$, as specified by Equations 3.8 and 3.9. The magnitude and direction of **B** are obtained from expressions equivalent to Equations 3.10 and 3.11. Thus, we can express the components of a vector in *any* coordinate system that is convenient for a particular situation.

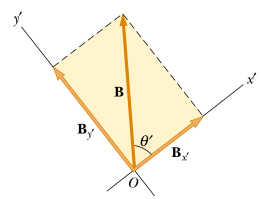


Figure 3.15 The component vectors of **B** in a coordinate system that is tilted.

Unit Vectors

Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols

i, **j**, and **k** to represent unit vectors pointing in the positive *x*, *y*, and *z* directions, respectively. The unit vectors **i**, **j**, and **k** form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure 3.16a. The magnitude of each unit vector equals 1; that is, $|\mathbf{i}| = |\mathbf{j}| = |\mathbf{k}| = 1$.

Consider a vector **A** lying in the *xy* plane, as shown in Figure 3.16b. The product of the component A_x and the unit vector **i** is the vector $A_x\mathbf{i}$, which lies on the *x* axis and has magnitude $|A_x|$. (The vector $A_x\mathbf{i}$ is an alternative representation of vector $\mathbf{A}_x$.) Likewise, $A_y\mathbf{j}$ is a vector of magnitude $|A_y|$ lying on the *y* axis. (Again, vector $A_y\mathbf{j}$ is an alternative representation of vector $\mathbf{A}_y$.) Thus, the unit–vector notation for the vector **A** is

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} \tag{3.12}$$

For example, consider a point lying in the *xy* plane and having cartesian coordinates (x, y), as in Figure 3.17. The point can be specified by the **position vector r**, which in unit–vector form is given by

Position vector

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \tag{3.13}$$

This notation tells us that the components of **r** are the lengths *x* and *y*.

Now let us see how to use components to add vectors when the geometric method is not sufficiently accurate. Suppose we wish to add vector **B** to vector **A**, where vector **B** has components B_x and B_y. All we do is add the *x* and *y* components separately. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is therefore

$$\mathbf{R} = (A_x\mathbf{i} + A_y\mathbf{j}) + (B_x\mathbf{i} + B_y\mathbf{j})$$

or

$$\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} \tag{3.14}$$

Because $\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$, we see that the components of the resultant vector are

$$R_x = A_x + B_x \qquad R_y = A_y + B_y \tag{3.15}$$

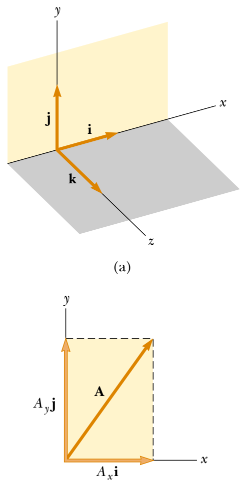


Figure 3.16 (a) The unit vectors **i**, **j**, and **k** are directed along the *x*, *y*, and *z* axes, respectively. (b) Vector $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$ lying in the *xy* plane has components A_x and A_y.

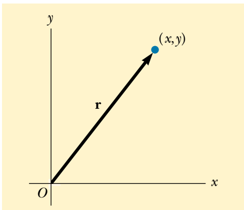


Figure 3.17 The point whose cartesian coordinates are (x, y) can be represented by the position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$.

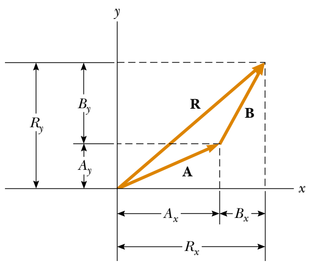


Figure 3.18 This geometric construction for the sum of two vectors shows the relationship between the components of the resultant **R** and the components of the individual vectors.

We obtain the magnitude of **R** and the angle it makes with the *x* axis from its components, using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \tag{3.16}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \tag{3.17}$$

We can check this addition by components with a geometric construction, as shown in Figure 3.18. Remember that you must note the *signs* of the components when using either the algebraic or the geometric method.

At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If **A** and **B** both have *x*, *y*, and *z* components, we express them in the form

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \tag{3.18}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} \tag{3.19}$$

The sum of **A** and **B** is

$$\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k} \tag{3.20}$$

Note that Equation 3.20 differs from Equation 3.14: in Equation 3.20, the resultant vector also has a *z* component $R_z = A_z + B_z$.

QuickLab

Write an expression for the vector describing the displacement of a fly that moves from one corner of the floor of the room that you are in to the opposite corner of the room, near the ceiling.

Quick Quiz 3.4

If one component of a vector is not zero, can the magnitude of the vector be zero? Explain.

Quick Quiz 3.5

If $\mathbf{A} + \mathbf{B} = 0$, what can you say about the components of the two vectors?

Problem-Solving Hints

Adding Vectors

When you need to add two or more vectors, use this step-by-step procedure:

- Select a coordinate system that is convenient. (Try to reduce the number of components you need to find by choosing axes that line up with as many vectors as possible.)
- Draw a labeled sketch of the vectors described in the problem.
- Find the *x* and *y* components of all vectors and the resultant components (the algebraic sum of the components) in the *x* and *y* directions.
- If necessary, use the Pythagorean theorem to find the magnitude of the resultant vector and select a suitable trigonometric function to find the angle that the resultant vector makes with the *x* axis.

EXAMPLE 3.3 The Sum of Two Vectors

Find the sum of two vectors **A** and **B** lying in the *xy* plane and given by

$$\mathbf{A} = (2.0\mathbf{i} + 2.0\mathbf{j})\text{ m} \quad \text{and} \quad \mathbf{B} = (2.0\mathbf{i} - 4.0\mathbf{j})\text{ m}$$

Solution Comparing this expression for **A** with the general expression $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$, we see that $A_x = 2.0$ m and that $A_y = 2.0$ m. Likewise, $B_x = 2.0$ m and $B_y = -4.0$ m. We obtain the resultant vector **R**, using Equation 3.14:

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\mathbf{i}\text{ m} + (2.0 - 4.0)\mathbf{j}\text{ m}$$
$$= (4.0\mathbf{i} - 2.0\mathbf{j})\text{ m}$$

or

$$R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$$

The magnitude of **R** is given by Equation 3.16:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0\text{ m})^2 + (-2.0\text{ m})^2} = \sqrt{20}\text{ m}$$
$$= 4.5\text{ m}$$

We can find the direction of **R** from Equation 3.17:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\text{ m}}{4.0\text{ m}} = -0.50$$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the *x* axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\theta = 333°$.

EXAMPLE 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\mathbf{d}_1 = (15\mathbf{i} + 30\mathbf{j} + 12\mathbf{k})$ cm, $\mathbf{d}_2 = (23\mathbf{i} - 14\mathbf{j} - 5.0\mathbf{k})$ cm, and $\mathbf{d}_3 = (-13\mathbf{i} + 15\mathbf{j})$ cm. Find the components of the resultant displacement and its magnitude.

Solution Rather than looking at a sketch on flat paper, visualize the problem as follows: Start with your fingertip at the front left corner of your horizontal desktop. Move your fingertip 15 cm to the right, then 30 cm toward the far side of the desk, then 12 cm vertically upward, then 23 cm to the right, then 14 cm horizontally toward the front edge of the desk, then 5.0 cm vertically toward the desk, then 13 cm to the left, and (finally!) 15 cm toward the back of the desk. The mathematical calculation keeps track of this motion along the three perpendicular axes:

$$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$$
$$= (15 + 23 - 13)\mathbf{i}\text{ cm} + (30 - 14 + 15)\mathbf{j}\text{ cm}$$
$$+ (12 - 5.0 + 0)\mathbf{k}\text{ cm}$$
$$= (25\mathbf{i} + 31\mathbf{j} + 7.0\mathbf{k})\text{ cm}$$

The resultant displacement has components $R_x = 25$ cm, $R_y = 31$ cm, and $R_z = 7.0$ cm. Its magnitude is

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$= \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = 40\text{ cm}$$

EXAMPLE 3.5 Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower. (a) Determine the components of the hiker's displacement for each day.

Solution If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.19. Displacement **A** has a magnitude of 25.0 km and is directed 45.0° below the positive *x* axis. From Equations 3.8 and 3.9, its components are

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = -(25.0\text{ km})(0.707) = -17.7\text{ km}$$

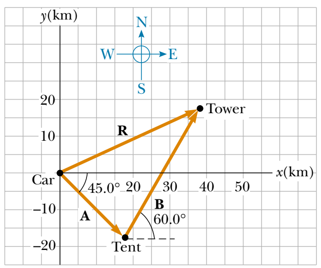


Figure 3.19 The total displacement of the hiker is the vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

The negative value of A_y indicates that the hiker walks in the negative *y* direction on the first day. The signs of A_x and A_y also are evident from Figure 3.19.

The second displacement **B** has a magnitude of 40.0 km and is 60.0° north of east. Its components are

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = 20.0\text{ km}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = 34.6\text{ km}$$

(b) Determine the components of the hiker's resultant displacement **R** for the trip. Find an expression for **R** in terms of unit vectors.

Solution The resultant displacement for the trip $\mathbf{R} = \mathbf{A} + \mathbf{B}$ has components given by Equation 3.15:

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = 37.7\text{ km}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = 16.9\text{ km}$$

In unit–vector form, we can write the total displacement as

$$\mathbf{R} = (37.7\mathbf{i} + 16.9\mathbf{j})\text{ km}$$

Exercise Determine the magnitude and direction of the total displacement.

Answer 41.3 km, 24.1° north of east from the car.

EXAMPLE 3.6 Let's Fly Away!

A commuter airplane takes the route shown in Figure 3.20. First, it flies from the origin of the coordinate system shown to city A, located 175 km in a direction 30.0° north of east. Next, it flies 153 km 20.0° west of north to city B. Finally, it flies 195 km due west to city C. Find the location of city C relative to the origin.

Solution It is convenient to choose the coordinate system shown in Figure 3.20, where the *x* axis points to the east and the *y* axis points to the north. Let us denote the three consecutive displacements by the vectors **a**, **b**, and **c**. Displacement **a** has a magnitude of 175 km and the components

$$a_x = a\cos(30.0°) = (175\text{ km})(0.866) = 152\text{ km}$$
$$a_y = a\sin(30.0°) = (175\text{ km})(0.500) = 87.5\text{ km}$$

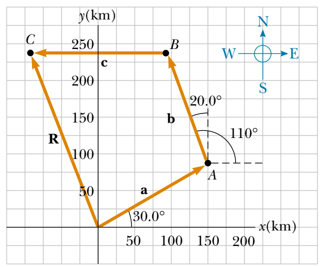


Figure 3.20 The airplane starts at the origin, flies first to city A, then to city B, and finally to city C.

Displacement **b**, whose magnitude is 153 km, has the components

$$b_x = b\cos(110°) = (153\text{ km})(-0.342) = -52.3\text{ km}$$
$$b_y = b\sin(110°) = (153\text{ km})(0.940) = 144\text{ km}$$

Finally, displacement **c**, whose magnitude is 195 km, has the components

$$c_x = c\cos(180°) = (195\text{ km})(-1) = -195\text{ km}$$
$$c_y = c\sin(180°) = 0$$

Therefore, the components of the position vector **R** from the starting point to city C are

$$R_x = a_x + b_x + c_x = 152\text{ km} - 52.3\text{ km} - 195\text{ km}$$
$$= -95.3\text{ km}$$

$$R_y = a_y + b_y + c_y = 87.5\text{ km} + 144\text{ km} + 0$$
$$= 232\text{ km}$$

In unit–vector notation, $\mathbf{R} = (-95.3\mathbf{i} + 232\mathbf{j})$ km. That is, the airplane can reach city C from the starting point by first traveling 95.3 km due west and then by traveling 232 km due north.

Exercise Find the magnitude and direction of **R**.

Answer 251 km, 22.3° west of north.

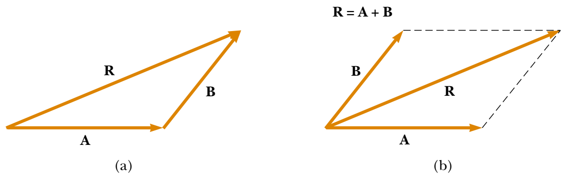


Figure 3.21 (a) Vector addition by the triangle method. (b) Vector addition by the parallelogram rule.

Summary

Scalar quantities are those that have only magnitude and no associated direction. **Vector quantities** have both magnitude and direction and obey the laws of vector addition.

We can add two vectors **A** and **B** graphically, using either the triangle method or the parallelogram rule. In the triangle method (Fig. 3.21a), the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ runs from the tail of **A** to the tip of **B**. In the parallelogram method (Fig. 3.21b), **R** is the diagonal of a parallelogram having **A** and **B** as two of its sides. You should be able to add or subtract vectors, using these graphical methods.

The x component A_x of the vector **A** is equal to the projection of **A** along the x axis of a coordinate system, as shown in Figure 3.22, where $A_x = A\cos\theta$. The y component A_y of **A** is the projection of **A** along the y axis, where $A_y = A\sin\theta$. Be sure you can determine which trigonometric functions you should use in all situations, especially when θ is defined as something other than the counterclockwise angle from the positive x axis.

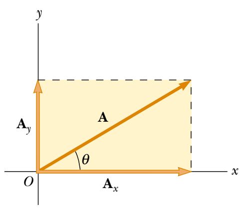


Figure 3.22 The addition of the two vectors $\mathbf{A}_x$ and $\mathbf{A}_y$ gives vector **A**. Note that $\mathbf{A}_x = A_x\mathbf{i}$ and $\mathbf{A}_y = A_y\mathbf{j}$, where A_x and A_y are the *components* of vector **A**.

If a vector **A** has an x component A_x and a y component A_y, the vector can be expressed in unit–vector form as $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$. In this notation, **i** is a unit vector pointing in the positive x direction, and **j** is a unit vector pointing in the positive y direction. Because **i** and **j** are unit vectors, $|\mathbf{i}| = |\mathbf{j}| = 1$.

We can find the resultant of two or more vectors by resolving all vectors into their x and y components, adding their resultant x and y components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. We can find the angle that the resultant vector makes with respect to the x axis by using a suitable trigonometric function.

Questions

1. Two vectors have unequal magnitudes. Can their sum be zero? Explain.
2. Can the magnitude of a particle's displacement be greater than the distance traveled? Explain.
3. The magnitudes of two vectors **A** and **B** are $A = 5$ units and $B = 2$ units. Find the largest and smallest values possible for the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$.
4. Vector **A** lies in the xy plane. For what orientations of vector **A** will both of its components be negative? For what orientations will its components have opposite signs?
5. If the component of vector **A** along the direction of vector **B** is zero, what can you conclude about these two vectors?
6. Can the magnitude of a vector have a negative value? Explain.
7. Which of the following are vectors and which are not: force, temperature, volume, ratings of a television show, height, velocity, age?
8. Under what circumstances would a nonzero vector lying in the xy plane ever have components that are equal in magnitude?
9. Is it possible to add a vector quantity to a scalar quantity? Explain.

Problems

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 3.1 Coordinate Systems

WEB 1. The polar coordinates of a point are $r = 5.50$ m and $\theta = 240°$. What are the cartesian coordinates of this point?

2. Two points in the xy plane have cartesian coordinates $(2.00, -4.00)$ m and $(-3.00, 3.00)$ m. Determine (a) the distance between these points and (b) their polar coordinates.

3. If the cartesian coordinates of a point are given by $(2, y)$ and its polar coordinates are $(r, 30°)$, determine y and r.

4. Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m, 120.0°). Determine (a) the cartesian coordinates of these points and (b) the distance between them.

5. A fly lands on one wall of a room. The lower left-hand corner of the wall is selected as the origin of a two-dimensional cartesian coordinate system. If the fly is located at the point having coordinates (2.00, 1.00) m, (a) how far is it from the corner of the room? (b) what is its location in polar coordinates?

6. If the polar coordinates of the point (x, y) are (r, θ), determine the polar coordinates for the points (a) $(-x, y)$, (b) $(-2x, -2y)$, and (c) $(3x, -3y)$.

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

7. An airplane flies 200 km due west from city A to city B and then 300 km in the direction 30.0° north of west from city B to city C. (a) In straight-line distance, how far is city C from city A? (b) Relative to city A, in what direction is city C?

8. A pedestrian moves 6.00 km east and then 13.0 km north. Using the graphical method, find the magnitude and direction of the resultant displacement vector.

9. A surveyor measures the distance across a straight river by the following method: Starting directly across from a tree on the opposite bank, she walks 100 m along the riverbank to establish a baseline. Then she sights across to the tree. The angle from her baseline to the tree is 35.0°. How wide is the river?

10. A plane flies from base camp to lake A, a distance of 280 km at a direction 20.0° north of east. After dropping off supplies, it flies to lake B, which is 190 km and 30.0° west of north from lake A. Graphically determine the distance and direction from lake B to the base camp.

11. Vector **A** has a magnitude of 8.00 units and makes an angle of 45.0° with the positive x axis. Vector **B** also has a magnitude of 8.00 units and is directed along the negative x axis. Using graphical methods, find (a) the vector sum $\mathbf{A} + \mathbf{B}$ and (b) the vector difference $\mathbf{A} - \mathbf{B}$.

12. A force $\mathbf{F}_1$ of magnitude 6.00 units acts at the origin in a direction 30.0° above the positive x axis. A second force $\mathbf{F}_2$ of magnitude 5.00 units acts at the origin in the direction of the positive y axis. Find graphically the magnitude and direction of the resultant force $\mathbf{F}_1 + \mathbf{F}_2$.

WEB 13. A person walks along a circular path of radius 5.00 m. If the person walks around one half of the circle, find (a) the magnitude of the displacement vector and (b) how far the person walked. (c) What is the magnitude of the displacement if the person walks all the way around the circle?

14. A dog searching for a bone walks 3.50 m south, then 8.20 m at an angle 30.0° north of east, and finally 15.0 m west. Using graphical techniques, find the dog's resultant displacement vector.

WEB 15. Each of the displacement vectors **A** and **B** shown in Figure P3.15 has a magnitude of 3.00 m. Find graphically (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $\mathbf{B} - \mathbf{A}$, (d) $\mathbf{A} - 2\mathbf{B}$. Report all angles counterclockwise from the positive x axis.

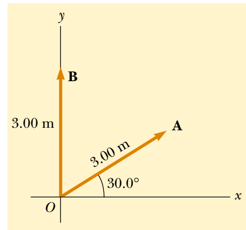


Figure P3.15 Problems 15 and 39.

16. Arbitrarily define the "instantaneous vector height" of a person as the displacement vector from the point halfway between the feet to the top of the head. Make an order-of-magnitude estimate of the total vector height of all the people in a city of population 100 000 (a) at 10 a.m. on a Tuesday and (b) at 5 a.m. on a Saturday. Explain your reasoning.

17. A roller coaster moves 200 ft horizontally and then rises 135 ft at an angle of 30.0° above the horizontal. It then travels 135 ft at an angle of 40.0° downward. What is its displacement from its starting point? Use graphical techniques.

18. The driver of a car drives 3.00 km north, 2.00 km northeast (45.0° east of north), 4.00 km west, and then

3.00 km southeast (45.0° east of south). Where does he end up relative to his starting point? Work out your answer graphically. Check by using components. (The car is not near the North Pole or the South Pole.)

19. Fox Mulder is trapped in a maze. To find his way out, he walks 10.0 m, makes a 90.0° right turn, walks 5.00 m, makes another 90.0° right turn, and walks 7.00 m. What is his displacement from his initial position?

Section 3.4 Components of a Vector and Unit Vectors

20. Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building following the path shown in Figure P3.20.

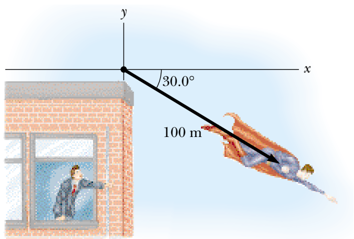


Figure P3.20

21. A person walks 25.0° north of east for 3.10 km. How far would she have to walk due north and due east to arrive at the same location?

22. While exploring a cave, a spelunker starts at the entrance and moves the following distances: She goes 75.0 m north, 250 m east, 125 m at an angle 30.0° north of east, and 150 m south. Find the resultant displacement from the cave entrance.

23. In the assembly operation illustrated in Figure P3.23, a robot first lifts an object upward along an arc that forms one quarter of a circle having a radius of 4.80 cm and lying in an east–west vertical plane. The robot then moves the object upward along a second arc that forms one quarter of a circle having a radius of 3.70 cm and lying in a north–south vertical plane. Find (a) the magnitude of the total displacement of the object and (b) the angle the total displacement makes with the vertical.

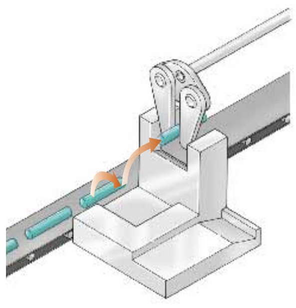


Figure P3.23

24. Vector **B** has x, y, and z components of 4.00, 6.00, and 3.00 units, respectively. Calculate the magnitude of **B** and the angles that **B** makes with the coordinate axes.

WEB **25.** A vector has an x component of -25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.

26. A map suggests that Atlanta is 730 mi in a direction 5.00° north of east from Dallas. The same map shows that Chicago is 560 mi in a direction 21.0° west of north from Atlanta. Assuming that the Earth is flat, use this information to find the displacement from Dallas to Chicago.

27. A displacement vector lying in the xy plane has a magnitude of 50.0 m and is directed at an angle of 120° to the positive x axis. Find the x and y components of this vector and express the vector in unit–vector notation.

28. If $\mathbf{A} = 2.00\mathbf{i} + 6.00\mathbf{j}$ and $\mathbf{B} = 3.00\mathbf{i} - 2.00\mathbf{j}$, (a) sketch the vector sum $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and the vector difference $\mathbf{D} = \mathbf{A} - \mathbf{B}$. (b) Find solutions for **C** and **D**, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the $+x$ axis.

29. Find the magnitude and direction of the resultant of three displacements having x and y components (3.00, 2.00) m, $(-5.00, 3.00)$ m, and (6.00, 1.00) m.

30. Vector **A** has x and y components of -8.70 cm and 15.0 cm, respectively; vector **B** has x and y components of 13.2 cm and -6.60 cm, respectively. If $\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$, what are the components of **C**?

31. Consider two vectors $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = -\mathbf{i} - 4\mathbf{j}$. Calculate (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $|\mathbf{A} + \mathbf{B}|$, (d) $|\mathbf{A} - \mathbf{B}|$, (e) the directions of $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$.

32. A boy runs 3.00 blocks north, 4.00 blocks northeast, and 5.00 blocks west. Determine the length and direction of the displacement vector that goes from the starting point to his final position.

33. Obtain expressions in component form for the position vectors having polar coordinates (a) 12.8 m, 150°; (b) 3.30 cm, 60.0°; (c) 22.0 in., 215°.

34. Consider the displacement vectors $\mathbf{A} = (3\mathbf{i} + 3\mathbf{j})$ m, $\mathbf{B} = (\mathbf{i} - 4\mathbf{j})$ m, and $\mathbf{C} = (-2\mathbf{i} + 5\mathbf{j})$ m. Use the component method to determine (a) the magnitude and direction of the vector $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$ and (b) the magnitude and direction of $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C}$.

35. A particle undergoes the following consecutive displacements: 3.50 m south, 8.20 m northeast, and 15.0 m west. What is the resultant displacement?

36. In a game of American football, a quarterback takes the ball from the line of scrimmage, runs backward for 10.0 yards, and then sideways parallel to the line of scrimmage for 15.0 yards. At this point, he throws a forward pass 50.0 yards straight downfield perpendicular to the line of scrimmage. What is the magnitude of the football's resultant displacement?

37. The helicopter view in Figure P3.37 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons.

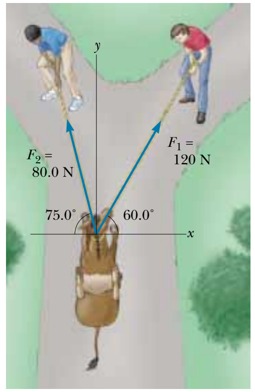


Figure P3.37

38. A novice golfer on the green takes three strokes to sink the ball. The successive displacements are 4.00 m to the north, 2.00 m northeast, and 1.00 m 30.0° west of south. Starting at the same initial point, an expert golfer could make the hole in what single displacement?

39. Find the x and y components of the vectors **A** and **B** shown in Figure P3.15; then derive an expression for the resultant vector $\mathbf{A} + \mathbf{B}$ in unit–vector notation.

40. You are standing on the ground at the origin of a coordinate system. An airplane flies over you with constant velocity parallel to the x axis and at a constant height of 7.60×10^3 m. At $t = 0$, the airplane is directly above you, so that the vector from you to it is given by $\mathbf{P}_0 = (7.60 \times 10^3 \text{ m})\mathbf{j}$. At $t = 30.0$ s, the position vector leading from you to the airplane is $\mathbf{P}_{30} = (8.04 \times 10^3 \text{ m})\mathbf{i} + (7.60 \times 10^3 \text{ m})\mathbf{j}$. Determine the magnitude and orientation of the airplane's position vector at $t = 45.0$ s.

41. A particle undergoes two displacements. The first has a magnitude of 150 cm and makes an angle of 120° with the positive x axis. The *resultant* displacement has a magnitude of 140 cm and is directed at an angle of 35.0° to the positive x axis. Find the magnitude and direction of the second displacement.

42. Vectors **A** and **B** have equal magnitudes of 5.00. If the sum of **A** and **B** is the vector 6.00 **j**, determine the angle between **A** and **B**.

43. The vector **A** has x, y, and z components of 8.00, 12.0, and -4.00 units, respectively. (a) Write a vector expression for **A** in unit–vector notation. (b) Obtain a unit–vector expression for a vector **B** one-fourth the length of **A** pointing in the same direction as **A**. (c) Obtain a unit–vector expression for a vector **C** three times the length of **A** pointing in the direction opposite the direction of **A**.

44. Instructions for finding a buried treasure include the following: Go 75.0 paces at 240°, turn to 135° and walk 125 paces, then travel 100 paces at 160°. The angles are measured counterclockwise from an axis pointing to the east, the $+x$ direction. Determine the resultant displacement from the starting point.

45. Given the displacement vectors $\mathbf{A} = (3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})$ m and $\mathbf{B} = (2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k})$ m, find the magnitudes of the vectors (a) $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and (b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B}$, also expressing each in terms of its x, y, and z components.

46. A radar station locates a sinking ship at range 17.3 km and bearing 136° clockwise from north. From the same station a rescue plane is at horizontal range 19.6 km, 153° clockwise from north, with elevation 2.20 km. (a) Write the vector displacement from plane to ship, letting **i** represent east, **j** north, and **k** up. (b) How far apart are the plane and ship?

47. As it passes over Grand Bahama Island, the eye of a hurricane is moving in a direction 60.0° north of west with a speed of 41.0 km/h. Three hours later, the course of the hurricane suddenly shifts due north and its speed slows to 25.0 km/h. How far from Grand Bahama is the eye 4.50 h after it passes over the island?

48. (a) Vector **E** has magnitude 17.0 cm and is directed 27.0° counterclockwise from the $+x$ axis. Express it in unit–vector notation. (b) Vector **F** has magnitude 17.0 cm and is directed 27.0° counterclockwise from the $+y$ axis. Express it in unit–vector notation. (c) Vector **G** has magnitude 17.0 cm and is directed 27.0° clockwise from the $+y$ axis. Express it in unit–vector notation.

49. Vector **A** has a negative x component 3.00 units in length and a positive y component 2.00 units in length. (a) Determine an expression for **A** in unit–vector notation. (b) Determine the magnitude and direction of **A**. (c) What vector **B**, when added to vector **A**, gives a resultant vector with no x component and a negative y component 4.00 units in length?

50. An airplane starting from airport A flies 300 km east, then 350 km at 30.0° west of north, and then 150 km north to arrive finally at airport B. (a) The next day, another plane flies directly from airport A to airport B in a straight line. In what direction should the pilot travel in this direct flight? (b) How far will the pilot travel in this direct flight? Assume there is no wind during these flights.

WEB **51.** Three vectors are oriented as shown in Figure P3.51, where $|\mathbf{A}| = 20.0$ units, $|\mathbf{B}| = 40.0$ units, and $|\mathbf{C}| = 30.0$ units. Find (a) the x and y components of the resultant vector (expressed in unit–vector notation) and (b) the magnitude and direction of the resultant vector.

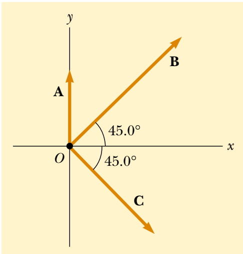


Figure P3.51

52. If $\mathbf{A} = (6.00\mathbf{i} - 8.00\mathbf{j})$ units, $\mathbf{B} = (-8.00\mathbf{i} + 3.00\mathbf{j})$ units, and $\mathbf{C} = (26.0\mathbf{i} + 19.0\mathbf{j})$ units, determine a and b such that $a\mathbf{A} + b\mathbf{B} + \mathbf{C} = 0$.

ADDITIONAL PROBLEMS

53. Two vectors **A** and **B** have precisely equal magnitudes. For the magnitude of $\mathbf{A} + \mathbf{B}$ to be 100 times greater than the magnitude of $\mathbf{A} - \mathbf{B}$, what must be the angle between them?

54. Two vectors **A** and **B** have precisely equal magnitudes. For the magnitude of $\mathbf{A} + \mathbf{B}$ to be greater than the magnitude of $\mathbf{A} - \mathbf{B}$ by the factor n, what must be the angle between them?

55. A vector is given by $\mathbf{R} = 2.00\mathbf{i} + 1.00\mathbf{j} + 3.00\mathbf{k}$. Find (a) the magnitudes of the x, y, and z components, (b) the magnitude of **R**, and (c) the angles between **R** and the x, y, and z axes.

56. Find the sum of these four vector forces: 12.0 N to the right at 35.0° above the horizontal, 31.0 N to the left at 55.0° above the horizontal, 8.40 N to the left at 35.0° below the horizontal, and 24.0 N to the right at 55.0° below the horizontal. (*Hint:* Make a drawing of this situation and select the best axes for x and y so that you have the least number of components. Then add the vectors, using the component method.)

57. A person going for a walk follows the path shown in Figure P3.57. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

58. In general, the instantaneous position of an object is specified by its position vector **P** leading from a fixed origin to the location of the object. Suppose that for a certain object the position vector is a function of time, given by $\mathbf{P} = 4\mathbf{i} + 3\mathbf{j} - 2t\mathbf{j}$, where P is in meters and t is in seconds. Evaluate $d\mathbf{P}/dt$. What does this derivative represent about the object?

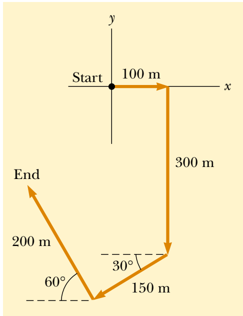


Figure P3.57

59. A jet airliner, moving initially at 300 mi/h to the east, suddenly enters a region where the wind is blowing at 100 mi/h in a direction 30.0° north of east. What are the new speed and direction of the aircraft relative to the ground?

60. A pirate has buried his treasure on an island with five trees located at the following points: A(30.0 m, −20.0 m), B(60.0 m, 80.0 m), C(−10.0 m, −10.0 m), D(40.0 m, −30.0 m), and E(−70.0 m, 60.0 m). All points are measured relative to some origin, as in Figure P3.60. Instructions on the map tell you to start at A and move toward B, but to cover only one-half the distance between A and B. Then, move toward C, covering one-third the distance between your current location and C. Next, move toward D, covering one-fourth the distance between where you are and D. Finally, move toward E, covering one-fifth the distance between you and E, stop, and dig. (a) What are the coordinates of the point where the pirate's treasure is buried? (b) Rearrange the order of the trees, (for instance, B(30.0 m, −20.0 m), A(60.0 m, 80.0 m), E(−10.0 m, −10.0 m), C(40.0 m, −30.0 m), and D(−70.0 m, 60.0 m), and repeat the calculation to show that the answer does not depend on the order of the trees.

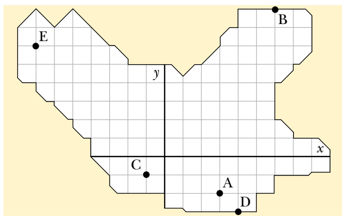


Figure P3.60

61. A rectangular parallelepiped has dimensions a, b, and c, as in Figure P3.61. (a) Obtain a vector expression for the face diagonal vector $\mathbf{R}_1$. What is the magnitude of this vector? (b) Obtain a vector expression for the body diagonal vector $\mathbf{R}_2$. Note that $\mathbf{R}_1$, $c\mathbf{k}$, and $\mathbf{R}_2$ make a right triangle, and prove that the magnitude of $\mathbf{R}_2$ is $\sqrt{a^2 + b^2 + c^2}$.

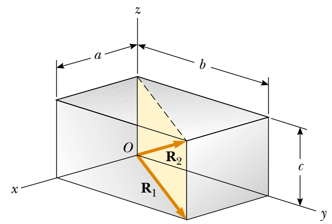


Figure P3.61

62. A point lying in the xy plane and having coordinates (x, y) can be described by the position vector given by $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$. (a) Show that the displacement vector for a particle moving from (x_1, y_1) to (x_2, y_2) is given by $\mathbf{d} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$. (b) Plot the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ and the displacement vector **d**, and verify by the graphical method that $\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$.

63. A point P is described by the coordinates (x, y) with respect to the normal cartesian coordinate system shown in Figure P3.63. Show that (x', y'), the coordinates of this point in the rotated coordinate system, are related to (x, y) and the rotation angle α by the expressions

$$x' = x\cos\alpha + y\sin\alpha$$

$$y' = -x\sin\alpha + y\cos\alpha$$

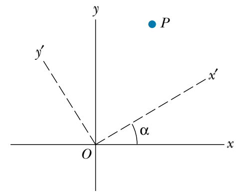


Figure P3.63

ANSWERS TO QUICK QUIZZES

3.1 The honeybee needs to communicate to the other honeybees how far it is to the flower and in what direction they must fly. This is exactly the kind of information that polar coordinates convey, as long as the origin of the coordinates is the beehive.

3.2 The resultant has magnitude $A + B$ when vector **A** is oriented in the same direction as vector **B**. The resultant vector is $\mathbf{A} + \mathbf{B} = 0$ when vector **A** is oriented in the direction opposite vector **B** and $A = B$.

3.3 No. In two dimensions, a vector and its components form a right triangle. The vector is the hypotenuse and must be longer than either side. Problem 61 extends this concept to three dimensions.

3.4 No. The magnitude of a vector **A** is equal to $\sqrt{A_x^2 + A_y^2 + A_z^2}$. Therefore, if any component is nonzero, A cannot be zero. This generalization of the Pythagorean theorem is left for you to prove in Problem 61.

3.5 The fact that $\mathbf{A} + \mathbf{B} = 0$ tells you that $\mathbf{A} = -\mathbf{B}$. Therefore, the components of the two vectors must have opposite signs and equal magnitudes: $A_x = -B_x$, $A_y = -B_y$, and $A_z = -B_z$.

PUZZLER

This airplane is used by NASA for astronaut training. When it flies along a certain curved path, anything inside the plane that is not strapped down begins to float. What causes this strange effect? *(NASA)*

web

For more information on microgravity in general and on this airplane, visit **http://microgravity.msfc.nasa.gov/** and **http://www.jsc.nasa.gov/coop/kc135/kc135.html**

chapter

4 Motion in Two Dimensions

Chapter Outline

In this chapter we deal with the kinematics of a particle moving in two dimensions. Knowing the basics of two-dimensional motion will allow us to examine—in future chapters—a wide variety of motions, ranging from the motion of satellites in orbit to the motion of electrons in a uniform electric field. We begin by studying in greater detail the vector nature of displacement, velocity, and acceleration. As in the case of one-dimensional motion, we derive the kinematic equations for two-dimensional motion from the fundamental definitions of these three quantities. We then treat projectile motion and uniform circular motion as special cases of motion in two dimensions. We also discuss the concept of relative motion, which shows why observers in different frames of reference may measure different displacements, velocities, and accelerations for a given particle.

4.1 THE DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

In Chapter 2 we found that the motion of a particle moving along a straight line is completely known if its position is known as a function of time. Now let us extend this idea to motion in the *xy* plane. We begin by describing the position of a particle by its position vector **r**, drawn from the origin of some coordinate system to the particle located in the *xy* plane, as in Figure 4.1. At time t_i the particle is at point Ⓐ, and at some later time t_f it is at point Ⓑ. The path from Ⓐ to Ⓑ is not necessarily a straight line. As the particle moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$, its position vector changes from $\mathbf{r}_i$ to $\mathbf{r}_f$. As we learned in Chapter 2, displacement is a vector, and the displacement of the particle is the difference between its final position and its initial position. We now formally define the **displacement vector Δr** for the particle of Figure 4.1 as being the difference between its final position vector and its initial position vector:

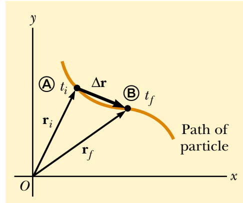


Figure 4.1 A particle moving in the *xy* plane is located with the position vector **r** drawn from the origin to the particle. The displacement of the particle as it moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$ is equal to the vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$.

Displacement vector

$$\Delta\mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i \qquad (4.1)$$

The direction of **Δr** is indicated in Figure 4.1. As we see from the figure, the magnitude of **Δr** is *less* than the distance traveled along the curved path followed by the particle.

As we saw in Chapter 2, it is often useful to quantify motion by looking at the ratio of a displacement divided by the time interval during which that displacement occurred. In two-dimensional (or three-dimensional) kinematics, everything is the same as in one-dimensional kinematics except that we must now use vectors rather than plus and minus signs to indicate the direction of motion.

We define the **average velocity** of a particle during the time interval Δt as the displacement of the particle divided by that time interval:

Average velocity

$$\bar{\mathbf{v}} \equiv \frac{\Delta\mathbf{r}}{\Delta t} \qquad (4.2)$$

Multiplying or dividing a vector quantity by a scalar quantity changes only the magnitude of the vector, not its direction. Because displacement is a vector quantity and the time interval is a scalar quantity, we conclude that the average velocity is a vector quantity directed along **Δr**.

Note that the average velocity between points is *independent of the path* taken. This is because average velocity is proportional to displacement, which depends

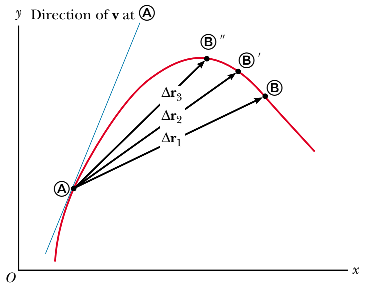


Figure 4.2 As a particle moves between two points, its average velocity is in the direction of the displacement vector $\Delta\mathbf{r}$. As the end point of the path is moved from Ⓑ to Ⓑ′ to Ⓑ″, the respective displacements and corresponding time intervals become smaller and smaller. In the limit that the end point approaches Ⓐ, Δt approaches zero, and the direction of $\Delta\mathbf{r}$ approaches that of the line tangent to the curve at Ⓐ. By definition, the instantaneous velocity at Ⓐ is in the direction of this tangent line.

only on the initial and final position vectors and not on the path taken. As we did with one-dimensional motion, we conclude that if a particle starts its motion at some point and returns to this point via any path, its average velocity is zero for this trip because its displacement is zero.

Consider again the motion of a particle between two points in the xy plane, as shown in Figure 4.2. As the time interval over which we observe the motion becomes smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at Ⓐ.

Instantaneous velocity

The **instantaneous velocity v** is defined as the limit of the average velocity $\Delta\mathbf{r}/\Delta t$ as Δt approaches zero:

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \tag{4.3}$$

That is, the instantaneous velocity equals the derivative of the position vector with respect to time. The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion (Fig. 4.3).

The magnitude of the instantaneous velocity vector $v = |\mathbf{v}|$ is called the *speed*, which, as you should remember, is a scalar quantity.

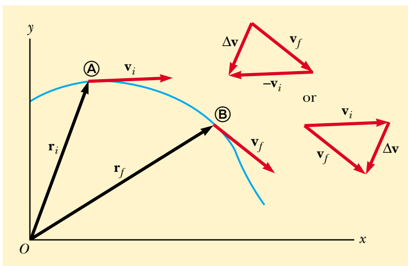


Figure 4.3 A particle moves from position Ⓐ to position Ⓑ. Its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. The vector diagrams at the upper right show two ways of determining the vector $\Delta\mathbf{v}$ from the initial and final velocities.

As a particle moves from one point to another along some path, its instantaneous velocity vector changes from $\mathbf{v}_i$ at time t_i to $\mathbf{v}_f$ at time t_f. Knowing the velocity at these points allows us to determine the average acceleration of the particle:

Average acceleration

The **average acceleration** of a particle as it moves from one position to another is defined as the change in the instantaneous velocity vector $\Delta\mathbf{v}$ divided by the time Δt during which that change occurred:

$$\bar{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t} \tag{4.4}$$

Because it is the ratio of a vector quantity $\Delta\mathbf{v}$ and a scalar quantity Δt, we conclude that average acceleration $\bar{\mathbf{a}}$ is a vector quantity directed along $\Delta\mathbf{v}$. As indicated in Figure 4.3, the direction of $\Delta\mathbf{v}$ is found by adding the vector $-\mathbf{v}_i$ (the negative of $\mathbf{v}_i$) to the vector $\mathbf{v}_f$, because by definition $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$.

When the average acceleration of a particle changes during different time intervals, it is useful to define its instantaneous acceleration **a**:

Instantaneous acceleration

The **instantaneous acceleration a** is defined as the limiting value of the ratio $\Delta\mathbf{v}/\Delta t$ as Δt approaches zero:

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \tag{4.5}$$

3.5 In other words, the instantaneous acceleration equals the derivative of the velocity vector with respect to time.

It is important to recognize that various changes can occur when a particle accelerates. First, the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion. Second, the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (two-dimensional) motion. Finally, both the magnitude and the direction of the velocity vector may change simultaneously.

Quick Quiz 4.1

The gas pedal in an automobile is called the *accelerator.* (a) Are there any other controls in an automobile that can be considered accelerators? (b) When is the gas pedal not an accelerator?

4.2 TWO-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Let us consider two-dimensional motion during which the acceleration remains constant in both magnitude and direction.

The position vector for a particle moving in the xy plane can be written

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \tag{4.6}$$

where x, y, and $\mathbf{r}$ change with time as the particle moves while $\mathbf{i}$ and $\mathbf{j}$ remain constant. If the position vector is known, the velocity of the particle can be obtained from Equations 4.3 and 4.6, which give

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} \tag{4.7}$$

Because $\mathbf{a}$ is assumed constant, its components a_x and a_y also are constants. Therefore, we can apply the equations of kinematics to the x and y components of the velocity vector. Substituting $v_{xf} = v_{xi} + a_x t$ and $v_{yf} = v_{yi} + a_y t$ into Equation 4.7 to determine the final velocity at any time t, we obtain

$$\mathbf{v}_f = (v_{xi} + a_x t)\mathbf{i} + (v_{yi} + a_y t)\mathbf{j}$$
$$= (v_{xi}\mathbf{i} + v_{yi}\mathbf{j}) + (a_x\mathbf{i} + a_y\mathbf{j})t$$

Velocity vector as a function of time

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \textbf{(4.8)}$$

This result states that the velocity of a particle at some time t equals the vector sum of its initial velocity $\mathbf{v}_i$ and the additional velocity $\mathbf{a}t$ acquired in the time t as a result of constant acceleration.

Similarly, from Equation 2.11 we know that the x and y coordinates of a particle moving with constant acceleration are

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

Substituting these expressions into Equation 4.6 (and labeling the final position vector $\mathbf{r}_f$) gives

$$\mathbf{r}_f = (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\mathbf{i} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\mathbf{j}$$
$$= (x_i\mathbf{i} + y_i\mathbf{j}) + (v_{xi}\mathbf{i} + v_{yi}\mathbf{j})t + \tfrac{1}{2}(a_x\mathbf{i} + a_y\mathbf{j})t^2$$

Position vector as a function of time

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \textbf{(4.9)}$$

This equation tells us that the displacement vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$ is the vector sum of a displacement $\mathbf{v}_i t$ arising from the initial velocity of the particle and a displacement $\tfrac{1}{2}\mathbf{a}t^2$ resulting from the uniform acceleration of the particle.

Graphical representations of Equations 4.8 and 4.9 are shown in Figure 4.4. For simplicity in drawing the figure, we have taken $\mathbf{r}_i = 0$ in Figure 4.4a. That is, we assume the particle is at the origin at $t = t_i = 0$. Note from Figure 4.4a that $\mathbf{r}_f$ is generally not along the direction of either $\mathbf{v}_i$ or $\mathbf{a}$ because the relationship between these quantities is a vector expression. For the same reason, from Figure 4.4b we see that $\mathbf{v}_f$ is generally not along the direction of $\mathbf{v}_i$ or $\mathbf{a}$. Finally, note that $\mathbf{v}_f$ and $\mathbf{r}_f$ are generally not in the same direction.

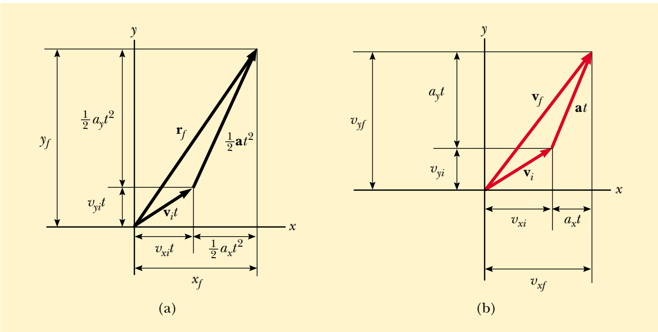


Figure 4.4 Vector representations and components of (a) the displacement and (b) the velocity of a particle moving with a uniform acceleration $\mathbf{a}$. To simplify the drawing, we have set $\mathbf{r}_i = 0$.

Because Equations 4.8 and 4.9 are vector expressions, we may write them in component form:

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases} \qquad \textbf{(4.8a)}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \begin{cases} x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \\ y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \end{cases} \qquad \textbf{(4.9a)}$$

These components are illustrated in Figure 4.4. The component form of the equations for $\mathbf{v}_f$ and $\mathbf{r}_f$ show us that two-dimensional motion at constant acceleration is equivalent to two *independent* motions—one in the x direction and one in the y direction—having constant accelerations a_x and a_y.

EXAMPLE 4.1 Motion in a Plane

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0\ \text{m/s}^2$. (a) Determine the components of the velocity vector at any time and the total velocity vector at any time.

Solution After carefully reading the problem, we realize we can set $v_{xi} = 20$ m/s, $v_{yi} = -15$ m/s, $a_x = 4.0\ \text{m/s}^2$, and $a_y = 0$. This allows us to sketch a rough motion diagram of the situation. The x component of velocity starts at 20 m/s and increases by 4.0 m/s every second. The y component of velocity never changes from its initial value of -15 m/s. From this information we sketch some velocity vectors as shown in Figure 4.5. Note that the spacing between successive images increases as time goes on because the velocity is increasing.

The equations of kinematics give

$$v_{xf} = v_{xi} + a_x t = (20 + 4.0t)\ \text{m/s}$$
$$v_{yf} = v_{yi} + a_y t = -15\ \text{m/s} + 0 = -15\ \text{m/s}$$

Therefore,

$$\mathbf{v}_f = v_{xf}\mathbf{i} + v_{yf}\mathbf{j} = [(20 + 4.0t)\mathbf{i} - 15\mathbf{j}]\ \text{m/s}$$

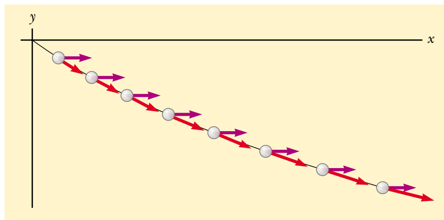


Figure 4.5 Motion diagram for the particle.

We could also obtain this result using Equation 4.8 directly, noting that $\mathbf{a} = 4.0\mathbf{i}\ \text{m/s}^2$ and $\mathbf{v}_i = (20\mathbf{i} - 15\mathbf{j})$ m/s. According to this result, the x component of velocity increases while the y component remains constant; this is consistent with what we predicted. After a long time, the x component will be so great that the y component will be negligible. If we were to extend the object's path in Figure 4.5, eventually it would become nearly parallel to the x axis. It is always helpful to make comparisons between final answers and initial stated conditions.

(b) Calculate the velocity and speed of the particle at $t = 5.0$ s.

Solution With $t = 5.0$ s, the result from part (a) gives

$$\mathbf{v}_f = \{[20 + 4.0(5.0)]\mathbf{i} - 15\mathbf{j}\}\ \text{m/s} = (40\mathbf{i} - 15\mathbf{j})\ \text{m/s}$$

This result tells us that at $t = 5.0$ s, $v_{xf} = 40$ m/s and $v_{yf} = -15$ m/s. Knowing these two components for this two-dimensional motion, we can find both the direction and the magnitude of the velocity vector. To determine the angle θ that $\mathbf{v}$ makes with the x axis at $t = 5.0$ s, we use the fact that $\tan\theta = v_{yf}/v_{xf}$:

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\ \text{m/s}}{40\ \text{m/s}}\right) = -21^\circ$$

where the minus sign indicates an angle of 21° below the positive x axis. The speed is the magnitude of $\mathbf{v}_f$:

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(40)^2 + (-15)^2}\ \text{m/s} = 43\ \text{m/s}$$

In looking over our result, we notice that if we calculate v_i from the x and y components of $\mathbf{v}_i$, we find that $v_f > v_i$. Does this make sense?

(c) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution Because $x_i = y_i = 0$ at $t = 0$, Equation 2.11 gives

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = (20t + 2.0t^2)\ \text{m}$$

$$y_f = v_{yi}t = (-15t)\ \text{m}$$

Therefore, the position vector at any time t is

$$\mathbf{r}_f = x_f\mathbf{i} + y_f\mathbf{j} = [(20t + 2.0t^2)\mathbf{i} - 15t\mathbf{j}]\ \text{m}$$

(Alternatively, we could obtain $\mathbf{r}_f$ by applying Equation 4.9 directly, with $\mathbf{v}_i = (20\mathbf{i} - 15\mathbf{j})$ m/s and $\mathbf{a} = 4.0\mathbf{i}$ m/s². Try it!) Thus, for example, at $t = 5.0$ s, $x = 150$ m, $y = -75$ m, and $\mathbf{r}_f = (150\mathbf{i} - 75\mathbf{j})$ m. The magnitude of the displacement of the particle from the origin at $t = 5.0$ s is the magnitude of $\mathbf{r}_f$ at this time:

$$r_f = |\mathbf{r}_f| = \sqrt{(150)^2 + (-75)^2}\ \text{m} = 170\ \text{m}$$

Note that this is *not* the distance that the particle travels in this time! Can you determine this distance from the available data?

4.3 PROJECTILE MOTION

Assumptions of projectile motion

Anyone who has observed a baseball in motion (or, for that matter, any other object thrown into the air) has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration **g** is constant over the range of motion and is directed downward,[1] and (2) the effect of air resistance is negligible.[2] With these assumptions, we find that the path of a projectile, which we call its *trajectory,* is *always* a parabola. **We use these assumptions throughout this chapter.**

To show that the trajectory of a projectile is a parabola, let us choose our reference frame such that the y direction is vertical and positive is upward. Because air resistance is neglected, we know that $a_y = -g$ (as in one-dimensional free fall) and that $a_x = 0$. Furthermore, let us assume that at $t = 0$, the projectile leaves the origin ($x_i = y_i = 0$) with speed v_i, as shown in Figure 4.6. The vector $\mathbf{v}_i$ makes an angle θ_i with the horizontal, where θ_i is the angle at which the projectile leaves the origin. From the definitions of the cosine and sine functions we have

3.5

$$\cos\theta_i = v_{xi}/v_i \qquad \sin\theta_i = v_{yi}/v_i$$

Therefore, the initial x and y components of velocity are

$$v_{xi} = v_i\cos\theta_i \qquad v_{yi} = v_i\sin\theta_i$$

Substituting the x component into Equation 4.9a with $x_i = 0$ and $a_x = 0$, we find that

Horizontal position component

$$x_f = v_{xi}t = (v_i\cos\theta_i)t \qquad \textbf{(4.10)}$$

Repeating with the y component and using $y_i = 0$ and $a_y = -g$, we obtain

Vertical position component

$$y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = (v_i\sin\theta_i)t - \tfrac{1}{2}gt^2 \qquad \textbf{(4.11)}$$

Next, we solve Equation 4.10 for $t = x_f/(v_i\cos\theta_i)$ and substitute this expression for t into Equation 4.11; this gives

$$y = (\tan\theta_i)x - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x^2 \qquad \textbf{(4.12)}$$

[1] This assumption is reasonable as long as the range of motion is small compared with the radius of the Earth (6.4×10^6 m). In effect, this assumption is equivalent to assuming that the Earth is flat over the range of motion considered.

[2] This assumption is generally *not* justified, especially at high velocities. In addition, any spin imparted to a projectile, such as that applied when a pitcher throws a curve ball, can give rise to some very interesting effects associated with aerodynamic forces, which will be discussed in Chapter 15.

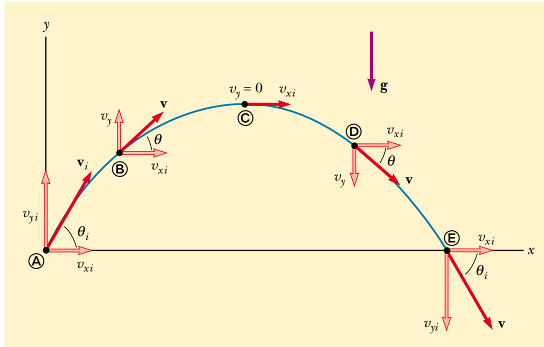


Figure 4.6 The parabolic path of a projectile that leaves the origin with a velocity $\mathbf{v}_i$. The velocity vector **v** changes with time in both magnitude and direction. This change is the result of acceleration in the negative y direction. The x component of velocity remains constant in time because there is no acceleration along the horizontal direction. The y component of velocity is zero at the peak of the path.

A welder cuts holes through a heavy metal construction beam with a hot torch. The sparks generated in the process follow parabolic paths.

This equation is valid for launch angles in the range $0 < \theta_i < \pi/2$. We have left the subscripts off the x and y because the equation is valid for any point (x, y) along the path of the projectile. The equation is of the form $y = ax - bx^2$, which is the equation of a parabola that passes through the origin. Thus, we have shown that the trajectory of a projectile is a parabola. Note that the trajectory is completely specified if both the initial speed v_i and the launch angle θ_i are known.

The vector expression for the position vector of the projectile as a function of time follows directly from Equation 4.9, with $\mathbf{r}_i = 0$ and $\mathbf{a} = \mathbf{g}$:

$$\mathbf{r} = \mathbf{v}_i t + \tfrac{1}{2}\mathbf{g}t^2$$

This expression is plotted in Figure 4.7.

QuickLab

Place two tennis balls at the edge of a tabletop. Sharply snap one ball horizontally off the table with one hand while gently tapping the second ball off with your other hand. Compare how long it takes the two to reach the floor. Explain your results.

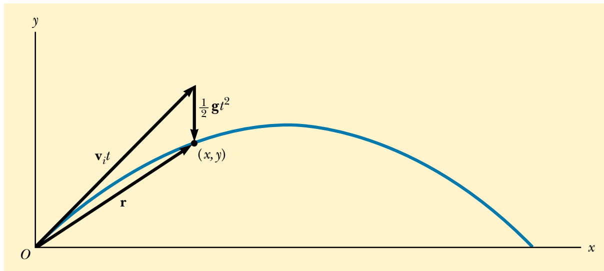


Figure 4.7 The position vector **r** of a projectile whose initial velocity at the origin is $\mathbf{v}_i$. The vector $\mathbf{v}_i t$ would be the displacement of the projectile if gravity were absent, and the vector $\frac{1}{2}\mathbf{g}t^2$ is its vertical displacement due to its downward gravitational acceleration.

Multiflash exposure of a tennis player executing a forehand swing. Note that the ball follows a parabolic path characteristic of a projectile. Such photographs can be used to study the quality of sports equipment and the performance of an athlete.

It is interesting to realize that the motion of a particle can be considered the superposition of the term $\mathbf{v}_i t$, the displacement if no acceleration were present, and the term $\frac{1}{2}\mathbf{g}t^2$, which arises from the acceleration due to gravity. In other words, if there were no gravitational acceleration, the particle would continue to move along a straight path in the direction of $\mathbf{v}_i$. Therefore, the vertical distance $\frac{1}{2}\mathbf{g}t^2$ through which the particle "falls" off the straight-line path is the same distance that a freely falling body would fall during the same time interval. We conclude that **projectile motion is the superposition of two motions:** (1) **constant-velocity motion in the horizontal direction** and (2) **free-fall motion in the vertical direction.** Except for t, the time of flight, the horizontal and vertical components of a projectile's motion are completely independent of each other.

EXAMPLE 4.2 Approximating Projectile Motion

A ball is thrown in such a way that its initial vertical and horizontal components of velocity are 40 m/s and 20 m/s, respectively. Estimate the total time of flight and the distance the ball is from its starting point when it lands.

Solution We start by remembering that the two velocity components are independent of each other. By considering the vertical motion first, we can determine how long the ball remains in the air. Then, we can use the time of flight to estimate the horizontal distance covered.

A motion diagram like Figure 4.8 helps us organize what we know about the problem. The acceleration vectors are all the same, pointing downward with a magnitude of nearly 10 m/s^2. The velocity vectors change direction. Their horizontal components are all the same: 20 m/s. Because the vertical motion is free fall, the vertical components of the velocity vectors change, second by second, from 40 m/s to roughly 30, 20, and 10 m/s in the upward direction, and then to 0 m/s. Subsequently, its velocity becomes 10, 20, 30, and 40 m/s in the downward direction. Thus it takes the ball about 4 s to go up and another 4 s to come back down, for a total time of flight of approximately 8 s. Because the horizontal component of velocity is 20 m/s, and because the ball travels at this speed for 8 s, it ends up approximately 160 m from its starting point.

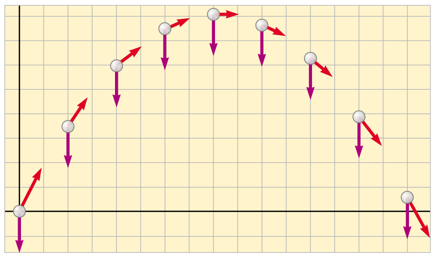


Figure 4.8 Motion diagram for a projectile.

Horizontal Range and Maximum Height of a Projectile

Let us assume that a projectile is fired from the origin at $t_i = 0$ with a positive v_{yi} component, as shown in Figure 4.9. Two points are especially interesting to analyze: the peak point Ⓐ, which has cartesian coordinates $(R/2, h)$, and the point Ⓑ, which has coordinates $(R, 0)$. The distance R is called the *horizontal range* of the projectile, and the distance h is its *maximum height.* Let us find h and R in terms of v_i, θ_i, and g.

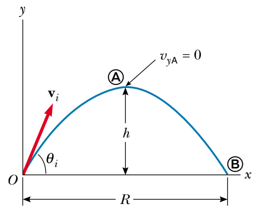


Figure 4.9 A projectile fired from the origin at $t_i = 0$ with an initial velocity $\mathbf{v}_i$. The maximum height of the projectile is h, and the horizontal range is R. At Ⓐ, the peak of the trajectory, the particle has coordinates $(R/2, h)$.

We can determine h by noting that at the peak, $v_{yA} = 0$. Therefore, we can use Equation 4.8a to determine the time t_A it takes the projectile to reach the peak:

$$v_{yf} = v_{yi} + a_y t$$
$$0 = v_i \sin\theta_i - g t_A$$
$$t_A = \frac{v_i \sin\theta_i}{g}$$

Substituting this expression for t_A into the y part of Equation 4.9a and replacing $y_f = y_A$ with h, we obtain an expression for h in terms of the magnitude and direction of the initial velocity vector:

$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g} \qquad \textbf{(4.13)}$$

Maximum height of projectile

The range R is the horizontal distance that the projectile travels in twice the time it takes to reach its peak, that is, in a time $t_B = 2t_A$. Using the x part of Equation 4.9a, noting that $v_{xi} = v_{xB} = v_i \cos\theta_i$, and setting $R \equiv x_B$ at $t = 2t_A$, we find that

$$R = v_{xi} t_B = (v_i \cos\theta_i) 2t_A$$
$$= (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Using the identity $\sin 2\theta = 2 \sin\theta \cos\theta$ (see Appendix B.4), we write R in the more compact form

$$R = \frac{v_i^2 \sin 2\theta_i}{g} \qquad \textbf{(4.14)}$$

Range of projectile

Keep in mind that Equations 4.13 and 4.14 are useful for calculating h and R only if v_i and θ_i are known (which means that only $\mathbf{v}_i$ has to be specified) and if the projectile lands at the same height from which it started, as it does in Figure 4.9.

The maximum value of R from Equation 4.14 is $R_{max} = v_i^2/g$. This result follows from the fact that the maximum value of $\sin 2\theta_i$ is 1, which occurs when $2\theta_i = 90°$. Therefore, R is a maximum when $\theta_i = 45°$.

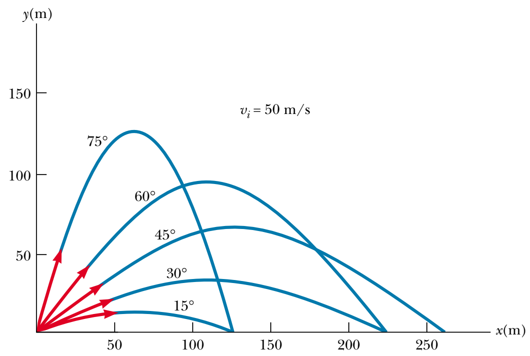


Figure 4.10 A projectile fired from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ_i result in the same value of x (range of the projectile).

QuickLab

To carry out this investigation, you need to be outdoors with a small ball, such as a tennis ball, as well as a wristwatch. Throw the ball straight up as hard as you can and determine the initial speed of your throw and the approximate maximum height of the ball, using only your watch. What happens when you throw the ball at some angle $\theta \neq 90°$? Does this change the time of flight (perhaps because it is easier to throw)? Can you still determine the maximum height and initial speed?

Figure 4.10 illustrates various trajectories for a projectile having a given initial speed but launched at different angles. As you can see, the range is a maximum for $\theta_i = 45°$. In addition, for any θ_i other than 45°, a point having cartesian coordinates $(R, 0)$ can be reached by using either one of two complementary values of θ_i, such as 75° and 15°. Of course, the maximum height and time of flight for one of these values of θ_i are different from the maximum height and time of flight for the complementary value.

Quick Quiz 4.2

As a projectile moves in its parabolic path, is there any point along the path where the velocity and acceleration vectors are (a) perpendicular to each other? (b) parallel to each other? (c) Rank the five paths in Figure 4.10 with respect to time of flight, from the shortest to the longest.

Problem-Solving Hints

Projectile Motion

We suggest that you use the following approach to solving projectile motion problems:

- Select a coordinate system and resolve the initial velocity vector into x and y components.
- Follow the techniques for solving constant-velocity problems to analyze the horizontal motion. Follow the techniques for solving constant-acceleration problems to analyze the vertical motion. The x and y motions share the same time of flight t.

EXAMPLE 4.3 The Long-Jump

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s. (a) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)

Solution Because the initial speed and launch angle are given, the most direct way of solving this problem is to use the range formula given by Equation 4.14. However, it is more instructive to take a more general approach and use Figure 4.9. As before, we set our origin of coordinates at the takeoff point and label the peak as Ⓐ and the landing point as Ⓑ. The horizontal motion is described by Equation 4.10:

$$x_f = x_B = (v_i \cos \theta_i) t_B = (11.0 \text{ m/s})(\cos 20.0°) t_B$$

The value of x_B can be found if the total time of the jump is known. We are able to find t_B by remembering that $a_y = -g$ and by using the y part of Equation 4.8a. We also note that at the top of the jump the vertical component of velocity v_{yA} is zero:

$$v_{yf} = v_{yA} = v_i \sin \theta_i - g t_A$$

$$0 = (11.0 \text{ m/s}) \sin 20.0° - (9.80 \text{ m/s}^2) t_A$$

$$t_A = 0.384 \text{ s}$$

This is the time needed to reach the *top* of the jump. Because of the symmetry of the vertical motion, an identical time interval passes before the jumper returns to the ground. Therefore, the *total time* in the air is $t_B = 2t_A = 0.768$ s. Substituting this value into the above expression for x_f gives

$$x_f = x_B = (11.0 \text{ m/s})(\cos 20.0°)(0.768 \text{ s}) = 7.94 \text{ m}$$

This is a reasonable distance for a world-class athlete.

(b) What is the maximum height reached?

Solution We find the maximum height reached by using Equation 4.11:

$$y_{max} = y_A = (v_i \sin \theta_i) t_A - \tfrac{1}{2} g t_A^2$$

$$= (11.0 \text{ m/s})(\sin 20.0°)(0.384 \text{ s}) - \tfrac{1}{2}(9.80 \text{ m/s}^2)(0.384 \text{ s})^2$$

$$= 0.722 \text{ m}$$

Treating the long-jumper as a particle is an oversimplification. Nevertheless, the values obtained are reasonable.

Exercise To check these calculations, use Equations 4.13 and 4.14 to find the maximum height and horizontal range.

In a long-jump event, 1993 United States champion Mike Powell can leap horizontal distances of at least 8 m.

EXAMPLE 4.4 A Bull's-Eye Every Time

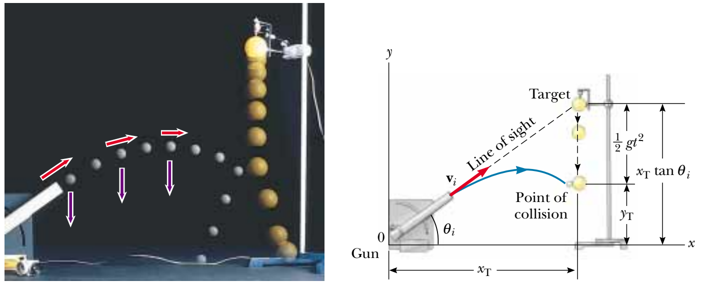
In a popular lecture demonstration, a projectile is fired at a target in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as shown in Figure 4.11. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.

Solution We can argue that a collision results under the conditions stated by noting that, as soon as they are released, the projectile and the target experience the same acceleration $a_y = -g$. First, note from Figure 4.11b that the initial y coordinate of the target is $x_T \tan \theta_i$ and that it falls through a distance $\frac{1}{2}gt^2$ in a time t. Therefore, the y coordinate of the target at any moment after release is

$$y_T = x_T \tan \theta_i - \tfrac{1}{2} g t^2$$

Now if we use Equation 4.9a to write an expression for the y coordinate of the projectile at any moment, we obtain

$$y_P = x_P \tan \theta_i - \tfrac{1}{2} g t^2$$



(a) (b)

Figure 4.11 (a) Multiflash photograph of projectile–target demonstration. If the gun is aimed directly at the target and is fired at the same instant the target begins to fall, the projectile will hit the target. Note that the velocity of the projectile (red arrows) changes in direction and magnitude, while the downward acceleration (violet arrows) remains constant. *(Central Scientific Company.)* (b) Schematic diagram of the projectile–target demonstration. Both projectile and target fall through the same vertical distance in a time t because both experience the same acceleration $a_y = -g$.

Thus, by comparing the two previous equations, we see that when the y coordinates of the projectile and target are the same, their x coordinates are the same and a collision results. That is, when $y_P = y_T$, $x_P = x_T$. You can obtain the same result, using expressions for the position vectors for the projectile and target.

Note that a collision will *not* always take place owing to a further restriction: A collision can result only when $v_i \sin\theta_i \geq \sqrt{gd/2}$, where d is the initial elevation of the target above the *floor*. If $v_i \sin\theta_i$ is less than this value, the projectile will strike the floor before reaching the target.

EXAMPLE 4.5 That's Quite an Arm!

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as shown in Figure 4.12. If the height of the building is 45.0 m, (a) how long is it before the stone hits the ground?

Solution We have indicated the various parameters in Figure 4.12. When working problems on your own, you should always make a sketch such as this and label it.

The initial x and y components of the stone's velocity are

$$v_{xi} = v_i \cos\theta_i = (20.0\text{ m/s})(\cos 30.0°) = 17.3\text{ m/s}$$

$$v_{yi} = v_i \sin\theta_i = (20.0\text{ m/s})(\sin 30.0°) = 10.0\text{ m/s}$$

To find t, we can use $y_f = v_{yi}t + \frac{1}{2}a_yt^2$ (Eq. 4.9a) with $y_f = -45.0$ m, $a_y = -g$, and $v_{yi} = 10.0$ m/s (there is a minus sign on the numerical value of y_f because we have chosen the top of the building as the origin):

$$-45.0\text{ m} = (10.0\text{ m/s})t - \tfrac{1}{2}(9.80\text{ m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t =$ 4.22 s. Does the negative root have any physical meaning? (Can you think of another way of finding t from the information given?)

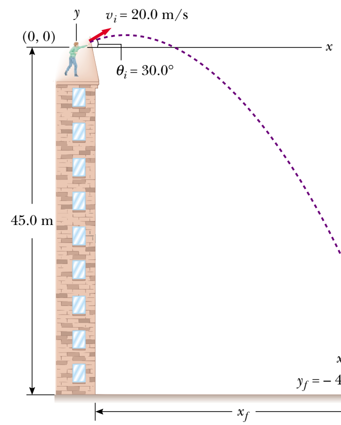


Figure 4.12

(b) What is the speed of the stone just before it strikes the ground?

Solution We can use Equation 4.8a, $v_{yf} = v_{yi} + a_yt$, with $t = 4.22$ s to obtain the y component of the velocity just before the stone strikes the ground:

$$v_{yf} = 10.0\text{ m/s} - (9.80\text{ m/s}^2)(4.22\text{ s}) = -31.4\text{ m/s}$$

The negative sign indicates that the stone is moving downward. Because $v_{xf} = v_{xi} = 17.3$ m/s, the required speed is

$$v_f = \sqrt{{v_{xf}}^2 + {v_{yf}}^2} = \sqrt{(17.3)^2 + (-31.4)^2}\text{ m/s} = 35.9\text{ m/s}$$

Exercise Where does the stone strike the ground?

Answer 73.0 m from the base of the building.

EXAMPLE 4.6 The Stranded Explorers

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in Figure 4.13. If the plane is traveling horizontally at 40.0 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it was released?

Solution For this problem we choose the coordinate system shown in Figure 4.13, in which the origin is at the point of release of the package. Consider first the horizontal motion of the package. The only equation available to us for finding the distance traveled along the horizontal direction is $x_f = v_{xi}t$ (Eq. 4.9a). The initial x component of the package velocity is the same as that of the plane when the package is released: 40.0 m/s. Thus, we have

$$x_f = (40.0\text{ m/s})t$$

If we know t, the length of time the package is in the air, then we can determine x_f, the distance the package travels in the horizontal direction. To find t, we use the equations that describe the vertical motion of the package. We know that at the instant the package hits the ground, its y coordinate is $y_f = -100$ m. We also know that the initial vertical component of the package velocity v_{yi} is zero because at the moment of release, the package had only a horizontal component of velocity.

From Equation 4.9a, we have

$$y_f = -\tfrac{1}{2}gt^2$$

$$-100\text{ m} = -\tfrac{1}{2}(9.80\text{ m/s}^2)t^2$$

$$t = 4.52\text{ s}$$

Substitution of this value for the time of flight into the equation for the x coordinate gives

$$x_f = (40.0\text{ m/s})(4.52\text{ s}) = 181\text{ m}$$

The package hits the ground 181 m to the right of the drop point.

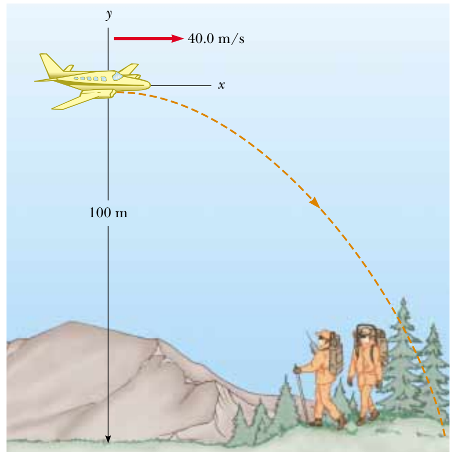


Figure 4.13

Exercise What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

Answer $v_{xf} = 40.0$ m/s; $v_{yf} = -44.3$ m/s.

Exercise Where is the plane when the package hits the ground? (Assume that the plane does not change its speed or course.)

Answer Directly over the package.

EXAMPLE 4.7 The End of the Ski Jump

A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure 4.14. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline?

Solution It is reasonable to expect the skier to be airborne for less than 10 s, and so he will not go farther than 250 m horizontally. We should expect the value of d, the distance traveled along the incline, to be of the same order of magnitude. It is convenient to select the beginning of the jump as the origin ($x_i = 0, y_i = 0$). Because $v_{xi} = 25.0$ m/s and $v_{yi} = 0$, the x and y component forms of Equation 4.9a are

(1) $x_f = v_{xi}t = (25.0 \text{ m/s})t$

(2) $y_f = \frac{1}{2}a_y t^2 = -\frac{1}{2}(9.80 \text{ m/s}^2)t^2$

From the right triangle in Figure 4.14, we see that the jumper's x and y coordinates at the landing point are $x_f = d \cos 35.0°$ and $y_f = -d \sin 35.0°$. Substituting these relationships into (1) and (2), we obtain

(3) $d \cos 35.0° = (25.0 \text{ m/s})t$

(4) $-d \sin 35.0° = -\frac{1}{2}(9.80 \text{ m/s}^2)t^2$

Solving (3) for t and substituting the result into (4), we find that $d = 109$ m. Hence, the x and y coordinates of the point at which he lands are

$$x_f = d \cos 35.0° = (109 \text{ m}) \cos 35.0° = \boxed{89.3 \text{ m}}$$

$$y_f = -d \sin 35.0° = -(109 \text{ m}) \sin 35.0° = \boxed{-62.5 \text{ m}}$$

Exercise Determine how long the jumper is airborne and his vertical component of velocity just before he lands.

Answer 3.57 s; −35.0 m/s.

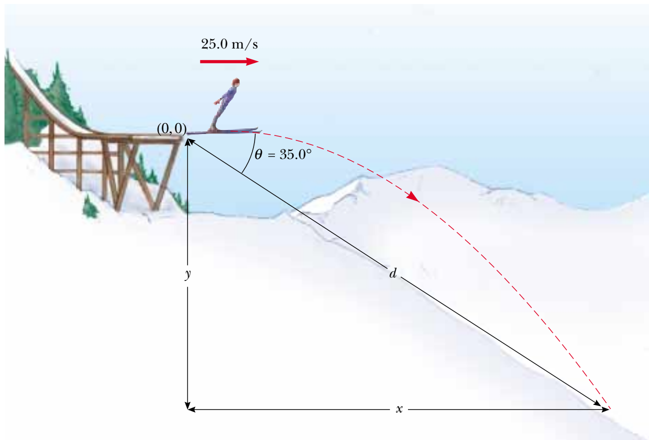


Figure 4.14

What would have occurred if the skier in the last example happened to be carrying a stone and let go of it while in midair? Because the stone has the same initial velocity as the skier, it will stay near him as he moves—that is, it floats alongside him. This is a technique that NASA uses to train astronauts. The plane pictured at the beginning of the chapter flies in the same type of projectile path that the skier and stone follow. The passengers and cargo in the plane fall along-

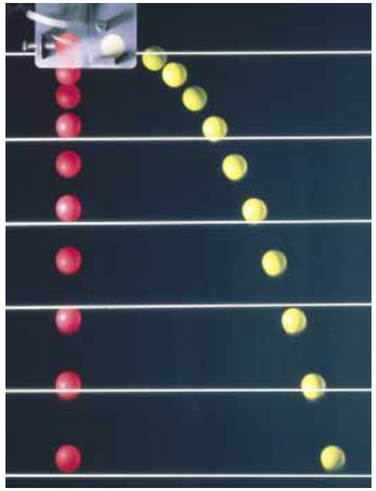


Figure 4.15 This multiflash photograph of two balls released simultaneously illustrates both free fall (red ball) and projectile motion (yellow ball). The yellow ball was projected horizontally, while the red ball was released from rest. *(Richard Megna/Fundamental Photographs)*

QuickLab

Armed with nothing but a ruler and the knowledge that the time between images was 1/30 s, find the horizontal speed of the yellow ball in Figure 4.15. (*Hint:* Start by analyzing the motion of the red ball. Because you know its vertical acceleration, you can calibrate the distances depicted in the photograph. Then you can find the horizontal speed of the yellow ball.)

side each other; that is, they have the same trajectory. An astronaut can release a piece of equipment and it will float freely alongside her hand. The same thing happens in the space shuttle. The craft and everything in it are falling as they orbit the Earth.

4.4 UNIFORM CIRCULAR MOTION

3.6 Figure 4.16a shows a car moving in a circular path with constant linear speed v. Such motion is called **uniform circular motion.** Because the car's direction of motion changes, the car has an acceleration, as we learned in Section 4.1. For any motion, the velocity vector is tangent to the path. Consequently, when an object moves in a circular path, its velocity vector is perpendicular to the radius of the circle.

We now show that the acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An ac-

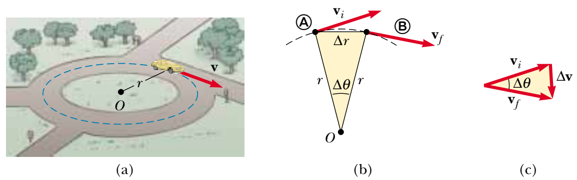


Figure 4.16 (a) A car moving along a circular path at constant speed experiences uniform circular motion. (b) As a particle moves from Ⓐ to Ⓑ, its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. (c) The construction for determining the direction of the change in velocity $\Delta\mathbf{v}$, which is toward the center of the circle for small $\Delta\mathbf{r}$.

celeration of this nature is called a **centripetal** (center-seeking) acceleration, and its magnitude is

$$a_r = \frac{v^2}{r} \tag{4.15}$$

where r is the radius of the circle and the notation a_r is used to indicate that the centripetal acceleration is along the radial direction.

To derive Equation 4.15, consider Figure 4.16b, which shows a particle first at point Ⓐ and then at point Ⓑ. The particle is at Ⓐ at time t_i, and its velocity at that time is $\mathbf{v}_i$. It is at Ⓑ at some later time t_f, and its velocity at that time is $\mathbf{v}_f$. Let us assume here that $\mathbf{v}_i$ and $\mathbf{v}_f$ differ only in direction; their magnitudes (speeds) are the same (that is, $v_i = v_f = v$). To calculate the acceleration of the particle, let us begin with the defining equation for average acceleration (Eq. 4.4):

$$\overline{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

This equation indicates that we must subtract $\mathbf{v}_i$ from $\mathbf{v}_f$, being sure to treat them as vectors, where $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ is the change in the velocity. Because $\mathbf{v}_i + \Delta\mathbf{v} = \mathbf{v}_f$, we can find the vector $\Delta\mathbf{v}$, using the vector triangle in Figure 4.16c.

Now consider the triangle in Figure 4.16b, which has sides Δr and r. This triangle and the one in Figure 4.16c, which has sides Δv and v, are similar. This fact enables us to write a relationship between the lengths of the sides:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

This equation can be solved for Δv and the expression so obtained substituted into $\overline{a} = \Delta v/\Delta t$ (Eq. 4.4) to give

$$\overline{a} = \frac{v\,\Delta r}{r\,\Delta t}$$

Now imagine that points Ⓐ and Ⓑ in Figure 4.16b are extremely close together. In this case $\Delta\mathbf{v}$ points toward the center of the circular path, and because the acceleration is in the direction of $\Delta\mathbf{v}$, it too points toward the center. Furthermore, as Ⓐ and Ⓑ approach each other, Δt approaches zero, and the ratio $\Delta r/\Delta t$ approaches the speed v. Hence, in the limit $\Delta t \rightarrow 0$, the magnitude of the acceleration is

$$a_r = \frac{v^2}{r}$$

Thus, we conclude that in uniform circular motion, the acceleration is directed toward the center of the circle and has a magnitude given by v^2/r, where v is the speed of the particle and r is the radius of the circle. You should be able to show that the dimensions of a_r are L/T^2. We shall return to the discussion of circular motion in Section 6.1.

4.5 TANGENTIAL AND RADIAL ACCELERATION

3.6 Now let us consider a particle moving along a curved path where the velocity changes both in direction and in magnitude, as shown in Figure 4.17. As is always the case, the velocity vector is tangent to the path, but now the direction of the acceleration vector **a** changes from point to point. This vector can be resolved into two component vectors: a radial component vector $\mathbf{a}_r$ and a tangential component vector $\mathbf{a}_t$. Thus, **a** can be written as the vector sum of these component vectors:

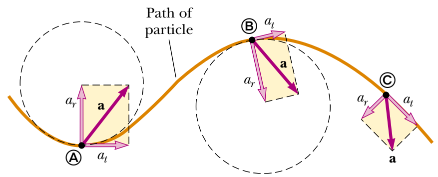


Figure 4.17 The motion of a particle along an arbitrary curved path lying in the xy plane. If the velocity vector **v** (always tangent to the path) changes in direction and magnitude, the component vectors of the acceleration **a** are a tangential component a_t and a radial component a_r.

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \tag{4.16}$$

Total acceleration

The tangential acceleration causes the change in the speed of the particle. It is parallel to the instantaneous velocity, and its magnitude is

$$a_t = \frac{d|\mathbf{v}|}{dt} \tag{4.17}$$

Tangential acceleration

The radial acceleration arises from the change in direction of the velocity vector as described earlier and has an absolute magnitude given by

$$a_r = \frac{v^2}{r} \tag{4.18}$$

Radial acceleration

where r is the radius of curvature of the path at the point in question. Because $\mathbf{a}_r$ and $\mathbf{a}_t$ are mutually perpendicular component vectors of **a**, it follows that $a = \sqrt{a_r^2 + a_t^2}$. As in the case of uniform circular motion, $\mathbf{a}_r$ in nonuniform circular motion always points toward the center of curvature, as shown in Figure 4.17. Also, at a given speed, a_r is large when the radius of curvature is small (as at points Ⓐ and Ⓑ in Figure 4.17) and small when r is large (such as at point Ⓒ). The direction of $\mathbf{a}_t$ is either in the same direction as **v** (if v is increasing) or opposite **v** (if v is decreasing).

In uniform circular motion, where v is constant, $a_t = 0$ and the acceleration is always completely radial, as we described in Section 4.4. (*Note:* Eq. 4.18 is identical to Eq. 4.15.) In other words, uniform circular motion is a special case of motion along a curved path. Furthermore, if the direction of **v** does not change, then there is no radial acceleration and the motion is one-dimensional (in this case, $a_r = 0$, but a_t may not be zero).

Quick Quiz 4.3

(a) Draw a motion diagram showing velocity and acceleration vectors for an object moving with constant speed counterclockwise around a circle. Draw similar diagrams for an object moving counterclockwise around a circle but (b) slowing down at constant tangential acceleration and (c) speeding up at constant tangential acceleration.

It is convenient to write the acceleration of a particle moving in a circular path in terms of unit vectors. We do this by defining the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ shown in

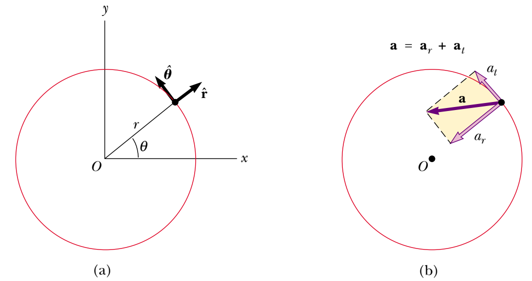


Figure 4.18 (a) Descriptions of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$. (b) The total acceleration **a** of a particle moving along a curved path (which at any instant is part of a circle of radius r) is the sum of radial and tangential components. The radial component is directed toward the center of curvature. If the tangential component of acceleration becomes zero, the particle follows uniform circular motion.

Figure 4.18a, where $\hat{\mathbf{r}}$ is a unit vector lying along the radius vector and directed radially outward from the center of the circle and $\hat{\boldsymbol{\theta}}$ is a unit vector tangent to the circle. The direction of $\hat{\boldsymbol{\theta}}$ is in the direction of increasing θ, where θ is measured counterclockwise from the positive x axis. Note that both $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ "move along with the particle" and so vary in time. Using this notation, we can express the total acceleration as

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r = \frac{d|\mathbf{v}|}{dt}\,\hat{\boldsymbol{\theta}} - \frac{v^2}{r}\,\hat{\mathbf{r}} \qquad \textbf{(4.19)}$$

These vectors are described in Figure 4.18b. The negative sign on the v^2/r term in Equation 4.19 indicates that the radial acceleration is always directed radially inward, *opposite* $\hat{\mathbf{r}}$.

Quick Quiz 4.4

Based on your experience, draw a motion diagram showing the position, velocity, and acceleration vectors for a pendulum that, from an initial position 45° to the right of a central vertical line, swings in an arc that carries it to a final position 45° to the left of the central vertical line. The arc is part of a circle, and you should use the center of this circle as the origin for the position vectors.

EXAMPLE 4.8 The Swinging Ball

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as shown in Figure 4.19. When the string makes an angle $\theta = 20°$ with the vertical, the ball has a speed of 1.5 m/s. (a) Find the magnitude of the radial component of acceleration at this instant.

Solution The diagram from the answer to Quick Quiz 4.4 (p. 109) applies to this situation, and so we have a good idea of how the acceleration vector varies during the motion. Figure 4.19 lets us take a closer look at the situation. The radial acceleration is given by Equation 4.18. With $v = 1.5$ m/s and $r = 0.50$ m, we find that

$$a_r = \frac{v^2}{r} = \frac{(1.5\text{ m/s})^2}{0.50\text{ m}} = \boxed{4.5\text{ m/s}^2}$$

(b) What is the magnitude of the tangential acceleration when $\theta = 20°$?

Solution When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude $g \sin\theta$ (the component of **g** tangent to the circle). Therefore, at $\theta = 20°$,

$$a_t = g \sin 20° = \boxed{3.4\text{ m/s}^2.}$$

(c) Find the magnitude and direction of the total acceleration **a** at $\theta = 20°$.

Solution Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of **a** at $\theta = 20°$ is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2}\text{ m/s}^2 = \boxed{5.6\text{ m/s}^2}$$

If ϕ is the angle between **a** and the string, then

$$\phi = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{3.4\text{ m/s}^2}{4.5\text{ m/s}^2}\right) = \boxed{37°}$$

Note that **a**, $\mathbf{a}_t$, and $\mathbf{a}_r$ all change in direction *and* magnitude as the ball swings through the circle. When the ball is at its lowest elevation ($\theta = 0$), $a_t = 0$ because there is no tangential component of **g** at this angle; also, a_r is a *maximum* because v is a maximum. If the ball has enough speed to reach its highest position ($\theta = 180°$), then a_t is again zero but a_r is a minimum because v is now a minimum. Finally, in the two horizontal positions ($\theta = 90°$ and 270°), $|\mathbf{a}_t| = g$ and a_r has a value between its minimum and maximum values.

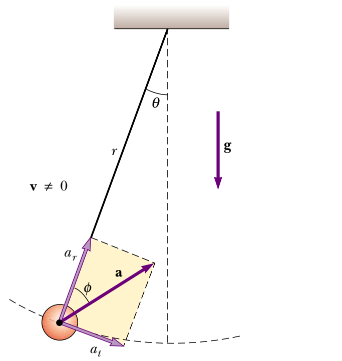


Figure 4.19 Motion of a ball suspended by a string of length r. The ball swings with nonuniform circular motion in a vertical plane, and its acceleration **a** has a radial component a_r and a tangential component a_t.

4.6 RELATIVE VELOCITY AND RELATIVE ACCELERATION

In this section, we describe how observations made by different observers in different frames of reference are related to each other. We find that observers in different frames of reference may measure different displacements, velocities, and accelerations for a given particle. That is, two observers moving relative to each other generally do not agree on the outcome of a measurement.

3.7 For example, suppose two cars are moving in the same direction with speeds of 50 mi/h and 60 mi/h. To a passenger in the slower car, the speed of the faster car is 10 mi/h. Of course, a stationary observer will measure the speed of the faster car to be 60 mi/h, not 10 mi/h. Which observer is correct? They both are! This simple example demonstrates that the velocity of an object depends on the frame of reference in which it is measured.

Suppose a person riding on a skateboard (observer A) throws a ball in such a way that it appears in this person's frame of reference to move first straight upward and then straight downward along the same vertical line, as shown in Figure 4.20a. A stationary observer B sees the path of the ball as a parabola, as illustrated in Figure 4.20b. Relative to observer B, the ball has a vertical component of velocity (resulting from the initial upward velocity and the downward acceleration of gravity) *and* a horizontal component.

Another example of this concept that of is a package dropped from an airplane flying with a constant velocity; this is the situation we studied in Example 4.6. An observer on the airplane sees the motion of the package as a straight line toward the Earth. The stranded explorer on the ground, however, sees the trajectory of the package as a parabola. If, once it drops the package, the airplane con-

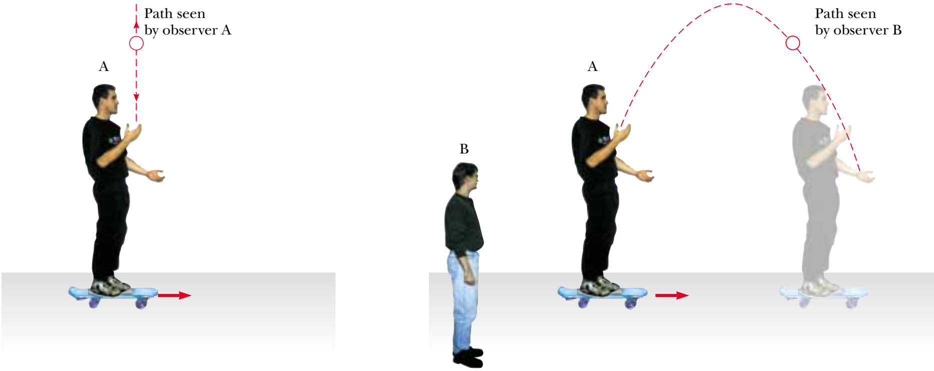


Figure 4.20 (a) Observer A on a moving vehicle throws a ball upward and sees it rise and fall in a straight-line path. (b) Stationary observer B sees a parabolic path for the same ball.

tinues to move horizontally with the same velocity, then the package hits the ground directly beneath the airplane (if we assume that air resistance is neglected)!

In a more general situation, consider a particle located at point Ⓐ in Figure 4.21. Imagine that the motion of this particle is being described by two observers, one in reference frame S, fixed relative to the Earth, and another in reference frame S', moving to the right relative to S (and therefore relative to the Earth) with a constant velocity $\mathbf{v}_0$. (Relative to an observer in S', S moves to the left with a velocity $-\mathbf{v}_0$.) Where an observer stands in a reference frame is irrelevant in this discussion, but for purposes of this discussion let us place each observer at her or his respective origin.

We label the position of the particle relative to the S frame with the position vector $\mathbf{r}$ and that relative to the S' frame with the position vector $\mathbf{r}'$, both after some time t. The vectors $\mathbf{r}$ and $\mathbf{r}'$ are related to each other through the expression $\mathbf{r} = \mathbf{r}' + \mathbf{v}_0 t$, or

Galilean coordinate transformation

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t \tag{4.20}$$

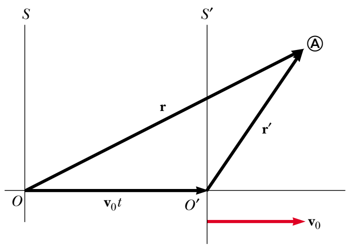
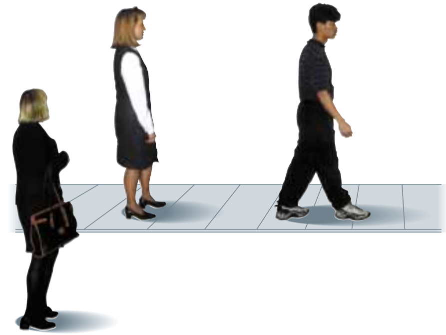


Figure 4.21 A particle located at Ⓐ is described by two observers, one in the fixed frame of reference S, and the other in the frame S', which moves to the right with a constant velocity $\mathbf{v}_0$. The vector $\mathbf{r}$ is the particle's position vector relative to S, and $\mathbf{r}'$ is its position vector relative to S'.



The woman standing on the beltway sees the walking man pass by at a slower speed than the woman standing on the stationary floor does.

That is, after a time t, the S' frame is displaced to the right of the S frame by an amount $\mathbf{v}_0 t$.

If we differentiate Equation 4.20 with respect to time and note that $\mathbf{v}_0$ is constant, we obtain

$$\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{v}_0$$

Galilean velocity transformation

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0 \tag{4.21}$$

where $\mathbf{v}'$ is the velocity of the particle observed in the S' frame and $\mathbf{v}$ is its velocity observed in the S frame. Equations 4.20 and 4.21 are known as **Galilean transformation equations.** They relate the coordinates and velocity of a particle as measured in a frame fixed relative to the Earth to those measured in a frame moving with uniform motion relative to the Earth.

Although observers in two frames measure different velocities for the particle, they measure the *same acceleration* when $\mathbf{v}_0$ is constant. We can verify this by taking the time derivative of Equation 4.21:

$$\frac{d\mathbf{v}'}{dt} = \frac{d\mathbf{v}}{dt} - \frac{d\mathbf{v}_0}{dt}$$

Because $\mathbf{v}_0$ is constant, $d\mathbf{v}_0/dt = 0$. Therefore, we conclude that $\mathbf{a}' = \mathbf{a}$ because $\mathbf{a}' = d\mathbf{v}'/dt$ and $\mathbf{a} = d\mathbf{v}/dt$. That is, **the acceleration of the particle measured by an observer in the Earth's frame of reference is the same as that measured by any other observer moving with constant velocity relative to the Earth's frame.**

Quick Quiz 4.5

A passenger in a car traveling at 60 mi/h pours a cup of coffee for the tired driver. Describe the path of the coffee as it moves from a Thermos bottle into a cup as seen by (a) the passenger and (b) someone standing beside the road and looking in the window of the car as it drives past. (c) What happens if the car accelerates while the coffee is being poured?

EXAMPLE 4.9 A Boat Crossing a River

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth. Determine the velocity of the boat relative to an observer standing on either bank.

Solution We know $\mathbf{v}_{br}$, the velocity of the *boat* relative to the *river*, and $\mathbf{v}_{rE}$, the velocity of the *river* relative to the *Earth*. What we need to find is $\mathbf{v}_{bE}$, the velocity of the *boat* relative to the *Earth*. The relationship between these three quantities is

$$\mathbf{v}_{bE} = \mathbf{v}_{br} + \mathbf{v}_{rE}$$

The terms in the equation must be manipulated as vector quantities; the vectors are shown in Figure 4.22. The quantity $\mathbf{v}_{br}$ is due north, $\mathbf{v}_{rE}$ is due east, and the vector sum of the two, $\mathbf{v}_{bE}$, is at an angle θ, as defined in Figure 4.22. Thus, we can find the speed v_{bE} of the boat relative to the Earth by using the Pythagorean theorem:

$$v_{bE} = \sqrt{{v_{br}}^2 + {v_{rE}}^2} = \sqrt{(10.0)^2 + (5.00)^2}\text{ km/h}$$

$$= \boxed{11.2\text{ km/h}}$$

The direction of $\mathbf{v}_{bE}$ is

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6°$$

The boat is moving at a speed of 11.2 km/h in the direction 26.6° east of north relative to the Earth.

Exercise If the width of the river is 3.0 km, find the time it takes the boat to cross it.

Answer 18 min.

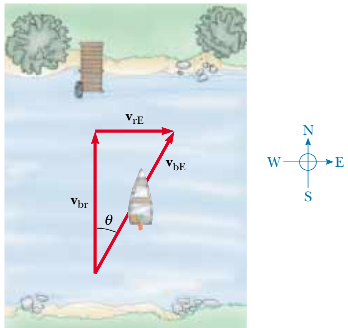


Figure 4.22

EXAMPLE 4.10 Which Way Should We Head?

If the boat of the preceding example travels with the same speed of 10.0 km/h relative to the river and is to travel due north, as shown in Figure 4.23, what should its heading be?

Solution As in the previous example, we know $\mathbf{v}_{rE}$ and the magnitude of the vector $\mathbf{v}_{br}$, and we want $\mathbf{v}_{bE}$ to be directed across the river. Figure 4.23 shows that the boat must head upstream in order to travel directly northward across the river. Note the difference between the triangle in Figure 4.22 and the one in Figure 4.23—specifically, that the hypotenuse in Figure 4.23 is no longer $\mathbf{v}_{bE}$. Therefore, when we use the Pythagorean theorem to find $\mathbf{v}_{bE}$ this time, we obtain

$$v_{bE} = \sqrt{{v_{br}}^2 - {v_{rE}}^2} = \sqrt{(10.0)^2 - (5.00)^2}\text{ km/h} = 8.66\text{ km/h}$$

Now that we know the magnitude of $\mathbf{v}_{bE}$, we can find the direction in which the boat is heading:

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{bE}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = \boxed{30.0°}$$

The boat must steer a course 30.0° west of north.

Exercise If the width of the river is 3.0 km, find the time it takes the boat to cross it.

Answer 21 min.

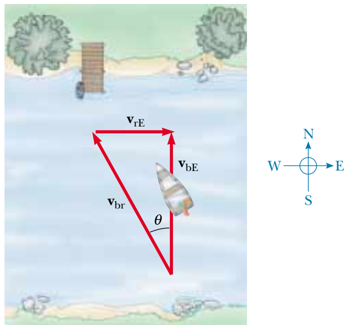


Figure 4.23

SUMMARY

If a particle moves with *constant* acceleration $\mathbf{a}$ and has velocity $\mathbf{v}_i$ and position $\mathbf{r}_i$ at $t = 0$, its velocity and position vectors at some later time t are

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \quad \textbf{(4.8)}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \quad \textbf{(4.9)}$$

For two-dimensional motion in the *xy* plane under constant acceleration, each of these vector expressions is equivalent to two component expressions—one for the motion in the *x* direction and one for the motion in the *y* direction. You should be able to break the two-dimensional motion of any object into these two components.

Projectile motion is one type of two-dimensional motion under constant acceleration, where $a_x = 0$ and $a_y = -g$. It is useful to think of projectile motion as the superposition of two motions: (1) constant-velocity motion in the x direction and (2) free-fall motion in the vertical direction subject to a constant downward acceleration of magnitude $g = 9.80\text{ m/s}^2$. You should be able to analyze the motion in terms of separate horizontal and vertical components of velocity, as shown in Figure 4.24.

A particle moving in a circle of radius r with constant speed v is in **uniform circular motion**. It undergoes a centripetal (or radial) acceleration $\mathbf{a}_r$ because the direction of $\mathbf{v}$ changes in time. The magnitude of $\mathbf{a}_r$ is

$$a_r = \frac{v^2}{r} \quad \textbf{(4.18)}$$

and its direction is always toward the center of the circle.

If a particle moves along a curved path in such a way that both the magnitude and the direction of $\mathbf{v}$ change in time, then the particle has an acceleration vector that can be described by two component vectors: (1) a radial component vector $\mathbf{a}_r$ that causes the change in direction of $\mathbf{v}$ and (2) a tangential component vector $\mathbf{a}_t$ that causes the change in magnitude of $\mathbf{v}$. The magnitude of $\mathbf{a}_r$ is v^2/r, and the magnitude of $\mathbf{a}_t$ is $d|\mathbf{v}|/dt$. You should be able to sketch motion diagrams for an object following a curved path and show how the velocity and acceleration vectors change as the object's motion varies.

The velocity $\mathbf{v}$ of a particle measured in a fixed frame of reference S can be related to the velocity $\mathbf{v}'$ of the same particle measured in a moving frame of reference S' by

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0 \quad \textbf{(4.21)}$$

where $\mathbf{v}_0$ is the velocity of S' relative to S. You should be able to translate back and forth between different frames of reference.

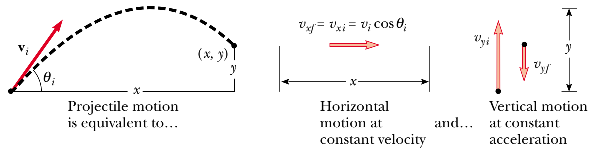


Figure 4.24 Analyzing projectile motion in terms of horizontal and vertical components.

QUESTIONS

1. Can an object accelerate if its speed is constant? Can an object accelerate if its velocity is constant?
2. If the average velocity of a particle is zero in some time interval, what can you say about the displacement of the particle for that interval?
3. If you know the position vectors of a particle at two points along its path and also know the time it took to get from one point to the other, can you determine the particle's instantaneous velocity? Its average velocity? Explain.
4. Describe a situation in which the velocity of a particle is always perpendicular to the position vector.
5. Explain whether or not the following particles have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.
6. Correct the following statement: "The racing car rounds the turn at a constant velocity of 90 mi/h."
7. Determine which of the following moving objects have an approximately parabolic trajectory: (a) a ball thrown in an arbitrary direction, (b) a jet airplane, (c) a rocket leaving the launching pad, (d) a rocket whose engines fail a few minutes after launch, (e) a tossed stone moving to the bottom of a pond.
8. A rock is dropped at the same instant that a ball at the same initial elevation is thrown horizontally. Which will have the greater speed when it reaches ground level?
9. A spacecraft drifts through space at a constant velocity. Suddenly, a gas leak in the side of the spacecraft causes a constant acceleration of the spacecraft in a direction perpendicular to the initial velocity. The orientation of the spacecraft does not change, and so the acceleration remains perpendicular to the original direction of the velocity. What is the shape of the path followed by the spacecraft in this situation?
10. A ball is projected horizontally from the top of a building. One second later another ball is projected horizontally from the same point with the same velocity. At what point in the motion will the balls be closest to each other? Will the first ball always be traveling faster than the second ball? How much time passes between the moment the first ball hits the ground and the moment the second one hits the ground? Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?
11. A student argues that as a satellite orbits the Earth in a circular path, the satellite moves with a constant velocity and therefore has no acceleration. The professor claims that the student is wrong because the satellite must have a centripetal acceleration as it moves in its circular orbit. What is wrong with the student's argument?
12. What is the fundamental difference between the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ and the unit vectors **i** and **j**?
13. At the end of its arc, the velocity of a pendulum is zero. Is its acceleration also zero at this point?
14. If a rock is dropped from the top of a sailboat's mast, will it hit the deck at the same point regardless of whether the boat is at rest or in motion at constant velocity?
15. A stone is thrown upward from the top of a building. Does the stone's displacement depend on the location of the origin of the coordinate system? Does the stone's velocity depend on the location of the origin?
16. Is it possible for a vehicle to travel around a curve without accelerating? Explain.
17. A baseball is thrown with an initial velocity of $(10\mathbf{i} + 15\mathbf{j})$ m/s. When it reaches the top of its trajectory, what are (a) its velocity and (b) its acceleration? Neglect the effect of air resistance.
18. An object moves in a circular path with constant speed v. (a) Is the velocity of the object constant? (b) Is its acceleration constant? Explain.
19. A projectile is fired at some angle to the horizontal with some initial speed v_i, and air resistance is neglected. Is the projectile a freely falling body? What is its acceleration in the vertical direction? What is its acceleration in the horizontal direction?
20. A projectile is fired at an angle of 30° from the horizontal with some initial speed. Firing at what other projectile angle results in the same range if the initial speed is the same in both cases? Neglect air resistance.
21. A projectile is fired on the Earth with some initial velocity. Another projectile is fired on the Moon with the same initial velocity. If air resistance is neglected, which projectile has the greater range? Which reaches the greater altitude? (Note that the free-fall acceleration on the Moon is about 1.6 $\mathrm{m/s^2}$.)
22. As a projectile moves through its parabolic trajectory, which of these quantities, if any, remain constant: (a) speed, (b) acceleration, (c) horizontal component of velocity, (d) vertical component of velocity?
23. A passenger on a train that is moving with constant velocity drops a spoon. What is the acceleration of the spoon relative to (a) the train and (b) the Earth?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 4.1 The Displacement, Velocity, and Acceleration Vectors

WEB 1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the total vector displacement, (b) the average speed, and (c) the average velocity. Use a coordinate system in which east is the positive x axis.

2. Suppose that the position vector for a particle is given as $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, with $x = at + b$ and $y = ct^2 + d$, where $a = 1.00$ m/s, $b = 1.00$ m, $c = 0.125\ \mathrm{m/s^2}$, and $d = 1.00$ m. (a) Calculate the average velocity during the time interval from $t = 2.00$ s to $t = 4.00$ s. (b) Determine the velocity and the speed at $t = 2.00$ s.

3. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates versus time are given by the following expressions:

$$x = (18.0\ \mathrm{m/s})t$$

and

$$y = (4.00\ \mathrm{m/s})t - (4.90\ \mathrm{m/s^2})t^2$$

(a) Write a vector expression for the ball's position as a function of time, using the unit vectors **i** and **j**. By taking derivatives of your results, write expressions for (b) the velocity vector as a function of time and (c) the acceleration vector as a function of time. Now use unit vector notation to write expressions for (d) the position, (e) the velocity, and (f) the acceleration of the ball, all at $t = 3.00$ s.

4. The coordinates of an object moving in the xy plane vary with time according to the equations

$$x = -(5.00\ \mathrm{m})\sin \omega t$$

and

$$y = (4.00\ \mathrm{m}) - (5.00\ \mathrm{m})\cos \omega t$$

where t is in seconds and ω has units of seconds^{-1}. (a) Determine the components of velocity and components of acceleration at $t = 0$. (b) Write expressions for the position vector, the velocity vector, and the acceleration vector at any time $t > 0$. (c) Describe the path of the object on an xy graph.

Section 4.2 Two-Dimensional Motion with Constant Acceleration

5. At $t = 0$, a particle moving in the xy plane with constant acceleration has a velocity of $\mathbf{v}_i = (3.00\mathbf{i} - 2.00\mathbf{j})$ m/s when it is at the origin. At $t = 3.00$ s, the particle's velocity is $\mathbf{v} = (9.00\mathbf{i} + 7.00\mathbf{j})$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

6. The vector position of a particle varies in time according to the expression $\mathbf{r} = (3.00\mathbf{i} - 6.00t^2\mathbf{j})$ m. (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at $t = 1.00$ s.

7. A fish swimming in a horizontal plane has velocity $\mathbf{v}_i = (4.00\mathbf{i} + 1.00\mathbf{j})$ m/s at a point in the ocean whose displacement from a certain rock is $\mathbf{r}_i = (10.0\mathbf{i} - 4.00\mathbf{j})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\mathbf{v} = (20.0\mathbf{i} - 5.00\mathbf{j})$ m/s. (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to the unit vector **i**? (c) Where is the fish at $t = 25.0$ s if it maintains its original acceleration and in what direction is it moving?

8. A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00\mathbf{j}\ \mathrm{m/s^2}$ and an initial velocity of $\mathbf{v}_i = 5.00\mathbf{i}$ m/s. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00$ s.

Section 4.3 Projectile Motion

(Neglect air resistance in all problems and take $g = 9.80\ \mathrm{m/s^2}$.)

WEB 9. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor 1.40 m from the base of the counter. If the height of the counter is 0.860 m, (a) with what velocity did the mug leave the counter and (b) what was the direction of the mug's velocity just before it hit the floor?

10. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor at distance d from the base of the counter. If the height of the counter is h, (a) with what velocity did the mug leave the counter and (b) what was the direction of the mug's velocity just before it hit the floor?

11. One strategy in a snowball fight is to throw a first snowball at a high angle over level ground. While your opponent is watching the first one, you throw a second one at a low angle and timed to arrive at your opponent before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second (low-angle) snowball be thrown if it is to land at the same point as the first? (b) How many seconds later should

the second snowball be thrown if it is to land at the same time as the first?

12. A tennis player standing 12.6 m from the net hits the ball at 3.00° above the horizontal. To clear the net, the ball must rise at least 0.330 m. If the ball just clears the net at the apex of its trajectory, how fast was the ball moving when it left the racket?

13. An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. What are the *x* and *y* coordinates of the shell where it explodes, relative to its firing point?

14. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free-fall acceleration on the planet?

15. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection? Give your answer to three significant figures.

16. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

17. A cannon with a muzzle speed of 1 000 m/s is used to start an avalanche on a mountain slope. The target is 2 000 m from the cannon horizontally and 800 m above the cannon. At what angle, above the horizontal, should the cannon be fired?

18. Consider a projectile that is launched from the origin of an *xy* coordinate system with speed v_i at initial angle θ_i above the horizontal. Note that at the apex of its trajectory the projectile is moving horizontally, so that the slope of its path is zero. Use the expression for the trajectory given in Equation 4.12 to find the *x* coordinate that corresponds to the maximum height. Use this *x* coordinate and the symmetry of the trajectory to determine the horizontal range of the projectile.

WEB **19.** A placekicker must kick a football from a point 36.0 m (about 40 yards) from the goal, and half the crowd hopes the ball will clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of 20.0 m/s at an angle of 53.0° to the horizontal. (a) By how much does the ball clear or fall short of clearing the crossbar? (b) Does the ball approach the crossbar while still rising or while falling?

20. A firefighter 50.0 m away from a burning building directs a stream of water from a fire hose at an angle of 30.0° above the horizontal, as in Figure P4.20. If the speed of the stream is 40.0 m/s, at what height will the water strike the building?

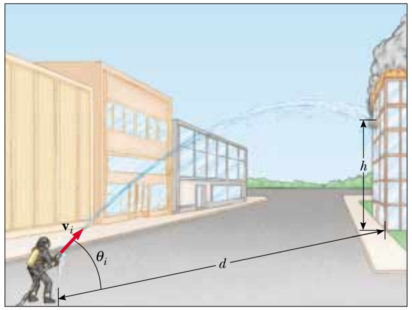


Figure P4.20 Problems 20 and 21. *(Frederick McKinney/FPG International)*

21. A firefighter a distance *d* from a burning building directs a stream of water from a fire hose at angle θ_i above the horizontal as in Figure P4.20. If the initial speed of the stream is v_i, at what height *h* does the water strike the building?

22. A soccer player kicks a rock horizontally off a cliff 40.0 m high into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

23. A basketball star covers 2.80 m horizontally in a jump to dunk the ball (Fig. P4.23). His motion through space can be modeled as that of a particle at a point called his center of mass (which we shall define in Chapter 9). His center of mass is at elevation 1.02 m when he leaves the floor. It reaches a maximum height of 1.85 m above the floor and is at elevation 0.900 m when he touches down again. Determine (a) his time of flight (his "hang time"), (b) his horizontal and (c) vertical velocity components at the instant of takeoff, and (d) his takeoff angle. (e) For comparison, determine the hang time of a whitetail deer making a jump with center-of-mass elevations $y_i = 1.20$ m, $y_{max} = 2.50$ m, $y_f = 0.700$ m.

Figure P4.23 *(Top, Ron Chapple/FPG International; bottom, Bill Lea/Dembinsky Photo Associates)*

Section 4.4 Uniform Circular Motion

24. The orbit of the Moon about the Earth is approximately circular, with a mean radius of 3.84×10^8 m. It takes 27.3 days for the Moon to complete one revolution about the Earth. Find (a) the mean orbital speed of the Moon and (b) its centripetal acceleration.

WEB **25.** The athlete shown in Figure P4.25 rotates a 1.00-kg discus along a circular path of radius 1.06 m. The maximum speed of the discus is 20.0 m/s. Determine the magnitude of the maximum radial acceleration of the discus.

Figure P4.25 *(Sam Sargent/Liaison International)*

26. From information on the endsheets of this book, compute, for a point located on the surface of the Earth at the equator, the radial acceleration due to the rotation of the Earth about its axis.

27. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge). (*Hint:* In one revolution, the stone travels a distance equal to the circumference of its path, $2\pi r$.)

28. During liftoff, Space Shuttle astronauts typically feel accelerations up to $1.4g$, where $g = 9.80$ m/s^2. In their training, astronauts ride in a device where they experience such an acceleration as a centripetal acceleration. Specifically, the astronaut is fastened securely at the end of a mechanical arm that then turns at constant speed in a horizontal circle. Determine the rotation rate, in revolutions per second, required to give an astronaut a centripetal acceleration of $1.40g$ while the astronaut moves in a circle of radius 10.0 m.

29. Young David who slew Goliath experimented with slings before tackling the giant. He found that he could revolve a sling of length 0.600 m at the rate of 8.00 rev/s. If he increased the length to 0.900 m, he could revolve the sling only 6.00 times per second. (a) Which rate of rotation gives the greater speed for the stone at the end of the sling? (b) What is the centripetal acceleration of the stone at 8.00 rev/s? (c) What is the centripetal acceleration at 6.00 rev/s?

30. The astronaut orbiting the Earth in Figure P4.30 is preparing to dock with a Westar VI satellite. The satellite is in a circular orbit 600 km above the Earth's surface, where the free-fall acceleration is 8.21 m/s^2. The radius of the Earth is 6 400 km. Determine the speed of the satellite and the time required to complete one orbit around the Earth.

Figure P4.30 *(Courtesy of NASA)*

Section 4.5 Tangential and Radial Acceleration

31. A train slows down as it rounds a sharp horizontal curve, slowing from 90.0 km/h to 50.0 km/h in the 15.0 s that it takes to round the curve. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume that the train slows down at a uniform rate during the 15.0-s interval.

32. An automobile whose speed is increasing at a rate of 0.600 m/s^2 travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find (a) the tangential acceleration component, (b) the radial acceleration component, and (c) the magnitude and direction of the total acceleration.

33. Figure P4.33 shows the total acceleration and velocity of a particle moving clockwise in a circle of radius 2.50 m at a given instant of time. At this instant, find (a) the radial acceleration, (b) the speed of the particle, and (c) its tangential acceleration.

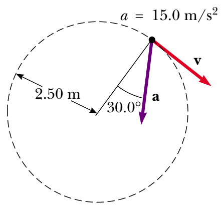


Figure P4.33

34. A student attaches a ball to the end of a string 0.600 m in length and then swings the ball in a vertical circle. The speed of the ball is 4.30 m/s at its highest point and 6.50 m/s at its lowest point. Find the acceleration of the ball when the string is vertical and the ball is at (a) its highest point and (b) its lowest point.

35. A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point and on its way up, its total acceleration is $(-22.5\mathbf{i} + 20.2\mathbf{j})$ m/s^2. At that instant, (a) sketch a vector diagram showing the components of this acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

Section 4.6 Relative Velocity and Relative Acceleration

36. Heather in her Corvette accelerates at the rate of $(3.00\mathbf{i} - 2.00\mathbf{j})$ m/s^2, while Jill in her Jaguar accelerates at $(1.00\mathbf{i} + 3.00\mathbf{j})$ m/s^2. They both start from rest at the origin of an *xy* coordinate system. After 5.00 s, (a) what is Heather's speed with respect to Jill, (b) how far apart are they, and (c) what is Heather's acceleration relative to Jill?

37. A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

38. How long does it take an automobile traveling in the left lane at 60.0 km/h to pull alongside a car traveling in the right lane at 40.0 km/h if the cars' front bumpers are initially 100 m apart?

39. The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. If there is a wind of 30.0 km/h toward the north, find the velocity of the airplane relative to the ground.

40. Two swimmers, Alan and Beth, start at the same point in a stream that flows with a speed v. Both move at the same speed c ($c > v$) relative to the stream. Alan swims downstream a distance L and then upstream the same distance. Beth swims such that her motion relative to the ground is perpendicular to the banks of the stream. She swims a distance L in this direction and then back. The result of the motions of Alan and Beth is that they both return to the starting point. Which swimmer returns first? (*Note:* First guess at the answer.)

41. A child in danger of drowning in a river is being carried downstream by a current that has a speed of 2.50 km/h. The child is 0.600 km from shore and 0.800 km upstream of a boat landing when a rescue boat sets out. (a) If the boat proceeds at its maximum speed of 20.0 km/h relative to the water, what heading relative to the shore should the pilot take? (b) What angle does the boat velocity make with the shore? (c) How long does it take the boat to reach the child?

42. A bolt drops from the ceiling of a train car that is accelerating northward at a rate of 2.50 m/s^2. What is the acceleration of the bolt relative to (a) the train car and (b) the Earth?

43. A science student is riding on a flatcar of a train traveling along a straight horizontal track at a constant speed of 10.0 m/s. The student throws a ball into the air along a path that he judges to make an initial angle of 60.0° with the horizontal and to be in line with the track. The student's professor, who is standing on the ground nearby, observes the ball to rise vertically. How high does she see the ball rise?

ADDITIONAL PROBLEMS

44. A ball is thrown with an initial speed v_i at an angle θ_i with the horizontal. The horizontal range of the ball is R, and the ball reaches a maximum height $R/6$. In terms of R and g, find (a) the time the ball is in motion, (b) the ball's speed at the peak of its path, (c) the initial vertical component of its velocity, (d) its initial speed, and (e) the angle θ_i. (f) Suppose the ball is thrown at the same initial speed found in part (d) but at the angle appropriate for reaching the maximum height. Find this height. (g) Suppose the ball is thrown at the same initial speed but at the angle necessary for maximum range. Find this range.

45. As some molten metal splashes, one droplet flies off to the east with initial speed v_i at angle θ_i above the horizontal, and another droplet flies off to the west with the same speed at the same angle above the horizontal, as in Figure P4.45. In terms of v_i and θ_i, find the distance between the droplets as a function of time.

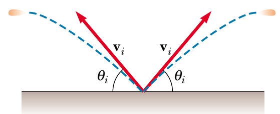


Figure P4.45

46. A ball on the end of a string is whirled around in a horizontal circle of radius 0.300 m. The plane of the circle is 1.20 m above the ground. The string breaks and the ball lands 2.00 m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the radial acceleration of the ball during its circular motion.

47. A projectile is fired up an incline (incline angle ϕ) with an initial speed v_i at an angle θ_i with respect to the horizontal ($\theta_i > \phi$), as shown in Figure P4.47. (a) Show that the projectile travels a distance d up the incline, where

$$d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$$

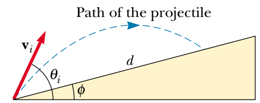


Figure P4.47

(b) For what value of θ_i is d a maximum, and what is that maximum value of d?

48. A student decides to measure the muzzle velocity of the pellets from his BB gun. He points the gun horizontally. On a vertical wall a distance x away from the gun, a target is placed. The shots hit the target a vertical distance y below the gun. (a) Show that the vertical displacement component of the pellets when traveling through the air is given by $y = Ax^2$, where A is a constant. (b) Express the constant A in terms of the initial velocity and the free-fall acceleration. (c) If $x = 3.00$ m and $y = 0.210$ m, what is the initial speed of the pellets?

49. A home run is hit in such a way that the baseball just clears a wall 21.0 m high, located 130 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Find (a) the initial speed of the ball, (b) the time it takes the ball to reach the wall, and (c) the velocity components and the speed of the ball when it reaches the wall. (Assume the ball is hit at a height of 1.00 m above the ground.)

50. An astronaut standing on the Moon fires a gun so that the bullet leaves the barrel initially moving in a horizontal direction. (a) What must be the muzzle speed of the bullet so that it travels completely around the Moon and returns to its original location? (b) How long does this trip around the Moon take? Assume that the free-fall acceleration on the Moon is one-sixth that on the Earth.

51. A pendulum of length 1.00 m swings in a vertical plane (Fig. 4.19). When the pendulum is in the two horizontal positions $\theta = 90°$ and $\theta = 270°$, its speed is 5.00 m/s. (a) Find the magnitude of the radial acceleration and tangential acceleration for these positions. (b) Draw a vector diagram to determine the direction of the total acceleration for these two positions. (c) Calculate the magnitude and direction of the total acceleration.

52. A basketball player who is 2.00 m tall is standing on the floor 10.0 m from the basket, as in Figure P4.52. If he shoots the ball at a 40.0° angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is 3.05 m.

53. A particle has velocity components

$$v_x = +4 \text{ m/s} \qquad v_y = -(6 \text{ m/s}^2)t + 4 \text{ m/s}$$

Calculate the speed of the particle and the direction $\theta = \tan^{-1}(v_y/v_x)$ of the velocity vector at $t = 2.00$ s.

54. When baseball players throw the ball in from the outfield, they usually allow it to take one bounce before it reaches the infielder on the theory that the ball arrives

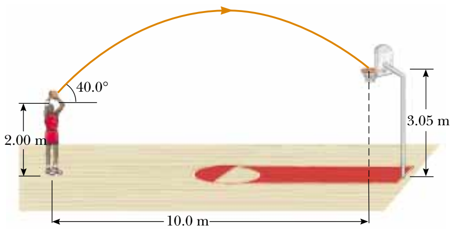


Figure P4.52

sooner that way. Suppose that the angle at which a bounced ball leaves the ground is the same as the angle at which the outfielder launched it, as in Figure P4.54, but that the ball's speed after the bounce is one half of what it was before the bounce. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the ball be thrown in order to go the same distance D with one bounce (blue path) as a ball thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.

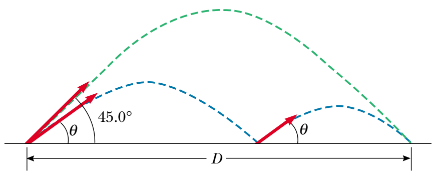


Figure P4.54

55. A boy can throw a ball a maximum horizontal distance of 40.0 m on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

56. A boy can throw a ball a maximum horizontal distance of R on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

57. A stone at the end of a sling is whirled in a vertical circle of radius 1.20 m at a constant speed $v_i = 1.50$ m/s as in Figure P4.57. The center of the string is 1.50 m above the ground. What is the range of the stone if it is released when the sling is inclined at 30.0° with the horizontal (a) at A? (b) at B? What is the acceleration of the stone (c) just before it is released at A? (d) just after it is released at A?

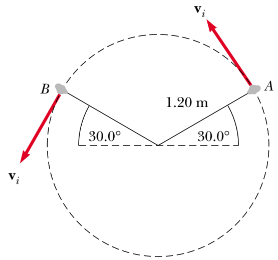


Figure P4.57

58. A quarterback throws a football straight toward a receiver with an initial speed of 20.0 m/s, at an angle of 30.0° above the horizontal. At that instant, the receiver is 20.0 m from the quarterback. In what direction and with what constant speed should the receiver run to catch the football at the level at which it was thrown?

59. A bomber is flying horizontally over level terrain, with a speed of 275 m/s relative to the ground, at an altitude of 3 000 m. Neglect the effects of air resistance. (a) How far will a bomb travel horizontally between its release from the plane and its impact on the ground? (b) If the plane maintains its original course and speed, where will it be when the bomb hits the ground? (c) At what angle from the vertical should the telescopic bombsight be set so that the bomb will hit the target seen in the sight at the time of release?

60. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\mathbf{v}_i$ as in Figure P4.60. (a) What must be its minimum initial speed if the ball is never to hit the rock after it is kicked? (b) With this initial speed, how far from the base of the rock does the ball hit the ground?

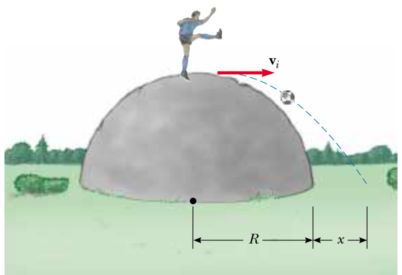


Figure P4.60

61. A hawk is flying horizontally at 10.0 m/s in a straight line, 200 m above the ground. A mouse it has been carrying struggles free from its grasp. The hawk continues on its path at the same speed for 2.00 s before attempting to retrieve its prey. To accomplish the retrieval, it dives in a straight line at constant speed and recaptures the mouse 3.00 m above the ground. (a) Assuming no air resistance, find the diving speed of the hawk. (b) What angle did the hawk make with the horizontal during its descent? (c) For how long did the mouse "enjoy" free fall?

62. A truck loaded with cannonball watermelons stops suddenly to avoid running over the edge of a washed-out bridge (Fig. P4.62). The quick stop causes a number of melons to fly off the truck. One melon rolls over the edge with an initial speed $v_i = 10.0$ m/s in the horizontal direction. A cross-section of the bank has the shape of the bottom half of a parabola with its vertex at the edge of the road, and with the equation $y^2 = 16x$, where x and y are measured in meters. What are the x and y coordinates of the melon when it splatters on the bank?

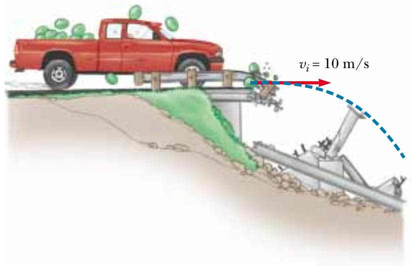


Figure P4.62

63. A catapult launches a rocket at an angle of 53.0° above the horizontal with an initial speed of 100 m/s. The rocket engine immediately starts a burn, and for 3.00 s the rocket moves along its initial line of motion with an acceleration of 30.0 m/s^2. Then its engine fails, and the rocket proceeds to move in free fall. Find (a) the maximum altitude reached by the rocket, (b) its total time of flight, and (c) its horizontal range.

64. A river flows with a uniform velocity $\mathbf{v}$. A person in a motorboat travels 1.00 km upstream, at which time she passes a log floating by. Always with the same throttle setting, the boater continues to travel upstream for another 60.0 min and then returns downstream to her starting point, which she reaches just as the same log does. Find the velocity of the river. (*Hint:* The time of travel of the boat after it meets the log equals the time of travel of the log.)

WEB **65.** A car is parked on a steep incline overlooking the ocean, where the incline makes an angle of 37.0° below the horizontal. The negligent driver leaves the car in neutral, and the parking brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s^2, traveling 50.0 m to the edge of a vertical cliff. The cliff is 30.0 m above the ocean. Find (a) the speed of the car when it reaches the edge of the cliff and the time it takes to get there, (b) the velocity of the car when it lands in the ocean, (c) the total time the car is in motion, and (d) the position of the car when it lands in the ocean, relative to the base of the cliff.

66. The determined coyote is out once more to try to capture the elusive roadrunner. The coyote wears a pair of Acme jet-powered roller skates, which provide a constant horizontal acceleration of 15.0 m/s^2 (Fig. P4.66). The coyote starts off at rest 70.0 m from the edge of a cliff at the instant the roadrunner zips past him in the direction of the cliff. (a) If the roadrunner moves with constant speed, determine the minimum speed he must have to reach the cliff before the coyote. At the brink of the cliff, the roadrunner escapes by making a sudden turn, while the coyote continues straight ahead. (b) If the cliff is 100 m above the floor of a canyon, determine where the coyote lands in the canyon (assume his skates remain horizontal and continue to operate when he is in "flight"). (c) Determine the components of the coyote's impact velocity.

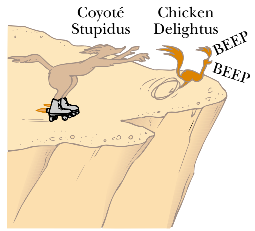


Figure P4.66

67. A skier leaves the ramp of a ski jump with a velocity of 10.0 m/s, 15.0° above the horizontal, as in Figure P4.67. The slope is inclined at 50.0°, and air resistance is negligible. Find (a) the distance from the ramp to where the jumper lands and (b) the velocity components just before the landing. (How do you think the results might be affected if air resistance were included? Note that jumpers lean forward in the shape of an airfoil, with their hands at their sides, to increase their distance. Why does this work?)

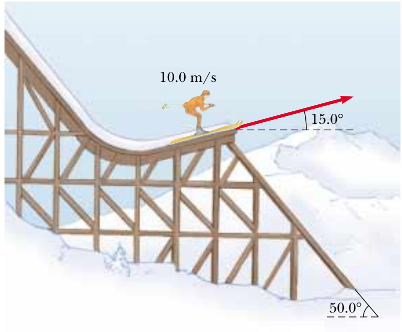


Figure P4.67

68. Two soccer players, Mary and Jane, begin running from nearly the same point at the same time. Mary runs in an easterly direction at 4.00 m/s, while Jane takes off in a direction 60.0° north of east at 5.40 m/s. (a) How long is it before they are 25.0 m apart? (b) What is the velocity of Jane relative to Mary? (c) How far apart are they after 4.00 s?

69. Do not hurt yourself; do not strike your hand against anything. Within these limitations, describe what you do to give your hand a large acceleration. Compute an order-of-magnitude estimate of this acceleration, stating the quantities you measure or estimate and their values.

70. An enemy ship is on the western side of a mountain island, as shown in Figure P4.70. The enemy ship can maneuver to within 2 500 m of the 1 800-m-high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the eastern shoreline is horizontally 300 m from the peak, what are the distances from the eastern shore at which a ship can be safe from the bombardment of the enemy ship?

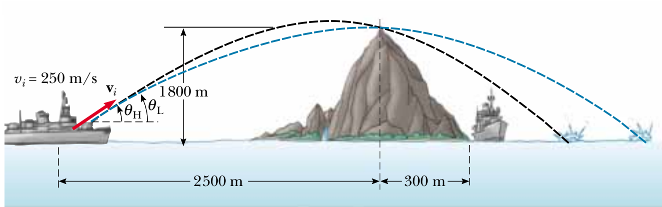


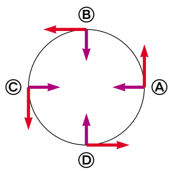
Figure P4.70

Answers to Quick Quizzes

4.1 (a) Because acceleration occurs whenever the velocity changes in any way—with an increase or decrease in speed, a change in direction, or both—the brake pedal can also be considered an accelerator because it causes the car to slow down. The steering wheel is also an accelerator because it changes the direction of the velocity vector. (b) When the car is moving with constant speed, the gas pedal is not causing an acceleration; it is an accelerator only when it causes a change in the speedometer reading.

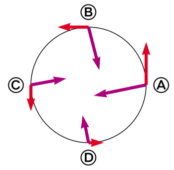
4.2 (a) At only one point—the peak of the trajectory—are the velocity and acceleration vectors perpendicular to each other. (b) If the object is thrown straight up or down, **v** and **a** are parallel to each other throughout the downward motion. Otherwise, the velocity and acceleration vectors are never parallel to each other. (c) The greater the maximum height, the longer it takes the projectile to reach that altitude and then fall back down from it. So, as the angle increases from 0° to 90°, the time of flight increases. Therefore, the 15° angle gives the shortest time of flight, and the 75° angle gives the longest.

4.3 (a) Because the object is moving with a constant speed, the velocity vector is always the same length; because the motion is circular, this vector is always tangent to the circle. The only acceleration is that which changes the direction of the velocity vector; it points radially inward.



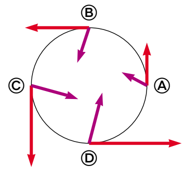
(a)

(b) Now there is a component of the acceleration vector that is tangent to the circle and points in the direction opposite the velocity. As a result, the acceleration vector does not point toward the center. The object is slowing down, and so the velocity vectors become shorter and shorter.



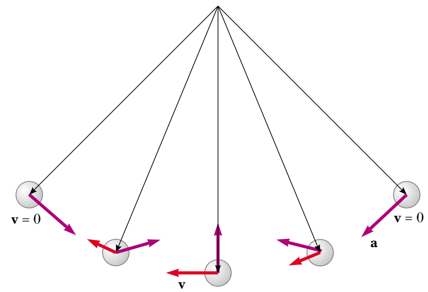
(b)

(c) Now the tangential component of the acceleration points in the same direction as the velocity. The object is speeding up, and so the velocity vectors become longer and longer.



(c)

4.4 The motion diagram is as shown below. Note that each position vector points from the pivot point at the center of the circle to the position of the ball.



4.5 (a) The passenger sees the coffee pouring nearly vertically into the cup, just as if she were standing on the ground pouring it. (b) The stationary observer sees the coffee moving in a parabolic path with a constant horizontal velocity of 60 mi/h (= 88 ft/s) and a downward acceleration of $-g$. If it takes the coffee 0.10 s to reach the cup, the stationary observer sees the coffee moving 8.8 ft horizontally before it hits the cup! (c) If the car slows suddenly, the coffee reaches the place where the cup *would have been* had there been no change in velocity and continues falling because the cup has not yet reached that location. If the car rapidly speeds up, the coffee falls behind the cup. If the car accelerates sideways, the coffee again ends up somewhere other than the cup.

PUZZLER

The *Spirit of Akron* is an airship that is more than 60 m long. When it is parked at an airport, one person can easily support it overhead using a single hand. Nonetheless, it is impossible for even a very strong adult to move the ship abruptly. What property of this huge airship makes it very difficult to cause any sudden changes in its motion? *(Courtesy of Edward E. Ogden)*

web

For more information about the airship, visit **http://www.goodyear.com/us/blimp/index.html**

chapter 5

The Laws of Motion

Chapter Outline

In Chapters 2 and 4, we described motion in terms of displacement, velocity, and acceleration without considering what might cause that motion. What might cause one particle to remain at rest and another particle to accelerate? In this chapter, we investigate what causes changes in motion. The two main factors we need to consider are the forces acting on an object and the mass of the object. We discuss the three basic laws of motion, which deal with forces and masses and were formulated more than three centuries ago by Isaac Newton. Once we understand these laws, we can answer such questions as "What mechanism changes motion?" and "Why do some objects accelerate more than others?"

5.1 THE CONCEPT OF FORCE

Everyone has a basic understanding of the concept of force from everyday experience. When you push your empty dinner plate away, you exert a force on it. Similarly, you exert a force on a ball when you throw or kick it. In these examples, the word *force* is associated with muscular activity and some change in the velocity of an object. Forces do not always cause motion, however. For example, as you sit reading this book, the force of gravity acts on your body and yet you remain stationary. As a second example, you can push (in other words, exert a force) on a large boulder and not be able to move it.

What force (if any) causes the Moon to orbit the Earth? Newton answered this and related questions by stating that forces are what cause any change in the velocity of an object. Therefore, if an object moves with uniform motion (constant velocity), no force is required for the motion to be maintained. The Moon's velocity is not constant because it moves in a nearly circular orbit around the Earth. We now know that this change in velocity is caused by the force exerted on the Moon by the Earth. Because only a force can cause a change in velocity, we can think of force as *that which causes a body to accelerate.* In this chapter, we are concerned with the relationship between the force exerted on an object and the acceleration of that object.

A body accelerates because of an external force

What happens when several forces act simultaneously on an object? In this case, the object accelerates only if the net force acting on it is not equal to zero. The **net force** acting on an object is defined as the vector sum of all forces acting on the object. (We sometimes refer to the net force as the *total force,* the *resultant force,* or the *unbalanced force.*) **If the net force exerted on an object is zero, then the acceleration of the object is zero and its velocity remains constant.** That is, if the net force acting on the object is zero, then the object either remains at rest or continues to move with constant velocity. When the velocity of an object is constant (including the case in which the object remains at rest), the object is said to be in **equilibrium.**

Definition of equilibrium

When a coiled spring is pulled, as in Figure 5.1a, the spring stretches. When a stationary cart is pulled suffcently hard that friction is overcome, as in Figure 5.1b, the cart moves. When a football is kicked, as in Figure 5.1c, it is both deformed and set in motion. These situations are all examples of a class of forces called *contact forces.* That is, they involve physical contact between two objects. Other examples of contact forces are the force exerted by gas molecules on the walls of a container and the force exerted by your feet on the floor.

Another class of forces, known as *field forces,* do not involve physical contact between two objects but instead act through empty space. The force of gravitational attraction between two objects, illustrated in Figure 5.1d, is an example of this class of force. This gravitational force keeps objects bound to the Earth. The plan-

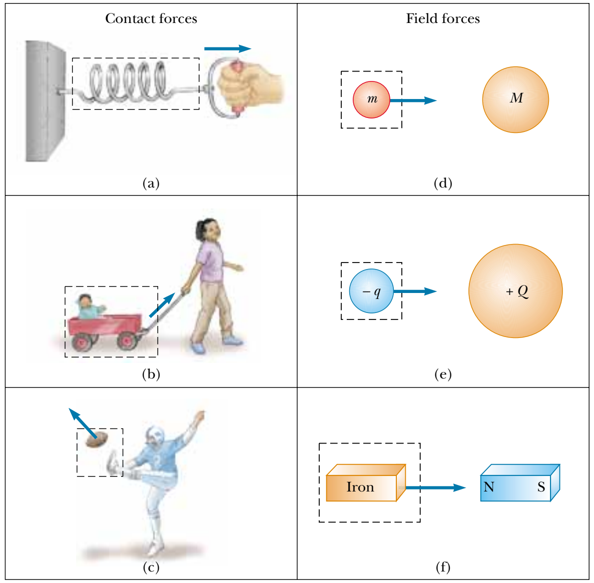


Figure 5.1 Some examples of applied forces. In each case a force is exerted on the object within the boxed area. Some agent in the environment external to the boxed area exerts a force on the object.

ets of our Solar System are bound to the Sun by the action of gravitational forces. Another common example of a field force is the electric force that one electric charge exerts on another, as shown in Figure 5.1e. These charges might be those of the electron and proton that form a hydrogen atom. A third example of a field force is the force a bar magnet exerts on a piece of iron, as shown in Figure 5.1f. The forces holding an atomic nucleus together also are field forces but are very short in range. They are the dominating interaction for particle separations of the order of 10^{-15} m.

Early scientists, including Newton, were uneasy with the idea that a force can act between two disconnected objects. To overcome this conceptual problem, Michael Faraday (1791–1867) introduced the concept of a *field.* According to this approach, when object 1 is placed at some point P near object 2, we say that object 1 interacts with object 2 by virtue of the gravitational field that exists at P. The gravitational field at P is created by object 2. Likewise, a gravitational field created by object 1 exists at the position of object 2. In fact, all objects create a gravitational field in the space around themselves.

The distinction between contact forces and field forces is not as sharp as you may have been led to believe by the previous discussion. When examined at the atomic level, all the forces we classify as contact forces turn out to be caused by

electric (field) forces of the type illustrated in Figure 5.1e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces. The only known *fundamental* forces in nature are all field forces: (1) gravitational forces between objects, (2) electromagnetic forces between electric charges, (3) strong nuclear forces between subatomic particles, and (4) weak nuclear forces that arise in certain radioactive decay processes. In classical physics, we are concerned only with gravitational and electromagnetic forces.

Measuring the Strength of a Force

It is convenient to use the deformation of a spring to measure force. Suppose we apply a vertical force to a spring scale that has a fixed upper end, as shown in Figure 5.2a. The spring elongates when the force is applied, and a pointer on the scale reads the value of the applied force. We can calibrate the spring by defining the unit force $\mathbf{F}_1$ as the force that produces a pointer reading of 1.00 cm. (Because force is a vector quantity, we use the bold-faced symbol $\mathbf{F}$.) If we now apply a different downward force $\mathbf{F}_2$ whose magnitude is 2 units, as seen in Figure 5.2b, the pointer moves to 2.00 cm. Figure 5.2c shows that the combined effect of the two collinear forces is the sum of the effects of the individual forces.

Now suppose the two forces are applied simultaneously with $\mathbf{F}_1$ downward and $\mathbf{F}_2$ horizontal, as illustrated in Figure 5.2d. In this case, the pointer reads $\sqrt{5}\text{ cm}^2 = 2.24$ cm. The single force $\mathbf{F}$ that would produce this same reading is the sum of the two vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, as described in Figure 5.2d. That is, $|\mathbf{F}| = \sqrt{F_1^2 + F_2^2} = 2.24$ units, and its direction is $\theta = \tan^{-1}(-0.500) = -26.6°$. **Because forces are vector quantities, you must use the rules of vector addition to obtain the net force acting on an object.**

QuickLab

Find a tennis ball, two drinking straws, and a friend. Place the ball on a table. You and your friend can each apply a force to the ball by blowing through the straws (held horizontally a few centimeters above the table) so that the air rushing out strikes the ball. Try a variety of configurations: Blow in opposite directions against the ball, blow in the same direction, blow at right angles to each other, and so forth. Can you verify the vector nature of the forces?

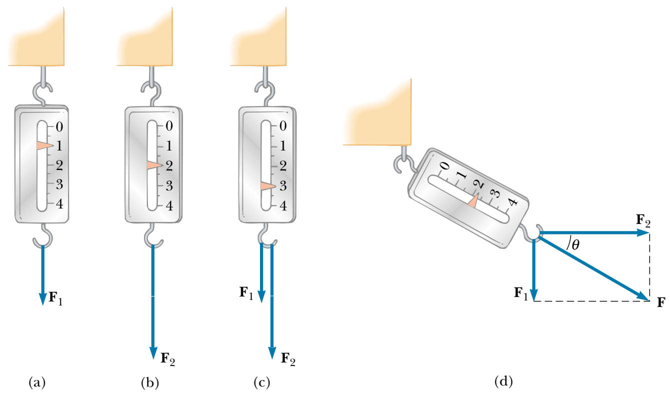


Figure 5.2 The vector nature of a force is tested with a spring scale. (a) A downward force $\mathbf{F}_1$ elongates the spring 1 cm. (b) A downward force $\mathbf{F}_2$ elongates the spring 2 cm. (c) When $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied simultaneously, the spring elongates by 3 cm. (d) When $\mathbf{F}_1$ is downward and $\mathbf{F}_2$ is horizontal, the combination of the two forces elongates the spring $\sqrt{1^2 + 2^2}$ cm $= \sqrt{5}$ cm.

5.2 NEWTON'S FIRST LAW AND INERTIAL FRAMES

4.2

Before we state Newton's first law, consider the following simple experiment. Suppose a book is lying on a table. Obviously, the book remains at rest. Now imagine that you push the book with a horizontal force great enough to overcome the force of friction between book and table. (This force you exert, the force of friction, and any other forces exerted on the book by other objects are referred to as *external forces*.) You can keep the book in motion with constant velocity by applying a force that is just equal in magnitude to the force of friction and acts in the opposite direction. If you then push harder so that the magnitude of your applied force exceeds the magnitude of the force of friction, the book accelerates. If you stop pushing, the book stops after moving a short distance because the force of friction retards its motion. Suppose you now push the book across a smooth, highly waxed floor. The book again comes to rest after you stop pushing but not as quickly as before. Now imagine a floor so highly polished that friction is absent; in this case, the book, once set in motion, moves until it hits a wall.

Before about 1600, scientists felt that the natural state of matter was the state of rest. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments, such as the one we just discussed for a book on a frictionless surface, and concluded that it is not the nature of an object to stop once set in motion: rather, it is its nature to *resist changes in its motion*. In his words, "Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed."

QuickLab

Use a drinking straw to impart a strong, short-duration burst of air against a tennis ball as it rolls along a tabletop. Make the force perpendicular to the ball's path. What happens to the ball's motion? What is different if you apply a continuous force (constant magnitude and direction) that is directed along the direction of motion?

This new approach to motion was later formalized by Newton in a form that has come to be known as **Newton's first law of motion:**

Newton's first law

> In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

In simpler terms, we can say that **when no force acts on an object, the acceleration of the object is zero.** If nothing acts to change the object's motion, then its velocity does not change. From the first law, we conclude that any *isolated object* (one that does not interact with its environment) is either at rest or moving with constant velocity. The tendency of an object to resist any attempt to change its velocity is called the **inertia** of the object. Figure 5.3 shows one dramatic example of a consequence of Newton's first law.

Definition of inertia

Another example of uniform (constant-velocity) motion on a nearly frictionless surface is the motion of a light disk on a film of air (the lubricant), as shown in Figure 5.4. If the disk is given an initial velocity, it coasts a great distance before stopping.

Finally, consider a spaceship traveling in space and far removed from any planets or other matter. The spaceship requires some propulsion system to change its velocity. However, if the propulsion system is turned off when the spaceship reaches a velocity **v**, the ship coasts at that constant velocity and the astronauts get a free ride (that is, no propulsion system is required to keep them moving at the velocity **v**).

Inertial Frames

As we saw in Section 4.6, a moving object can be observed from any number of reference frames. Newton's first law, sometimes called the *law of inertia*, defines a special set of reference frames called *inertial frames*. **An inertial frame of reference**

Definition of inertial frame

Figure 5.3 Unless a net external force acts on it, an object at rest remains at rest and an object in motion continues in motion with constant velocity. In this case, the wall of the building did not exert a force on the moving train that was large enough to stop it.

Isaac Newton **English physicist and mathematician (1642–1727)** Isaac Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today. *(Giraudon/Art Resource)*

is one that is not accelerating. Because Newton's first law deals only with objects that are not accelerating, it holds only in inertial frames. Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame. (The Galilean transformations given by Equations 4.20 and 4.21 relate positions and velocities between two inertial frames.)

A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame, and for our purposes we can consider planet Earth as being such a frame. The Earth is not really an inertial frame because of its orbital motion around the Sun and its rotational motion about its own axis. As the Earth travels in its nearly circular orbit around the Sun, it experiences an acceleration of about 4.4×10^{-3} m/s^2 directed toward the Sun. In addition, because the Earth rotates about its own axis once every 24 h, a point on the equator experiences an additional acceleration of 3.37×10^{-2} m/s^2 directed toward the center of the Earth. However, these accelerations are small compared with g and can often be neglected. For this reason, we assume that the Earth is an inertial frame, as is any other frame attached to it.

If an object is moving with constant velocity, an observer in one inertial frame (say, one at rest relative to the object) claims that the acceleration of the object and the resultant force acting on it are zero. An observer in *any other* inertial frame also finds that $\mathbf{a} = 0$ and $\Sigma\mathbf{F} = 0$ for the object. According to the first law, a body at rest and one moving with constant velocity are equivalent. A passenger in a car moving along a straight road at a constant speed of 100 km/h can easily pour coffee into a cup. But if the driver steps on the gas or brake pedal or turns the steering wheel while the coffee is being poured, the car accelerates and it is no longer an inertial frame. The laws of motion do not work as expected, and the coffee ends up in the passenger's lap!

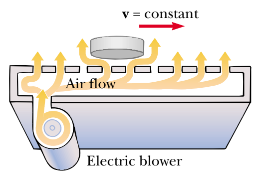


Figure 5.4 Air hockey takes advantage of Newton's first law to make the game more exciting.

Quick Quiz 5.1

True or false: (a) It is possible to have motion in the absence of a force. (b) It is possible to have force in the absence of motion.

5.3 MASS

4.3 Imagine playing catch with either a basketball or a bowling ball. Which ball is more likely to keep moving when you try to catch it? Which ball has the greater tendency to remain motionless when you try to throw it? Because the bowling ball is more resistant to changes in its velocity, we say it has greater inertia than the basketball. As noted in the preceding section, inertia is a measure of how an object responds to an external force.

Definition of mass

Mass is that property of an object that specifies how much inertia the object has, and as we learned in Section 1.1, the SI unit of mass is the kilogram. The greater the mass of an object, the less that object accelerates under the action of an applied force. For example, if a given force acting on a 3-kg mass produces an acceleration of 4 m/s^2, then the same force applied to a 6-kg mass produces an acceleration of 2 m/s^2.

To describe mass quantitatively, we begin by comparing the accelerations a given force produces on different objects. Suppose a force acting on an object of mass m_1 produces an acceleration $\mathbf{a}_1$, and the *same force* acting on an object of mass m_2 produces an acceleration $\mathbf{a}_2$. The ratio of the two masses is defined as the *inverse* ratio of the magnitudes of the accelerations produced by the force:

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1} \quad \textbf{(5.1)}$$

If one object has a known mass, the mass of the other object can be obtained from acceleration measurements.

Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it. Also, **mass is a scalar quantity** and thus obeys the rules of ordinary arithmetic. That is, several masses can be combined in simple numerical fashion. For example, if you combine a 3-kg mass with a 5-kg mass, their total mass is 8 kg. We can verify this result experimentally by comparing the acceleration that a known force gives to several objects separately with the acceleration that the same force gives to the same objects combined as one unit.

Mass and weight are different quantities

Mass should not be confused with weight. **Mass and weight are two different quantities.** As we see later in this chapter, the weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location. For example, a person who weighs 180 lb on the Earth weighs only about 30 lb on the Moon. On the other hand, the mass of a body is the same everywhere: an object having a mass of 2 kg on the Earth also has a mass of 2 kg on the Moon.

5.4 NEWTON'S SECOND LAW

4.4 Newton's first law explains what happens to an object when no forces act on it. It either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object that has a nonzero resultant force acting on it.

Imagine pushing a block of ice across a frictionless horizontal surface. When you exert some horizontal force **F**, the block moves with some acceleration **a**. If you apply a force twice as great, the acceleration doubles. If you increase the applied force to 3**F**, the acceleration triples, and so on. From such observations, we conclude that **the acceleration of an object is directly proportional to the resultant force acting on it.**

The acceleration of an object also depends on its mass, as stated in the preceding section. We can understand this by considering the following experiment. If you apply a force **F** to a block of ice on a frictionless surface, then the block undergoes some acceleration **a**. If the mass of the block is doubled, then the same applied force produces an acceleration **a**/2. If the mass is tripled, then the same applied force produces an acceleration **a**/3, and so on. According to this observation, we conclude that **the magnitude of the acceleration of an object is inversely proportional to its mass.**

These observations are summarized in **Newton's second law:**

Newton's second law

> The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Thus, we can relate mass and force through the following mathematical statement of Newton's second law:[1]

$$\sum \mathbf{F} = m\mathbf{a} \quad \textbf{(5.2)}$$

Note that this equation is a vector expression and hence is equivalent to three component equations:

Newton's second law—component form

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \quad \textbf{(5.3)}$$

Quick Quiz 5.2

Is there any relationship between the net force acting on an object and the direction in which the object moves?

Unit of Force

The SI unit of force is the **newton,** which is defined as the force that, when acting on a 1-kg mass, produces an acceleration of 1 m/s^2. From this definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time:

Definition of newton

$$1\text{ N} \equiv 1\text{ kg}\cdot\text{m/s}^2 \quad \textbf{(5.4)}$$

In the British engineering system, the unit of force is the **pound,** which is defined as the force that, when acting on a 1-slug mass,[2] produces an acceleration of 1 ft/s^2:

$$1\text{ lb} \equiv 1\text{ slug}\cdot\text{ft/s}^2 \quad \textbf{(5.5)}$$

A convenient approximation is that $1\text{ N} \approx \frac{1}{4}$ lb.

[1] Equation 5.2 is valid only when the speed of the object is much less than the speed of light. We treat the relativistic situation in Chapter 39.

[2] The *slug* is the unit of mass in the British engineering system and is that system's counterpart of the SI unit the *kilogram*. Because most of the calculations in our study of classical mechanics are in SI units, the slug is seldom used in this text.

TABLE 5.1 Units of Force, Mass, and Acceleration[a]

System of Units	Mass	Acceleration	Force
SI	kg	m/s^2	$N = kg \cdot m/s^2$
British engineering	slug	ft/s^2	$lb = slug \cdot ft/s^2$

[a] 1 N = 0.225 lb.

The units of force, mass, and acceleration are summarized in Table 5.1.

We can now understand how a single person can hold up an airship but is not able to change its motion abruptly, as stated at the beginning of the chapter. The mass of the blimp is greater than 6 800 kg. In order to make this large mass accelerate appreciably, a very large force is required—certainly one much greater than a human can provide.

EXAMPLE 5.1 An Accelerating Hockey Puck

A hockey puck having a mass of 0.30 kg slides on the horizontal, frictionless surface of an ice rink. Two forces act on the puck, as shown in Figure 5.5. The force $\mathbf{F}_1$ has a magnitude of 5.0 N, and the force $\mathbf{F}_2$ has a magnitude of 8.0 N. Determine both the magnitude and the direction of the puck's acceleration.

Solution The resultant force in the x direction is

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20°) + F_2 \cos 60°$$
$$= (5.0\ \text{N})(0.940) + (8.0\ \text{N})(0.500) = 8.7\ \text{N}$$

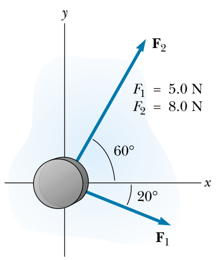


Figure 5.5 A hockey puck moving on a frictionless surface accelerates in the direction of the resultant force $\mathbf{F}_1 + \mathbf{F}_2$.

The resultant force in the y direction is

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20°) + F_2 \sin 60°$$
$$= (5.0\ \text{N})(-0.342) + (8.0\ \text{N})(0.866) = 5.2\ \text{N}$$

Now we use Newton's second law in component form to find the x and y components of acceleration:

$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7\ \text{N}}{0.30\ \text{kg}} = 29\ \text{m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{5.2\ \text{N}}{0.30\ \text{kg}} = 17\ \text{m/s}^2$$

The acceleration has a magnitude of

$$a = \sqrt{(29)^2 + (17)^2}\ \text{m/s}^2 = \boxed{34\ \text{m/s}^2}$$

and its direction relative to the positive x axis is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = \boxed{30°}$$

We can graphically add the vectors in Figure 5.5 to check the reasonableness of our answer. Because the acceleration vector is along the direction of the resultant force, a drawing showing the resultant force helps us check the validity of the answer.

Exercise Determine the components of a third force that, when applied to the puck, causes it to have zero acceleration.

Answer $F_{3x} = -8.7$ N, $F_{3y} = -5.2$ N.

5.5 THE FORCE OF GRAVITY AND WEIGHT

We are well aware that all objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **force of gravity $\mathbf{F}_g$**. This force is directed toward the center of the Earth,[3] and its magnitude is called the **weight** of the object.

Definition of weight

We saw in Section 2.6 that a freely falling object experiences an acceleration $\mathbf{g}$ acting toward the center of the Earth. Applying Newton's second law $\Sigma\mathbf{F} = m\mathbf{a}$ to a freely falling object of mass m, with $\mathbf{a} = \mathbf{g}$ and $\Sigma\mathbf{F} = \mathbf{F}_g$, we obtain

$$\mathbf{F}_g = m\mathbf{g} \qquad \textbf{(5.6)}$$

Thus, the weight of an object, being defined as the magnitude of $\mathbf{F}_g$, is mg. (You should not confuse the italicized symbol g for gravitational acceleration with the nonitalicized symbol g used as the abbreviation for "gram.")

Because it depends on g, weight varies with geographic location. Hence, weight, unlike mass, is not an inherent property of an object. Because g decreases with increasing distance from the center of the Earth, bodies weigh less at higher altitudes than at sea level. For example, a 1 000-kg palette of bricks used in the construction of the Empire State Building in New York City weighed about 1 N less by the time it was lifted from sidewalk level to the top of the building. As another example, suppose an object has a mass of 70.0 kg. Its weight in a location where $g = 9.80\ \text{m/s}^2$ is $F_g = mg = 686$ N (about 150 lb). At the top of a mountain, however, where $g = 9.77\ \text{m/s}^2$, its weight is only 684 N. Therefore, if you want to lose weight without going on a diet, climb a mountain or weigh yourself at 30 000 ft during an airplane flight!

Because weight $= F_g = mg$, we can compare the masses of two objects by measuring their weights on a spring scale. At a given location, the ratio of the weights of two objects equals the ratio of their masses.

QuickLab

Drop a pen and your textbook simultaneously from the same height and watch as they fall. How can they have the same acceleration when their weights are so different?

The life-support unit strapped to the back of astronaut Edwin Aldrin weighed 300 lb on the Earth. During his training, a 50-lb mock-up was used. Although this effectively simulated the reduced weight the unit would have on the Moon, it did not correctly mimic the unchanging mass. It was just as difficult to accelerate the unit (perhaps by jumping or twisting suddenly) on the Moon as on the Earth.

[3] This statement ignores the fact that the mass distribution of the Earth is not perfectly spherical.

Conceptual Example 5.2 How Much Do You Weigh in an Elevator?

You have most likely had the experience of standing in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force magnitude that is greater than your weight. Thus, you have tactile and measured evidence that leads you to believe you are heavier in this situation. *Are* you heavier?

Solution No, your weight is unchanged. To provide the acceleration upward, the floor or scale must exert on your feet an upward force that is greater in magnitude than your weight. It is this greater force that you feel, which you interpret as feeling heavier. The scale reads this upward force, not your weight, and so its reading increases.

Quick Quiz 5.3

A baseball of mass m is thrown upward with some initial speed. If air resistance is neglected, what forces are acting on the ball when it reaches (a) half its maximum height and (b) its maximum height?

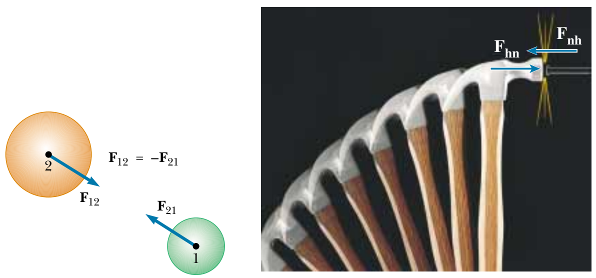
5.6 NEWTON'S THIRD LAW

4.5 If you press against a corner of this textbook with your fingertip, the book pushes back and makes a small dent in your skin. If you push harder, the book does the same and the dent in your skin gets a little larger. This simple experiment illustrates a general principle of critical importance known as **Newton's third law:**

Newton's third law

> If two objects interact, the force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1:
>
> $$\mathbf{F}_{12} = -\mathbf{F}_{21} \quad (5.7)$$

This law, which is illustrated in Figure 5.6a, states that a force that affects the motion of an object must come from a second, *external,* object. The external object, in turn, is subject to an equal-magnitude but oppositely directed force exerted on it.



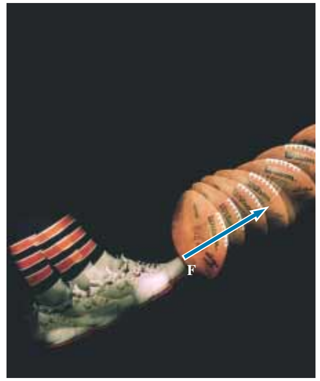
(a) (b)

Figure 5.6 Newton's third law. (a) The force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1. (b) The force $\mathbf{F}_{hn}$ exerted by the hammer on the nail is equal to and opposite the force $\mathbf{F}_{nh}$ exerted by the nail on the hammer.

This is equivalent to stating that **a single isolated force cannot exist.** The force that object 1 exerts on object 2 is sometimes called the *action force,* while the force object 2 exerts on object 1 is called the *reaction force.* In reality, either force can be labeled the action or the reaction force. **The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects.** For example, the force acting on a freely falling projectile is $\mathbf{F}_g = m\mathbf{g}$, which is the force of gravity exerted by the Earth on the projectile. The reaction to this force is the force exerted by the projectile on the Earth, $\mathbf{F}_g' = -\mathbf{F}_g$. The reaction force $\mathbf{F}_g'$ accelerates the Earth toward the projectile just as the action force $\mathbf{F}_g$ accelerates the projectile toward the Earth. However, because the Earth has such a great mass, its acceleration due to this reaction force is negligibly small.

Another example of Newton's third law is shown in Figure 5.6b. The force exerted by the hammer on the nail (the action force $\mathbf{F}_{hn}$) is equal in magnitude and opposite in direction to the force exerted by the nail on the hammer (the reaction force $\mathbf{F}_{nh}$). It is this latter force that causes the hammer to stop its rapid forward motion when it strikes the nail.

You experience Newton's third law directly whenever you slam your fist against a wall or kick a football. You should be able to identify the action and reaction forces in these cases.



Compression of a football as the force exerted by a player's foot sets the ball in motion.

Quick Quiz 5.4

A person steps from a boat toward a dock. Unfortunately, he forgot to tie the boat to the dock, and the boat scoots away as he steps from it. Analyze this situation in terms of Newton's third law.

The force of gravity $\mathbf{F}_g$ was defined as the attractive force the Earth exerts on an object. If the object is a TV at rest on a table, as shown in Figure 5.7a, why does the TV not accelerate in the direction of $\mathbf{F}_g$? The TV does not accelerate because the table holds it up. What is happening is that the table exerts on the TV an upward force $\mathbf{n}$ called the **normal force.**[4] The normal force is a contact force that prevents the TV from falling through the table and can have any magnitude needed to balance the downward force $\mathbf{F}_g$, up to the point of breaking the table. If someone stacks books on the TV, the normal force exerted by the table on the TV increases. If someone lifts up on the TV, the normal force exerted by the table on the TV decreases. (The normal force becomes zero if the TV is raised off the table.)

Definition of normal force

The two forces in an action–reaction pair **always act on different objects.** For the hammer-and-nail situation shown in Figure 5.6b, one force of the pair acts on the hammer and the other acts on the nail. For the unfortunate person stepping out of the boat in Quick Quiz 5.4, one force of the pair acts on the person, and the other acts on the boat.

For the TV in Figure 5.7, the force of gravity $\mathbf{F}_g$ and the normal force $\mathbf{n}$ are *not* an action–reaction pair because they act on the same body—the TV. The two reaction forces in this situation—$\mathbf{F}_g'$ and $\mathbf{n}'$—are exerted on objects other than the TV. Because the reaction to $\mathbf{F}_g$ is the force $\mathbf{F}_g'$ exerted by the TV on the Earth and the reaction to $\mathbf{n}$ is the force $\mathbf{n}'$ exerted by the TV on the table, we conclude that

$$\mathbf{F}_g = -\mathbf{F}_g' \quad \text{and} \quad \mathbf{n} = -\mathbf{n}'$$

[4] *Normal* in this context means *perpendicular.*

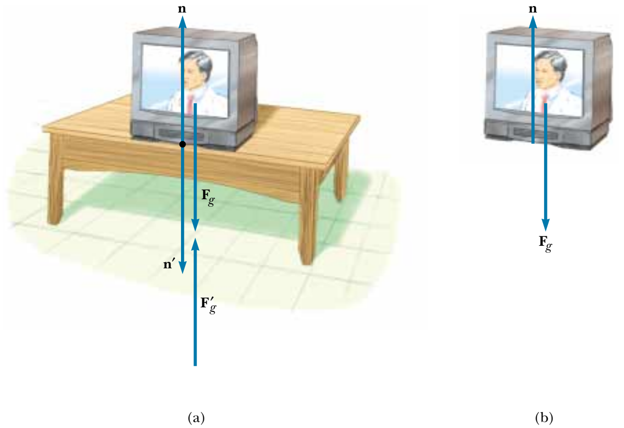


Figure 5.7 When a TV is at rest on a table, the forces acting on the TV are the normal force **n** and the force of gravity $\mathbf{F}_g$, as illustrated in part (b). The reaction to **n** is the force **n**′ exerted by the TV on the table. The reaction to $\mathbf{F}_g$ is the force $\mathbf{F}_g'$ exerted by the TV on the Earth.

The forces **n** and **n**′ have the same magnitude, which is the same as that of $\mathbf{F}_g$ until the table breaks. From the second law, we see that, because the TV is in equilibrium ($\mathbf{a} = 0$), it follows[5] that $F_g = n = mg$.

Quick Quiz 5.5

If a fly collides with the windshield of a fast-moving bus, (a) which experiences the greater impact force: the fly or the bus, or is the same force experienced by both? (b) Which experiences the greater acceleration: the fly or the bus, or is the same acceleration experienced by both?

Conceptual Example 5.3 You Push Me and I'll Push You

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. (a) Who moves away with the higher speed?

Solution This situation is similar to what we saw in Quick Quiz 5.5. According to Newton's third law, the force exerted by the man on the boy and the force exerted by the boy on the man are an action–reaction pair, and so they must be equal in magnitude. (A bathroom scale placed between their hands would read the same, regardless of which way it faced.) Therefore, the boy, having the lesser mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

(b) Who moves farther while their hands are in contact?

Solution Because the boy has the greater acceleration, he moves farther during the interval in which the hands are in contact.

[5] Technically, we should write this equation in the component form $F_{gy} = n_y = mg_y$. This component notation is cumbersome, however, and so in situations in which a vector is parallel to a coordinate axis, we usually do not include the subscript for that axis because there is no other component.

5.7 SOME APPLICATIONS OF NEWTON'S LAWS

4.6 In this section we apply Newton's laws to objects that are either in equilibrium ($\mathbf{a} = 0$) or accelerating along a straight line under the action of constant external forces. We assume that the objects behave as particles so that we need not worry about rotational motion. We also neglect the effects of friction in those problems involving motion; this is equivalent to stating that the surfaces are *frictionless.* Finally, we usually neglect the mass of any ropes involved. In this approximation, the magnitude of the force exerted at any point along a rope is the same at all points along the rope. In problem statements, the synonymous terms *light, lightweight,* and *of negligible mass* are used to indicate that a mass is to be ignored when you work the problems.

When we apply Newton's laws to an object, we are interested only in external forces that act on the object. For example, in Figure 5.7 the only external forces acting on the TV are **n** and $\mathbf{F}_g$. The reactions to these forces, **n**′ and $\mathbf{F}_g'$, act on the table and on the Earth, respectively, and therefore do not appear in Newton's second law applied to the TV.

Tension

When a rope attached to an object is pulling on the object, the rope exerts a force **T** on the object, and the magnitude of that force is called the **tension** in the rope. Because it is the magnitude of a vector quantity, tension is a scalar quantity.

Consider a crate being pulled to the right on a frictionless, horizontal surface, as shown in Figure 5.8a. Suppose you are asked to find the acceleration of the crate and the force the floor exerts on it. First, note that the horizontal force being applied to the crate acts through the rope. Use the symbol **T** to denote the force exerted by the rope on the crate. The magnitude of **T** is equal to the tension in the rope. A dotted circle is drawn around the crate in Figure 5.8a to remind you that you are interested only in the forces acting on the crate. These are illustrated in Figure 5.8b. In addition to the force **T**, this force diagram for the crate includes the force of gravity $\mathbf{F}_g$ and the normal force **n** exerted by the floor on the crate. Such a force diagram, referred to as a **free-body diagram,** shows all external forces acting on the object. The construction of a correct free-body diagram is an important step in applying Newton's laws. The *reactions* to the forces we have listed—namely, the force exerted by the crate on the rope, the force exerted by the crate on the Earth, and the force exerted by the crate on the floor—are *not* included in the free-body diagram because they act on *other* bodies and not on the crate.

We can now apply Newton's second law in component form to the crate. The only force acting in the x direction is **T**. Applying $\Sigma F_x = ma_x$ to the horizontal motion gives

$$\sum F_x = T = ma_x \qquad \text{or} \qquad a_x = \frac{T}{m}$$

No acceleration occurs in the y direction. Applying $\Sigma F_y = ma_y$ with $a_y = 0$ yields

$$n + (-F_g) = 0 \qquad \text{or} \qquad n = F_g$$

That is, the normal force has the same magnitude as the force of gravity but is in the opposite direction.

If **T** is a constant force, then the acceleration $a_x = T/m$ also is constant. Hence, the constant-acceleration equations of kinematics from Chapter 2 can be used to obtain the crate's displacement Δx and velocity v_x as functions of time. Be-

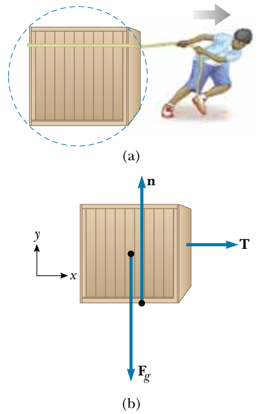


Figure 5.8 (a) A crate being pulled to the right on a frictionless surface. (b) The free-body diagram representing the external forces acting on the crate.

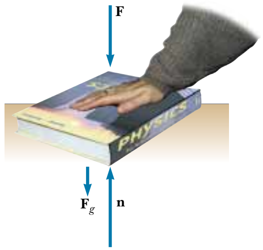


Figure 5.9 When one object pushes downward on another object with a force **F**, the normal force **n** is greater than the force of gravity: $n = F_g + F$.

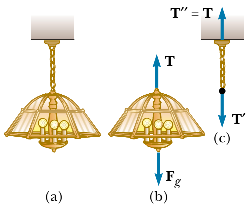


Figure 5.10 (a) A lamp suspended from a ceiling by a chain of negligible mass. (b) The forces acting on the lamp are the force of gravity $\mathbf{F}_g$ and the force exerted by the chain **T**. (c) The forces acting on the chain are the force exerted by the lamp **T**′ and the force exerted by the ceiling **T**″.

cause $a_x = T/m =$ constant, Equations 2.8 and 2.11 can be written as

$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t$$

$$\Delta x = v_{xi}t + \tfrac{1}{2}\left(\frac{T}{m}\right)t^2$$

In the situation just described, the magnitude of the normal force **n** is equal to the magnitude of $\mathbf{F}_g$, but this is not always the case. For example, suppose a book is lying on a table and you push down on the book with a force **F**, as shown in Figure 5.9. Because the book is at rest and therefore not accelerating, $\Sigma F_y = 0$, which gives $n - F_g - F = 0$, or $n = F_g + F$. Other examples in which $n \neq F_g$ are presented later.

Consider a lamp suspended from a light chain fastened to the ceiling, as in Figure 5.10a. The free-body diagram for the lamp (Figure 5.10b) shows that the forces acting on the lamp are the downward force of gravity $\mathbf{F}_g$ and the upward force **T** exerted by the chain. If we apply the second law to the lamp, noting that $\mathbf{a} = 0$, we see that because there are no forces in the x direction, $\Sigma F_x = 0$ provides no helpful information. The condition $\Sigma F_y = ma_y = 0$ gives

$$\sum F_y = T - F_g = 0 \qquad \text{or} \qquad T = F_g$$

Again, note that **T** and $\mathbf{F}_g$ are *not* an action–reaction pair because they act on the same object—the lamp. The reaction force to **T** is **T**′, the downward force exerted by the lamp on the chain, as shown in Figure 5.10c. The ceiling exerts on the chain a force **T**″ that is equal in magnitude to the magnitude of **T**′ and points in the opposite direction.

Problem-Solving Hints

Applying Newton's Laws

The following procedure is recommended when dealing with problems involving Newton's laws:

- Draw a simple, neat diagram of the system.
- Isolate the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw *separate* free-body diagrams for each object. *Do not* include in the free-body diagram forces exerted by the object on its surroundings. Establish convenient coordinate axes for each object and find the components of the forces along these axes.
- Apply Newton's second law, $\Sigma\mathbf{F} = m\mathbf{a}$, in component form. Check your dimensions to make sure that all terms have units of force.
- Solve the component equations for the unknowns. Remember that you must have as many independent equations as you have unknowns to obtain a complete solution.
- Make sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

EXAMPLE 5.4 A Traffic Light at Rest

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension in the three cables.

Solution Figure 5.11a shows the type of drawing we might make of this situation. We then construct two free-body diagrams—one for the traffic light, shown in Figure 5.11b, and one for the knot that holds the three cables together, as seen in Figure 5.11c. This knot is a convenient object to choose because all the forces we are interested in act through it. Because the acceleration of the system is zero, we know that the net force on the light and the net force on the knot are both zero.

In Figure 5.11b the force $\mathbf{T}_3$ exerted by the vertical cable supports the light, and so $T_3 = F_g =$ 125 N. Next, we choose the coordinate axes shown in Figure 5.11c and resolve the forces acting on the knot into their components:

Force	x Component	y Component
$\mathbf{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\mathbf{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\mathbf{T}_3$	0	-125 N

Knowing that the knot is in equilibrium ($\mathbf{a} = 0$) allows us to write

$$(1) \qquad \sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$(2) \qquad \sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-125\ \text{N}) = 0$$

From (1) we see that the horizontal components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must be equal in magnitude, and from (2) we see that the sum of the vertical components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must balance the weight of the light. We solve (1) for T_2 in terms of T_1 to obtain

$$T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33T_1$$

This value for T_2 is substituted into (2) to yield

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 125\ \text{N} = 0$$

$$T_1 = 75.1\ \text{N}$$

$$T_2 = 1.33T_1 = 99.9\ \text{N}$$

This problem is important because it combines what we have learned about vectors with the new topic of forces. The general approach taken here is very powerful, and we will repeat it many times.

Exercise In what situation does $T_1 = T_2$?

Answer When the two cables attached to the support make equal angles with the horizontal.

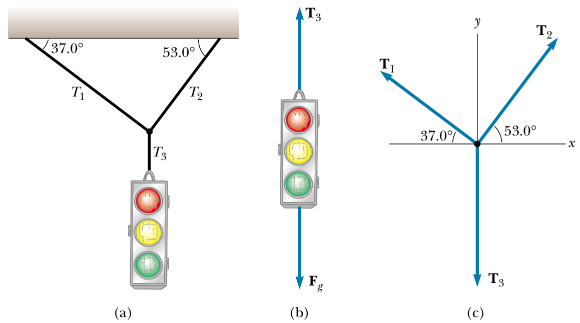


Figure 5.11 (a) A traffic light suspended by cables. (b) Free-body diagram for the traffic light. (c) Free-body diagram for the knot where the three cables are joined.

CONCEPTUAL EXAMPLE 5.5 Forces Between Cars in a Train

In a train, the cars are connected by *couplers,* which are under tension as the locomotive pulls the train. As you move down the train from locomotive to caboose, does the tension in the couplers increase, decrease, or stay the same as the train speeds up? When the engineer applies the brakes, the couplers are under compression. How does this compression force vary from locomotive to caboose? (Assume that only the brakes on the wheels of the engine are applied.)

Solution As the train speeds up, the tension decreases from the front of the train to the back. The coupler between the locomotive and the first car must apply enough force to accelerate all of the remaining cars. As you move back along the train, each coupler is accelerating less mass behind it. The last coupler has to accelerate only the caboose, and so it is under the least tension.

When the brakes are applied, the force again decreases from front to back. The coupler connecting the locomotive to the first car must apply a large force to slow down all the remaining cars. The final coupler must apply a force large enough to slow down only the caboose.

EXAMPLE 5.6 Crate on a Frictionless Incline

A crate of mass m is placed on a frictionless inclined plane of angle θ. (a) Determine the acceleration of the crate after it is released.

Solution Because we know the forces acting on the crate, we can use Newton's second law to determine its acceleration. (In other words, we have classified the problem; this gives us a hint as to the approach to take.) We make a sketch as in Figure 5.12a and then construct the free-body diagram for the crate, as shown in Figure 5.12b. The only forces acting on the crate are the normal force $\mathbf{n}$ exerted by the inclined plane, which acts perpendicular to the plane, and the force of gravity $\mathbf{F}_g = m\mathbf{g}$, which acts vertically downward. For problems involving inclined planes, it is convenient to choose the coordinate axes with x downward along the incline and y perpendicular to it, as shown in Figure 5.12b. (It is possible to solve the problem with "standard" horizontal and vertical axes. You may want to try this, just for practice.) Then, we replace the force of gravity by a component of magnitude $mg\sin\theta$ along the positive x axis and by one of magnitude $mg\cos\theta$ along the negative y axis.

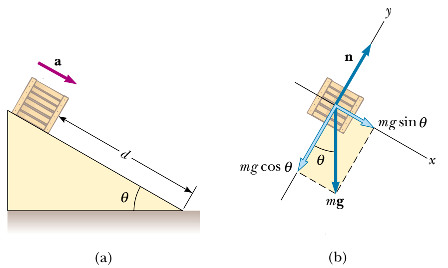


Figure 5.12 (a) A crate of mass m sliding down a frictionless incline. (b) The free-body diagram for the crate. Note that its acceleration along the incline is $g\sin\theta$.

Now we apply Newton's second law in component form, noting that $a_y = 0$:

$$(1)\qquad \sum F_x = mg\sin\theta = ma_x$$

$$(2)\qquad \sum F_y = n - mg\cos\theta = 0$$

Solving (1) for a_x, we see that the acceleration along the incline is caused by the component of $\mathbf{F}_g$ directed down the incline:

$$(3)\qquad a_x = g\sin\theta$$

Note that this acceleration component is independent of the mass of the crate! It depends only on the angle of inclination and on g.

From (2) we conclude that the component of $\mathbf{F}_g$ perpendicular to the incline is balanced by the normal force; that is, $n = mg\cos\theta$. This is one example of a situation in which the normal force is *not* equal in magnitude to the weight of the object.

Special Cases Looking over our results, we see that in the extreme case of $\theta = 90°$, $a_x = g$ and $n = 0$. This condition corresponds to the crate's being in free fall. When $\theta = 0$, $a_x = 0$ and $n = mg$ (its maximum value); in this case, the crate is sitting on a horizontal surface.

(b) Suppose the crate is released from rest at the top of the incline, and the distance from the front edge of the crate to the bottom is d. How long does it take the front edge to reach the bottom, and what is its speed just as it gets there?

Solution Because a_x = constant, we can apply Equation 2.11, $x_f - x_i = v_{xi}t + \frac{1}{2}a_xt^2$, to analyze the crate's motion.

With the displacement $x_f - x_i = d$ and $v_{xi} = 0$, we obtain

$$d = \tfrac{1}{2}a_xt^2$$

$$(4)\qquad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$$

Using Equation 2.12, $v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$, with $v_{xi} = 0$, we find that

$$v_{xf}{}^2 = 2a_xd$$

$$(5)\qquad v_{xf} = \sqrt{2a_xd} = \sqrt{2gd\sin\theta}$$

We see from equations (4) and (5) that the time t needed to reach the bottom and the speed v_{xf}, like acceleration, are independent of the crate's mass. This suggests a simple method you can use to measure g, using an inclined air track: Measure the angle of inclination, some distance traveled by a cart along the incline, and the time needed to travel that distance. The value of g can then be calculated from (4).

EXAMPLE 5.7 One Block Pushes Another

Two blocks of masses m_1 and m_2 are placed in contact with each other on a frictionless horizontal surface. A constant horizontal force $\mathbf{F}$ is applied to the block of mass m_1. (a) Determine the magnitude of the acceleration of the two-block system.

Solution Common sense tells us that both blocks must experience the same acceleration because they remain in contact with each other. Just as in the preceding example, we make a labeled sketch and free-body diagrams, which are shown in Figure 5.13. In Figure 5.13a the dashed line indicates that we treat the two blocks together as a system. Because $\mathbf{F}$ is the only external horizontal force acting on the system (the two blocks), we have

$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$

$$(1)\qquad a_x = \frac{F}{m_1 + m_2}$$

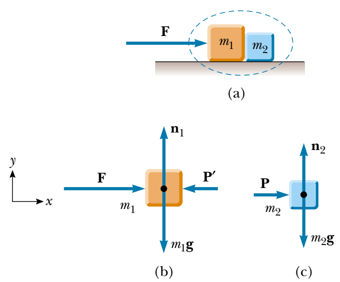


Figure 5.13

Treating the two blocks together as a system simplifies the solution but does not provide information about internal forces.

(b) Determine the magnitude of the contact force between the two blocks.

Solution To solve this part of the problem, we must treat each block separately with its own free-body diagram, as in Figures 5.13b and 5.13c. We denote the contact force by $\mathbf{P}$. From Figure 5.13c, we see that the only horizontal force acting on block 2 is the contact force $\mathbf{P}$ (the force exerted by block 1 on block 2), which is directed to the right. Applying Newton's second law to block 2 gives

$$(2)\qquad \sum F_x = P = m_2a_x$$

Substituting into (2) the value of a_x given by (1), we obtain

$$(3)\qquad P = m_2a_x = \left(\frac{m_2}{m_1 + m_2}\right)F$$

From this result, we see that the contact force $\mathbf{P}$ exerted by block 1 on block 2 is *less* than the applied force $\mathbf{F}$. This is consistent with the fact that the force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

It is instructive to check this expression for P by considering the forces acting on block 1, shown in Figure 5.13b. The horizontal forces acting on this block are the applied force $\mathbf{F}$ to the right and the contact force $\mathbf{P}'$ to the left (the force exerted by block 2 on block 1). From Newton's third law, $\mathbf{P}'$ is the reaction to $\mathbf{P}$, so that $|\mathbf{P}'| = |\mathbf{P}|$. Applying Newton's second law to block 1 produces

$$(4)\qquad \sum F_x = F - P' = F - P = m_1a_x$$

Substituting into (4) the value of a_x from (1), we obtain

$$P = F - m_1 a_x = F - \frac{m_1 F}{m_1 + m_2} = \left(\frac{m_2}{m_1 + m_2}\right) F$$

This agrees with (3), as it must.

Exercise If $m_1 = 4.00$ kg, $m_2 = 3.00$ kg, and $F = 9.00$ N, find the magnitude of the acceleration of the system and the magnitude of the contact force.

Answer $a_x = 1.29\ \text{m/s}^2$; $P = 3.86$ N.

EXAMPLE 5.8 Weighing a Fish in an Elevator

A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator, as illustrated in Figure 5.14. Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

Solution The external forces acting on the fish are the downward force of gravity $\mathbf{F}_g = m\mathbf{g}$ and the force $\mathbf{T}$ exerted by the scale. By Newton's third law, the tension T is also the reading of the scale. If the elevator is either at rest or moving at constant velocity, the fish is not accelerating, and so $\sum F_y = T - mg = 0$ or $T = mg$ (remember that the scalar mg is the weight of the fish).

If the elevator moves upward with an acceleration **a** relative to an observer standing outside the elevator in an inertial frame (see Fig. 5.14a), Newton's second law applied to the fish gives the net force on the fish:

$$(1) \qquad \sum F_y = T - mg = ma_y$$

where we have chosen upward as the positive direction. Thus, we conclude from (1) that the scale reading T is greater than the weight mg if **a** is upward, so that a_y is positive, and that the reading is less than mg if **a** is downward, so that a_y is negative.

For example, if the weight of the fish is 40.0 N and **a** is upward, so that $a_y = +2.00\ \text{m/s}^2$, the scale reading from (1) is

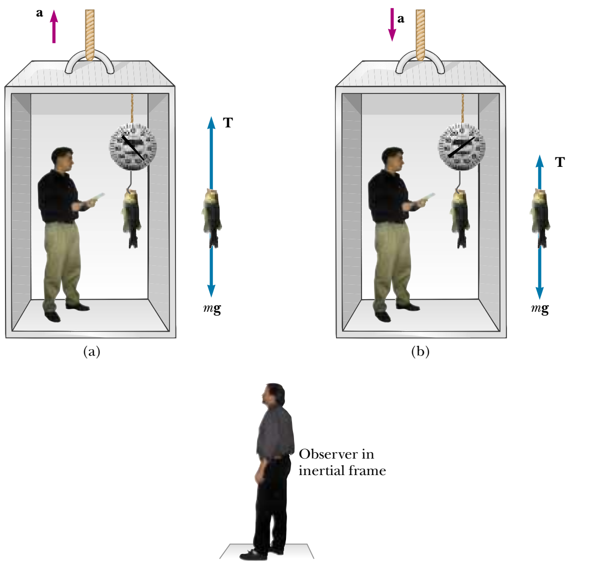


Figure 5.14 Apparent weight versus true weight. (a) When the elevator accelerates upward, the spring scale reads a value greater than the weight of the fish. (b) When the elevator accelerates downward, the spring scale reads a value less than the weight of the fish.

$$(2) \qquad T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right)$$

$$= (40.0\ \text{N})\left(\frac{2.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} + 1\right)$$

$$= \boxed{48.2\ \text{N}}$$

If **a** is downward so that $a_y = -2.00\ \text{m/s}^2$, then (2) gives us

$$T = mg\left(\frac{a_y}{g} + 1\right) = (40.0\ \text{N})\left(\frac{-2.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} + 1\right)$$

$$= \boxed{31.8\ \text{N}}$$

Hence, if you buy a fish by weight in an elevator, make sure the fish is weighed while the elevator is either at rest or accelerating downward! Furthermore, note that from the information given here one cannot determine the direction of motion of the elevator.

Special Cases If the elevator cable breaks, the elevator falls freely and $a_y = -g$. We see from (2) that the scale reading T is zero in this case; that is, the fish appears to be weightless. If the elevator accelerates downward with an acceleration greater than g, the fish (along with the person in the elevator) eventually hits the ceiling because the acceleration of fish and person is still that of a freely falling object relative to an outside observer.

EXAMPLE 5.9 Atwood's Machine

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as shown in Figure 5.15a, the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to measure the free-fall acceleration. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

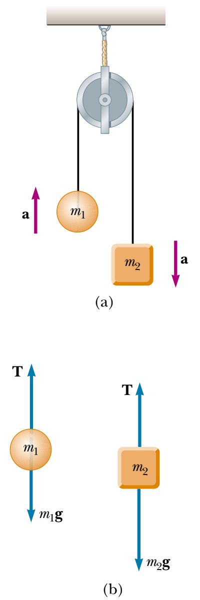


Figure 5.15 Atwood's machine. (a) Two objects ($m_2 > m_1$) connected by a cord of negligible mass strung over a frictionless pulley. (b) Free-body diagrams for the two objects.

Solution If we were to define our system as being made up of both objects, as we did in Example 5.7, we would have to determine an *internal* force (tension in the cord). We must define two systems here—one for each object—and apply Newton's second law to each. The free-body diagrams for the two objects are shown in Figure 5.15b. Two forces act on each object: the upward force **T** exerted by the cord and the downward force of gravity.

We need to be very careful with signs in problems such as this, in which a string or rope passes over a pulley or some other structure that causes the string or rope to bend. In Figure 5.15a, notice that if object 1 accelerates upward, then object 2 accelerates downward. Thus, for consistency with signs, if we define the upward direction as positive for object 1, we must define the downward direction as positive for object 2. With this sign convention, both objects accelerate in the same direction as defined by the choice of sign. With this sign convention applied to the forces, the y component of the net force exerted on object 1 is $T - m_1 g$, and the y component of the net force exerted on object 2 is $m_2 g - T$. Because the objects are connected by a cord, their accelerations must be equal in magnitude. (Otherwise the cord would stretch or break as the distance between the objects increased.) If we assume $m_2 > m_1$, then object 1 must accelerate upward and object 2 downward.

When Newton's second law is applied to object 1, we obtain

$$(1) \qquad \sum F_y = T - m_1 g = m_1 a_y$$

Similarly, for object 2 we find

$$(2) \qquad \sum F_y = m_2 g - T = m_2 a_y$$

When (2) is added to (1), T drops out and we get

$$-m_1g + m_2g = m_1a_y + m_2a_y$$

$$(3) \qquad a_y = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

When (3) is substituted into (1), we obtain

$$(4) \qquad T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

The result for the acceleration in (3) can be interpreted as the ratio of the unbalanced force on the system $(m_2g - m_1g)$ to the total mass of the system $(m_1 + m_2)$, as expected from Newton's second law.

Special Cases When $m_1 = m_2$, then $a_y = 0$ and $T = m_1g$, as we would expect for this balanced case. If $m_2 >> m_1$, then $a_y \approx g$ (a freely falling body) and $T \approx 2m_1g$.

Exercise Find the magnitude of the acceleration and the string tension for an Atwood machine in which $m_1 = 2.00$ kg and $m_2 = 4.00$ kg.

Answer $a_y = 3.27\ \text{m/s}^2$, $T = 26.1$ N.

EXAMPLE 5.10 Acceleration of Two Objects Connected by a Cord

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as shown in Figure 5.16a. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.

Solution Because the objects are connected by a cord (which we assume does not stretch), their accelerations have the same magnitude. The free-body diagrams are shown in Figures 5.16b and 5.16c. Applying Newton's second law in component form to the ball, with the choice of the upward direction as positive, yields

$$(1) \qquad \sum F_x = 0$$

$$(2) \qquad \sum F_y = T - m_1g = m_1a_y = m_1a$$

Note that in order for the ball to accelerate upward, it is necessary that $T > m_1g$. In (2) we have replaced a_y with a because the acceleration has only a y component.

For the block it is convenient to choose the positive x' axis along the incline, as shown in Figure 5.16c. Here we choose the positive direction to be down the incline, in the $+x'$ direction. Applying Newton's second law in component form to the block gives

$$(3) \qquad \sum F_{x'} = m_2g\sin\theta - T = m_2a_{x'} = m_2a$$

$$(4) \qquad \sum F_{y'} = n - m_2g\cos\theta = 0$$

In (3) we have replaced $a_{x'}$ with a because that is the acceleration's only component. In other words, the two objects have accelerations of the same magnitude a, which is what we are trying to find. Equations (1) and (4) provide no information regarding the acceleration. However, if we solve (2) for T and then substitute this value for T into (3) and solve for a, we obtain

$$(5) \qquad a = \frac{m_2g\sin\theta - m_1g}{m_1 + m_2}$$

When this value for a is substituted into (2), we find

$$(6) \qquad T = \frac{m_1m_2g(\sin\theta + 1)}{m_1 + m_2}$$

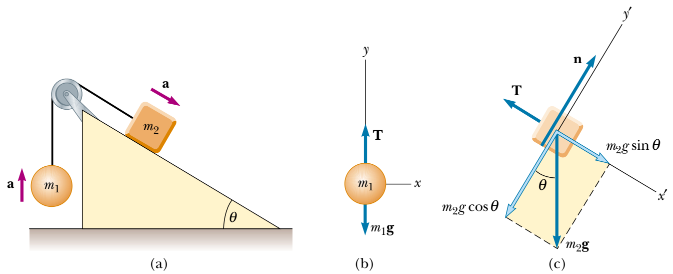


Figure 5.16 (a) Two objects connected by a lightweight cord strung over a frictionless pulley. (b) Free-body diagram for the ball. (c) Free-body diagram for the block. (The incline is frictionless.)

Note that the block accelerates down the incline only if $m_2\sin\theta > m_1$ (that is, if **a** is in the direction we assumed). If $m_1 > m_2\sin\theta$, then the acceleration is up the incline for the block and downward for the ball. Also note that the result for the acceleration (5) can be interpreted as the resultant force acting on the system divided by the total mass of the system; this is consistent with Newton's second law. Finally, if $\theta = 90°$, then the results for a and T are identical to those of Example 5.9.

Exercise If $m_1 = 10.0$ kg, $m_2 = 5.00$ kg, and $\theta = 45.0°$, find the acceleration of each object.

Answer $a = -4.22\ \text{m/s}^2$, where the negative sign indicates that the block accelerates up the incline and the ball accelerates downward.

5.8 FORCES OF FRICTION

When a body is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the body interacts with its surroundings. We call such resistance a **force of friction**. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Have you ever tried to move a heavy desk across a rough floor? You push harder and harder until all of a sudden the desk seems to "break free" and subsequently moves relatively easily. It takes a greater force to start the desk moving than it does to keep it going once it has started sliding. To understand why this happens, consider a book on a table, as shown in Figure 5.17a. If we apply an external horizontal force **F** to the book, acting to the right, the book remains stationary if **F** is not too great. The force that counteracts **F** and keeps the book from moving acts to the left and is called the **frictional force f.**

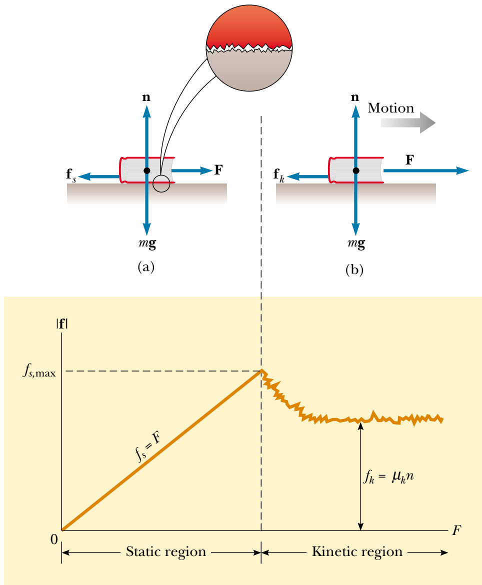
Force of static friction

As long as the book is not moving, $f = F$. Because the book is stationary, we call this frictional force the **force of static friction $\mathbf{f}_s$**. Experiments show that this force arises from contacting points that protrude beyond the general level of the surfaces in contact, even for surfaces that are apparently very smooth, as shown in the magnified view in Figure 5.17a. (If the surfaces are clean and smooth at the atomic level, they are likely to weld together when contact is made.) The frictional force arises in part from one peak's physically blocking the motion of a peak from the opposing surface, and in part from chemical bonding of opposing points as they come into contact. If the surfaces are rough, bouncing is likely to occur, further complicating the analysis. Although the details of friction are quite complex at the atomic level, this force ultimately involves an electrical interaction between atoms or molecules.

Force of kinetic friction

If we increase the magnitude of **F**, as shown in Figure 5.17b, the magnitude of $\mathbf{f}_s$ increases along with it, keeping the book in place. The force $\mathbf{f}_s$ cannot increase indefinitely, however. Eventually the surfaces in contact can no longer supply sufficient frictional force to counteract **F**, and the book accelerates. When it is on the verge of moving, f_s is a maximum, as shown in Figure 5.17c. When F exceeds $f_{s,\text{max}}$, the book accelerates to the right. Once the book is in motion, the retarding frictional force becomes less than $f_{s,\text{max}}$ (see Fig. 5.17c). When the book is in motion, we call the retarding force the **force of kinetic friction $\mathbf{f}_k$**. If $F = f_k$, then the book moves to the right with constant speed. If $F > f_k$, then there is an unbalanced force $F - f_k$ in the positive x direction, and this force accelerates the book to the right. If the applied force **F** is removed, then the frictional force $\mathbf{f}_k$ acting to the left accelerates the book in the negative x direction and eventually brings it to rest.

Experimentally, we find that, to a good approximation, both $f_{s,\text{max}}$ and f_k are proportional to the normal force acting on the book. The following empirical laws of friction summarize the experimental observations:



(c)

Figure 5.17 The direction of the force of friction **f** between a book and a rough surface is opposite the direction of the applied force **F**. Because the two surfaces are both rough, contact is made only at a few points, as illustrated in the "magnified" view. (a) The magnitude of the force of static friction equals the magnitude of the applied force. (b) When the magnitude of the applied force exceeds the magnitude of the force of kinetic friction, the book accelerates to the right. (c) A graph of frictional force versus applied force. Note that $f_{s,\max} > f_k$.

- The direction of the force of static friction between any two surfaces in contact with each other is opposite the direction of relative motion and can have values

$$f_s \le \mu_s n \tag{5.8}$$

where the dimensionless constant μ_s is called the **coefficient of static friction** and n is the magnitude of the normal force. The equality in Equation 5.8 holds when one object is on the verge of moving, that is, when $f_s = f_{s,\max} = \mu_s n$. The inequality holds when the applied force is less than $\mu_s n$.

- The direction of the force of kinetic friction acting on an object is opposite the direction of the object's sliding motion relative to the surface applying the frictional force and is given by

$$f_k = \mu_k n \tag{5.9}$$

where μ_k is the **coefficient of kinetic friction.**

- The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s. Typical values range from around 0.03 to 1.0. Table 5.2 lists some reported values.

TABLE 5.2 Coefficients of Friction[a]

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

[a] All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

- The coefficients of friction are nearly independent of the area of contact between the surfaces. To understand why, we must examine the difference between the *apparent contact area,* which is the area we see with our eyes, and the *real contact area,* represented by two irregular surfaces touching, as shown in the magnified view in Figure 5.17a. It seems that increasing the apparent contact area does not increase the real contact area. When we increase the apparent area (without changing anything else), there is less force per unit area driving the jagged points together. This decrease in force counteracts the effect of having more points involved.

If you would like to learn more about this subject, read the article "Friction at the Atomic Scale" by J. Krim in the October 1996 issue of *Scientific American.*

Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations in this text. We can easily demonstrate the approximate nature of the equations by trying to get a block to slip down an incline at constant speed. Especially at low speeds, the motion is likely to be characterized by alternate episodes of sticking and movement.

QuickLab

Can you apply the ideas of Example 5.12 to determine the coefficients of static and kinetic friction between the cover of your book and a quarter? What should happen to those coefficients if you make the measurements between your book and *two* quarters taped one on top of the other?

Quick Quiz 5.6

A crate is sitting in the center of a flatbed truck. The truck accelerates to the right, and the crate moves with it, not sliding at all. What is the direction of the frictional force exerted by the truck on the crate? (a) To the left. (b) To the right. (c) No frictional force because the crate is not sliding.

CONCEPTUAL EXAMPLE 5.11 Why Does the Sled Accelerate?

A horse pulls a sled along a level, snow-covered road, causing the sled to accelerate, as shown in Figure 5.18a. Newton's third law states that the sled exerts an equal and opposite force on the horse. In view of this, how can the sled accelerate? Under what condition does the system (horse plus sled) move with constant velocity?

Solution It is important to remember that the forces described in Newton's third law act on different objects—the horse exerts a force on the sled, and the sled exerts an equal-magnitude and oppositely directed force on the horse. Because we are interested only in the motion of the sled, we do not consider the forces it exerts on the horse. When deter-

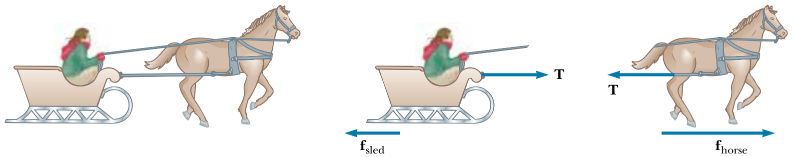


Figure 5.18

mining the motion of an object, you must add only the forces on that object. The horizontal forces exerted on the sled are the forward force **T** exerted by the horse and the backward force of friction $\mathbf{f}_{\text{sled}}$ between sled and snow (see Fig. 5.18b). When the forward force exceeds the backward force, the sled accelerates to the right.

The force that accelerates the system (horse plus sled) is the frictional force $\mathbf{f}_{\text{horse}}$ exerted by the Earth on the horse's feet. The horizontal forces exerted on the horse are the forward force $\mathbf{f}_{\text{horse}}$ exerted by the Earth and the backward tension force **T** exerted by the sled (Fig. 5.18c). The resultant of these two forces causes the horse to accelerate. When $\mathbf{f}_{\text{horse}}$ balances $\mathbf{f}_{\text{sled}}$, the system moves with constant velocity.

Exercise Are the normal force exerted by the snow on the horse and the gravitational force exerted by the Earth on the horse a third-law pair?

Answer No, because they act on the same object. Third-law force pairs are equal in magnitude and opposite in direction, and the forces act on *different* objects.

EXAMPLE 5.12 Experimental Determination of μ_s and μ_k

The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure 5.19. The incline angle is increased until the block starts to move. Let us show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s.

Solution The only forces acting on the block are the force of gravity $m\mathbf{g}$, the normal force **n**, and the force of static friction $\mathbf{f}_s$. These forces balance when the block is on the verge

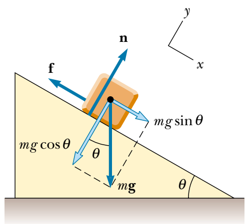


Figure 5.19 The external forces exerted on a block lying on a rough incline are the force of gravity $m\mathbf{g}$, the normal force **n**, and the force of friction **f**. For convenience, the force of gravity is resolved into a component along the incline $mg\sin\theta$ and a component perpendicular to the incline $mg\cos\theta$.

of slipping but has not yet moved. When we take x to be parallel to the plane and y perpendicular to it, Newton's second law applied to the block for this balanced situation gives

Static case: (1) $\sum F_x = mg\sin\theta - f_s = ma_x = 0$

(2) $\sum F_y = n - mg\cos\theta = ma_y = 0$

We can eliminate mg by substituting $mg = n/\cos\theta$ from (2) into (1) to get

$$(3)\quad f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

When the incline is at the critical angle θ_c, we know that $f_s = f_{s,\max} = \mu_s n$, and so at this angle, (3) becomes

$$\mu_s n = n\tan\theta_c$$

Static case: $\mu_s = \tan\theta_c$

For example, if the block just slips at $\theta_c = 20°$, then we find that $\mu_s = \tan 20° = 0.364$.

Once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k n$. However, if θ is reduced to a value less than θ_c, it may be possible to find an angle θ_c' such that the block moves down the incline with constant speed ($a_x = 0$). In this case, using (1) and (2) with f_s replaced by f_k gives

Kinetic case: $\mu_k = \tan\theta_c'$

where $\theta_c' < \theta_c$.

EXAMPLE 5.13 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Solution The forces acting on the puck after it is in motion are shown in Figure 5.20. If we assume that the force of kinetic friction f_k remains constant, then this force produces a uniform acceleration of the puck in the direction opposite its velocity, causing the puck to slow down. First, we find this acceleration in terms of the coefficient of kinetic friction, using Newton's second law. Knowing the acceleration of the puck and the distance it travels, we can then use the equations of kinematics to find the coefficient of kinetic friction.

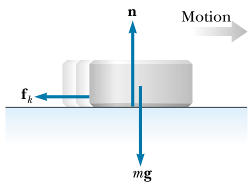


Figure 5.20 *After* the puck is given an initial velocity to the right, the only external forces acting on it are the force of gravity $m\mathbf{g}$, the normal force **n**, and the force of kinetic friction $\mathbf{f}_k$.

Defining rightward and upward as our positive directions, we apply Newton's second law in component form to the puck and obtain

(1) $\sum F_x = -f_k = ma_x$

(2) $\sum F_y = n - mg = 0 \quad (a_y = 0)$

But $f_k = \mu_k n$, and from (2) we see that $n = mg$. Therefore, (1) becomes

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left; this means that the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume that μ_k remains constant.

Because the acceleration is constant, we can use Equation 2.12, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_{xf} = 0$:

$$v_{xi}^2 + 2ax_f = v_{xi}^2 - 2\mu_k g x_f = 0$$

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(115\ \text{m})} = \boxed{0.177}$$

Note that μ_k is dimensionless.

EXAMPLE 5.14 Acceleration of Two Connected Objects When Friction Is Present

A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a lightweight cord over a lightweight, frictionless pulley, as shown in Figure 5.21a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.

Solution We start by drawing free-body diagrams for the two objects, as shown in Figures 5.21b and 5.21c. (Are you beginning to see the similarities in all these examples?) Next, we apply Newton's second law in component form to each object and use Equation 5.9, $f_k = \mu_k n$. Then we can solve for the acceleration in terms of the parameters given.

The applied force **F** has x and y components $F\cos\theta$ and $F\sin\theta$, respectively. Applying Newton's second law to both objects and assuming the motion of the block is to the right, we obtain

Motion of block: (1) $\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

(2) $\sum F_y = n + F\sin\theta - m_1 g = m_1 a_y = 0$

Motion of ball: $\sum F_x = m_2 a_x = 0$

(3) $\sum F_y = T - m_2 g = m_2 a_y = m_2 a$

Note that because the two objects are connected, we can equate the magnitudes of the x component of the acceleration of the block and the y component of the acceleration of the ball. From Equation 5.9 we know that $f_k = \mu_k n$, and from (2) we know that $n = m_1 g - F\sin\theta$ (note that in this case n is *not* equal to $m_1 g$); therefore,

(4) $f_k = \mu_k(m_1 g - F\sin\theta)$

That is, the frictional force is reduced because of the positive

y component of **F**. Substituting (4) and the value of T from (3) into (1) gives

$$F\cos\theta - \mu_k(m_1g - F\sin\theta) - m_2(a+g) = m_1a$$

Solving for a, we obtain

$$(5)\qquad a = \frac{F(\cos\theta + \mu_k\sin\theta) - g(m_2 + \mu_k m_1)}{m_1 + m_2}$$

Note that the acceleration of the block can be either to the right or to the left,[6] depending on the sign of the numerator in (5). If the motion is to the left, then we must reverse the sign of f_k in (1) because the force of kinetic friction must oppose the motion. In this case, the value of a is the same as in (5), with μ_k replaced by $-\mu_k$.

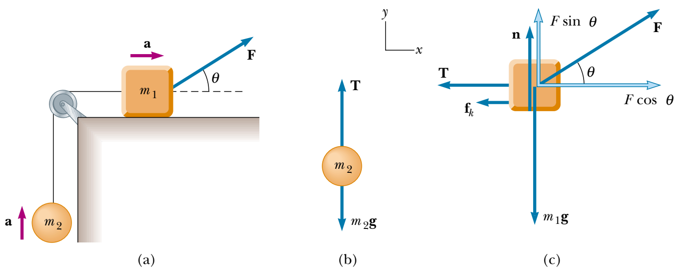


Figure 5.21 (a) The external force **F** applied as shown can cause the block to accelerate to the right. (b) and (c) The free-body diagrams, under the assumption that the block accelerates to the right and the ball accelerates upward. The magnitude of the force of kinetic friction in this case is given by $f_k = \mu_k n = \mu_k(m_1g - F\sin\theta)$.

APPLICATION Automobile Antilock Braking Systems (ABS)

If an automobile tire is rolling and not slipping on a road surface, then the maximum frictional force that the road can exert on the tire is the force of static friction $\mu_s n$. One must use static friction in this situation because at the point of contact between the tire and the road, no sliding of one surface over the other occurs if the tire is not skidding. However, if the tire starts to skid, the frictional force exerted on it is reduced to the force of kinetic friction $\mu_k n$. Thus, to maximize the frictional force and minimize stopping distance, the wheels must maintain pure rolling motion and not skid. An additional benefit of maintaining wheel rotation is that directional control is not lost as it is in skidding.

Unfortunately, in emergency situations drivers typically press down as hard as they can on the brake pedal, "locking the brakes." This stops the wheels from rotating, ensuring a skid and reducing the frictional force from the static to the kinetic case. To address this problem, automotive engineers have developed antilock braking systems (ABS) that very briefly release the brakes when a wheel is just about to stop turning. This maintains rolling contact between the tire and the pavement. When the brakes are released momentarily, the stopping distance is greater than it would be if the brakes were being applied continuously. However, through the use of computer control, the "brake-off" time is kept to a minimum. As a result, the stopping distance is much less than what it would be if the wheels were to skid.

Let us model the stopping of a car by examining real data. In a recent issue of *AutoWeek*,[7] the braking performance for a Toyota Corolla was measured. These data correspond to the braking force acquired by a highly trained, professional driver. We begin by assuming constant acceleration. (Why do we need to make this assumption?) The magazine provided the initial speed and stopping distance in non-SI units. After converting these values to SI we use $v_{xf}^2 = v_{xi}^2 + 2a_xx$ to determine the acceleration at different speeds. These do not vary greatly, and so our assumption of constant acceleration is reasonable.

[6] Equation 5 shows that when $\mu_k m_1 > m_2$, there is a range of values of F for which no motion occurs at a given angle θ.

[7] *AutoWeek* magazine, 48:22–23, 1998.

Initial Speed		Stopping Distance		Acceleration
(mi/h)	(m/s)	(ft)	(m)	(m/s^2)
30	13.4	34	10.4	−8.67
60	26.8	143	43.6	−8.25
80	35.8	251	76.5	−8.36

We take an average value of acceleration of -8.4 m/s^2, which is approximately $0.86g$. We then calculate the coefficient of friction from $\Sigma F = \mu_s mg = ma$; this gives $\mu_s = 0.86$ for the Toyota. This is lower than the rubber-to-concrete value given in Table 5.2. Can you think of any reasons for this?

Let us now estimate the stopping distance of the car if the wheels were skidding. Examining Table 5.2 again, we see that the difference between the coefficients of static and kinetic friction for rubber against concrete is about 0.2. Let us therefore assume that our coefficients differ by the same amount, so that $\mu_k \approx 0.66$. This allows us to calculate estimated stopping distances for the case in which the wheels are locked and the car skids across the pavement. The results illustrate the advantage of not allowing the wheels to skid.

Initial Speed (mi/h)	Stopping Distance no skid (m)	Stopping distance skidding (m)
30	10.4	13.9
60	43.6	55.5
80	76.5	98.9

An ABS keeps the wheels rotating, with the result that the higher coefficient of static friction is maintained between the tires and road. This approximates the technique of a professional driver who is able to maintain the wheels at the point of maximum frictional force. Let us estimate the ABS performance by assuming that the magnitude of the acceleration is not quite as good as that achieved by the professional driver but instead is reduced by 5%.

We now plot in Figure 5.22 vehicle speed versus distance from where the brakes were applied (at an initial speed of 80 mi/h = 37.5 m/s) for the three cases of amateur driver, professional driver, and estimated ABS performance (amateur driver). We find that a markedly shorter distance is necessary for stopping without locking the wheels and skidding and a satisfactory value of stopping distance when the ABS computer maintains tire rotation.

The purpose of the ABS is to help typical drivers (whose tendency is to lock the wheels in an emergency) to better maintain control of their automobiles and minimize stopping distance.

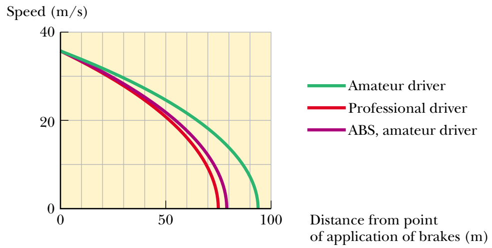


Figure 5.22 This plot of vehicle speed versus distance from where the brakes were applied shows that an antilock braking system (ABS) approaches the performance of a trained professional driver.

SUMMARY

Newton's first law states that, in the absence of an external force, a body at rest remains at rest and a body in uniform motion in a straight line maintains that motion. An **inertial frame** is one that is not accelerating.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The net force acting on an object equals the product of its mass and its acceleration: $\Sigma\mathbf{F} = m\mathbf{a}$. You should be able to apply the x and y component forms of this equation to describe the acceleration of any object acting under the influence of speci-

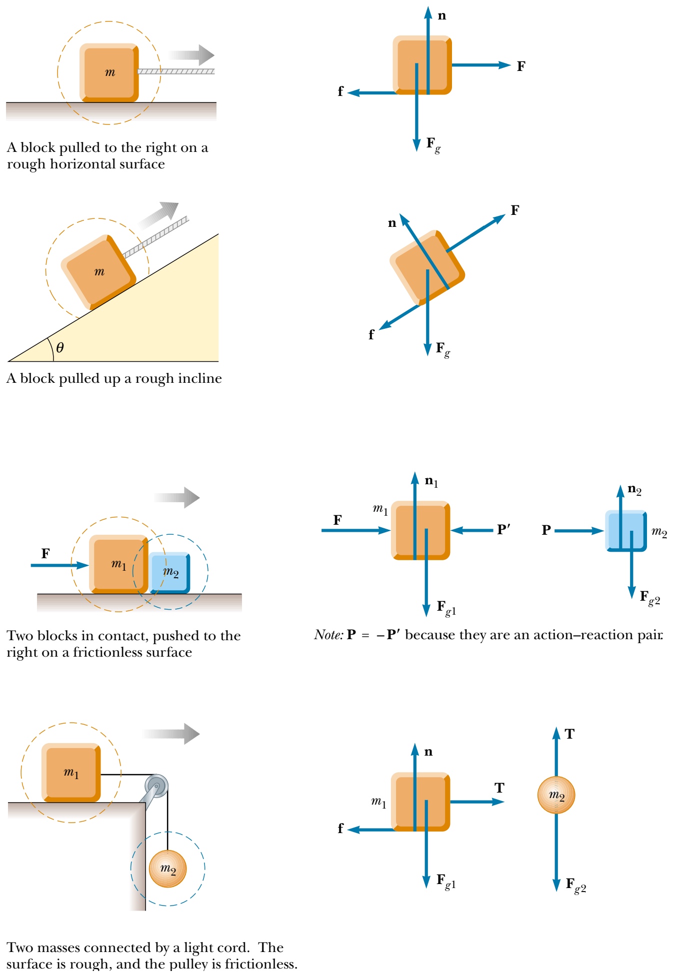


Figure 5.23 Various systems *(left)* and the corresponding free-body diagrams *(right)*.

fied forces. If the object is either stationary or moving with constant velocity, then the forces must vectorially cancel each other.

The **force of gravity** exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration: $\mathbf{F}_g = m\mathbf{g}$. The **weight** of an object is the magnitude of the force of gravity acting on the object.

Newton's third law states that if two objects interact, then the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1. Thus, an isolated force cannot exist in nature. Make sure you can identify third-law pairs and the two objects upon which they act.

The **maximum force of static friction** $\mathbf{f}_{s,\text{max}}$ between an object and a surface is proportional to the normal force acting on the object. In general, $f_s \leq \mu_s n$, where μ_s is the **coefficient of static friction** and n is the magnitude of the normal force. When an object slides over a surface, the direction of the **force of kinetic friction** $\mathbf{f}_k$ is opposite the direction of sliding motion and is also proportional to the magnitude of the normal force. The magnitude of this force is given by $f_k = \mu_k n$, where μ_k is the **coefficient of kinetic friction.**

More on Free-Body Diagrams

To be successful in applying Newton's second law to a system, you must be able to recognize all the forces acting on the system. That is, you must be able to construct the correct free-body diagram. The importance of constructing the free-body diagram cannot be overemphasized. In Figure 5.23 a number of systems are presented together with their free-body diagrams. You should examine these carefully and then construct free-body diagrams for other systems described in the end-of-chapter problems. When a system contains more than one element, it is important that you construct a separate free-body diagram for *each* element.

As usual, $\mathbf{F}$ denotes some applied force, $\mathbf{F}_g = m\mathbf{g}$ is the force of gravity, $\mathbf{n}$ denotes a normal force, $\mathbf{f}$ is the force of friction, and $\mathbf{T}$ is the force whose magnitude is the tension exerted on an object.

QUESTIONS

1. A passenger sitting in the rear of a bus claims that he was injured when the driver slammed on the brakes, causing a suitcase to come flying toward the passenger from the front of the bus. If you were the judge in this case, what disposition would you make? Why?
2. A space explorer is in a spaceship moving through space far from any planet or star. She notices a large rock, taken as a specimen from an alien planet, floating around the cabin of the spaceship. Should she push it gently toward a storage compartment or kick it toward the compartment? Why?
3. A massive metal object on a rough metal surface may undergo contact welding to that surface. Discuss how this affects the frictional force between object and surface.
4. The observer in the elevator of Example 5.8 would claim that the weight of the fish is T, the scale reading. This claim is obviously wrong. Why does this observation differ from that of a person in an inertial frame outside the elevator?
5. Identify the action–reaction pairs in the following situations: a man takes a step; a snowball hits a woman in the back; a baseball player catches a ball; a gust of wind strikes a window.
6. A ball is held in a person's hand. (a) Identify all the external forces acting on the ball and the reaction to each. (b) If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Neglect air resistance.)
7. If a car is traveling westward with a constant speed of 20 m/s, what is the resultant force acting on it?
8. "When the locomotive in Figure 5.3 broke through the wall of the train station, the force exerted by the locomotive on the wall was greater than the force the wall could exert on the locomotive." Is this statement true or in need of correction? Explain your answer.
9. A rubber ball is dropped onto the floor. What force causes the ball to bounce?
10. What is wrong with the statement, "Because the car is at rest, no forces are acting on it"? How would you correct this statement?

11. Suppose you are driving a car along a highway at a high speed. Why should you avoid slamming on your brakes if you want to stop in the shortest distance? That is, why should you keep the wheels turning as you brake?

12. If you have ever taken a ride in an elevator of a high-rise building, you may have experienced a nauseating sensation of "heaviness" and "lightness" depending on the direction of the acceleration. Explain these sensations. Are we truly weightless in free-fall?

13. The driver of a speeding empty truck slams on the brakes and skids to a stop through a distance *d*. (a) If the truck carried a heavy load such that its mass were doubled, what would be its skidding distance? (b) If the initial speed of the truck is halved, what would be its skidding distance?

14. In an attempt to define Newton's third law, a student states that the action and reaction forces are equal in magnitude and opposite in direction to each other. If this is the case, how can there ever be a net force on an object?

15. What force causes (a) a propeller-driven airplane to move? (b) a rocket? (c) a person walking?

16. Suppose a large and spirited Freshman team is beating the Sophomores in a tug-of-war contest. The center of the rope being tugged is gradually accelerating toward the Freshman team. State the relationship between the strengths of these two forces: First, the force the Freshmen exert on the Sophomores; and second, the force the Sophomores exert on the Freshmen.

17. If you push on a heavy box that is at rest, you must exert some force to start its motion. However, once the box is sliding, you can apply a smaller force to maintain that motion. Why?

18. A weight lifter stands on a bathroom scale. He pumps a barbell up and down. What happens to the reading on the scale as this is done? Suppose he is strong enough to actually *throw* the barbell upward. How does the reading on the scale vary now?

19. As a rocket is fired from a launching pad, its speed *and* acceleration increase with time as its engines continue to operate. Explain why this occurs even though the force of the engines exerted on the rocket remains constant.

20. In the motion picture *It Happened One Night* (Columbia Pictures, 1934), Clark Gable is standing inside a stationary bus in front of Claudette Colbert, who is seated. The bus suddenly starts moving forward, and Clark falls into Claudette's lap. Why did this happen?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
□ = paired numerical/symbolic problems

Sections 5.1 through 5.6

1. A force **F** applied to an object of mass m_1 produces an acceleration of 3.00 m/s². The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s². (a) What is the value of the ratio m_1/m_2? (b) If m_1 and m_2 are combined, find their acceleration under the action of the force **F**.

2. A force of 10.0 N acts on a body of mass 2.00 kg. What are (a) the body's acceleration, (b) its weight in newtons, and (c) its acceleration if the force is doubled?

3. A 3.00-kg mass undergoes an acceleration given by $\mathbf{a} = (2.00\mathbf{i} + 5.00\mathbf{j})$ m/s². Find the resultant force $\Sigma\mathbf{F}$ and its magnitude.

4. A heavy freight train has a mass of 15 000 metric tons. If the locomotive can pull with a force of 750 000 N, how long does it take to increase the speed from 0 to 80.0 km/h?

5. A 5.00-g bullet leaves the muzzle of a rifle with a speed of 320 m/s. The expanding gases behind it exert what force on the bullet while it is traveling down the barrel of the rifle, 0.820 m long? Assume constant acceleration and negligible friction.

6. After uniformly accelerating his arm for 0.090 0 s, a pitcher releases a baseball of weight 1.40 N with a velocity of 32.0 m/s horizontally forward. If the ball starts from rest, (a) through what distance does the ball accelerate before its release? (b) What force does the pitcher exert on the ball?

7. After uniformly accelerating his arm for a time *t*, a pitcher releases a baseball of weight $-F_g\mathbf{j}$ with a velocity $v\mathbf{i}$. If the ball starts from rest, (a) through what distance does the ball accelerate before its release? (b) What force does the pitcher exert on the ball?

8. Define one pound as the weight of an object of mass 0.453 592 37 kg at a location where the acceleration due to gravity is 32.174 0 ft/s². Express the pound as one quantity with one SI unit.

WEB **9.** A 4.00-kg object has a velocity of 3.00**i** m/s at one instant. Eight seconds later, its velocity has increased to $(8.00\mathbf{i} + 10.0\mathbf{j})$ m/s. Assuming the object was subject to a constant total force, find (a) the components of the force and (b) its magnitude.

10. The average speed of a nitrogen molecule in air is about 6.70×10^2 m/s, and its mass is 4.68×10^{-26} kg. (a) If it takes 3.00×10^{-13} s for a nitrogen molecule to hit a wall and rebound with the same speed but moving in the opposite direction, what is the average acceleration of the molecule during this time interval? (b) What average force does the molecule exert on the wall?

11. An electron of mass 9.11×10^{-31} kg has an initial speed of 3.00×10^5 m/s. It travels in a straight line, and its speed increases to 7.00×10^5 m/s in a distance of 5.00 cm. Assuming its acceleration is constant, (a) determine the force exerted on the electron and (b) compare this force with the weight of the electron, which we neglected.

12. A woman weighs 120 lb. Determine (a) her weight in newtons and (b) her mass in kilograms.

13. If a man weighs 900 N on the Earth, what would he weigh on Jupiter, where the acceleration due to gravity is 25.9 m/s²?

14. The distinction between mass and weight was discovered after Jean Richer transported pendulum clocks from Paris to French Guiana in 1671. He found that they ran slower there quite systematically. The effect was reversed when the clocks returned to Paris. How much weight would you personally lose in traveling from Paris, where $g = 9.809\ 5$ m/s², to Cayenne, where $g = 9.780\ 8$ m/s²? (We shall consider how the free-fall acceleration influences the period of a pendulum in Section 13.4.)

15. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a 5.00-kg mass. If $F_1 = 20.0$ N and $F_2 = 15.0$ N, find the accelerations in (a) and (b) of Figure P5.15.

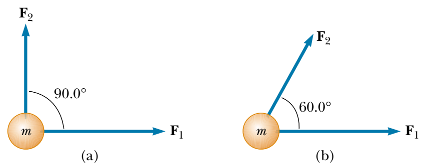


Figure P5.15

16. Besides its weight, a 2.80-kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.20\text{ m})\mathbf{i} - (3.30\text{ m})\mathbf{j}$, where the direction of **j** is the upward vertical direction. Determine the other force.

17. You stand on the seat of a chair and then hop off. (a) During the time you are in flight down to the floor, the Earth is lurching up toward you with an acceleration of what order of magnitude? In your solution explain your logic. Visualize the Earth as a perfectly solid object. (b) The Earth moves up through a distance of what order of magnitude?

18. Forces of 10.0 N north, 20.0 N east, and 15.0 N south are simultaneously applied to a 4.00-kg mass as it rests on an air table. Obtain the object's acceleration.

19. A boat moves through the water with two horizontal forces acting on it. One is a 2000-N forward push caused by the motor; the other is a constant 1800-N resistive force caused by the water. (a) What is the acceleration of the 1 000-kg boat? (b) If it starts from rest, how far will it move in 10.0 s? (c) What will be its speed at the end of this time?

20. Three forces, given by $\mathbf{F}_1 = (-2.00\mathbf{i} + 2.00\mathbf{j})$ N, $\mathbf{F}_2 = (5.00\mathbf{i} - 3.00\mathbf{j})$ N, and $\mathbf{F}_3 = (-45.0\mathbf{i})$ N, act on an object to give it an acceleration of magnitude 3.75 m/s². (a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed after 10.0 s? (d) What are the velocity components of the object after 10.0 s?

21. A 15.0-lb block rests on the floor. (a) What force does the floor exert on the block? (b) If a rope is tied to the block and run vertically over a pulley, and the other end is attached to a free-hanging 10.0-lb weight, what is the force exerted by the floor on the 15.0-lb block? (c) If we replace the 10.0-lb weight in part (b) with a 20.0-lb weight, what is the force exerted by the floor on the 15.0-lb block?

Section 5.7 Some Applications of Newton's Laws

22. A 3.00-kg mass is moving in a plane, with its *x* and *y* coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where *x* and *y* are in meters and *t* is in seconds. Find the magnitude of the net force acting on this mass at $t = 2.00$ s.

23. The distance between two telephone poles is 50.0 m. When a 1.00-kg bird lands on the telephone wire midway between the poles, the wire sags 0.200 m. Draw a free-body diagram of the bird. How much tension does the bird produce in the wire? Ignore the weight of the wire.

24. A bag of cement of weight 325 N hangs from three wires as shown in Figure P5.24. Two of the wires make angles $\theta_1 = 60.0°$ and $\theta_2 = 25.0°$ with the horizontal. If the system is in equilibrium, find the tensions T_1, T_2, and T_3 in the wires.

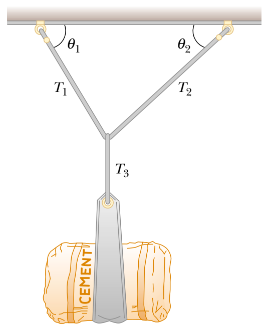


Figure P5.24 Problems 24 and 25.

25. A bag of cement of weight F_g hangs from three wires as shown in Figure P5.24. Two of the wires make angles θ_1 and θ_2 with the horizontal. If the system is in equilibrium, show that the tension in the left-hand wire is

$$T_1 = F_g \cos \theta_2 / \sin(\theta_1 + \theta_2)$$

26. You are a judge in a children's kite-flying contest, and two children will win prizes for the kites that pull most strongly and least strongly on their strings. To measure string tensions, you borrow a weight hanger, some slotted weights, and a protractor from your physics teacher and use the following protocol, illustrated in Figure P5.26: Wait for a child to get her kite well-controlled, hook the hanger onto the kite string about 30 cm from her hand, pile on weights until that section of string is horizontal, record the mass required, and record the angle between the horizontal and the string running up to the kite. (a) Explain how this method works. As you construct your explanation, imagine that the children's parents ask you about your method, that they might make false assumptions about your ability without concrete evidence, and that your explanation is an opportunity to give them confidence in your evaluation technique. (b) Find the string tension if the mass required to make the string horizontal is 132 g and the angle of the kite string is 46.3°.

Figure P5.26

27. The systems shown in Figure P5.27 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Neglect the masses of the pulleys and strings, and assume the incline is frictionless.)

28. A fire helicopter carries a 620-kg bucket of water at the end of a cable 20.0 m long. As the aircraft flies back from a fire at a constant speed of 40.0 m/s, the cable makes an angle of 40.0° with respect to the vertical. (a) Determine the force of air resistance on the bucket. (b) After filling the bucket with sea water, the pilot returns to the fire at the same speed with the bucket now making an angle of 7.00° with the vertical. What is the mass of the water in the bucket?

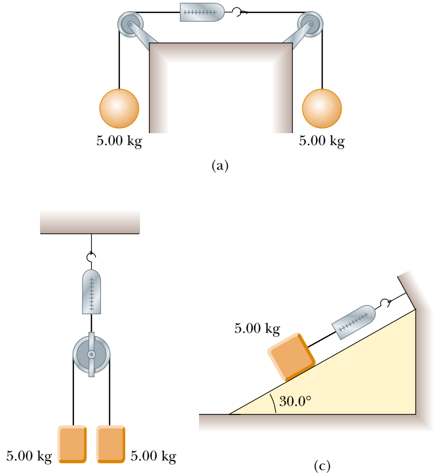


Figure P5.27

29. A 1.00-kg mass is observed to accelerate at 10.0 m/s^2 in a direction 30.0° north of east (Fig. P5.29). The force $\mathbf{F}_2$ acting on the mass has a magnitude of 5.00 N and is directed north. Determine the magnitude and direction of the force $\mathbf{F}_1$ acting on the mass.

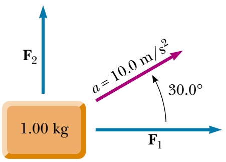


Figure P5.29

30. A simple accelerometer is constructed by suspending a mass m from a string of length L that is tied to the top of a cart. As the cart is accelerated the string-mass system makes a constant angle θ with the vertical. (a) Assuming that the string mass is negligible compared with m, derive an expression for the cart's acceleration in terms of θ and show that it is independent of the mass m and the length L. (b) Determine the acceleration of the cart when $\theta = 23.0°$.

31. Two people pull as hard as they can on ropes attached to a boat that has a mass of 200 kg. If they pull in the same direction, the boat has an acceleration of 1.52 m/s^2 to the right. If they pull in opposite directions, the boat has an acceleration of 0.518 m/s^2 to the left. What is the force exerted by each person on the boat? (Disregard any other forces on the boat.)

32. Draw a free-body diagram for a block that slides down a frictionless plane having an inclination of $\theta = 15.0°$ (Fig. P5.32). If the block starts from rest at the top and the length of the incline is 2.00 m, find (a) the acceleration of the block and (b) its speed when it reaches the bottom of the incline.

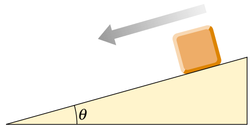


Figure P5.32

33. A block is given an initial velocity of 5.00 m/s up a frictionless 20.0° incline. How far up the incline does the block slide before coming to rest?

34. Two masses are connected by a light string that passes over a frictionless pulley, as in Figure P5.34. If the incline is frictionless and if $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$, find (a) the accelerations of the masses, (b) the tension in the string, and (c) the speed of each mass 2.00 s after being released from rest.

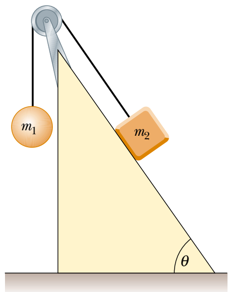


Figure P5.34

35. Two masses m_1 and m_2 situated on a frictionless, horizontal surface are connected by a light string. A force $\mathbf{F}$ is exerted on one of the masses to the right (Fig. P5.35). Determine the acceleration of the system and the tension T in the string.

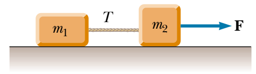


Figure P5.35 Problems 35 and 51.

36. Two masses of 3.00 kg and 5.00 kg are connected by a light string that passes over a frictionless pulley, as was shown in Figure 5.15a. Determine (a) the tension in the string, (b) the acceleration of each mass, and (c) the distance each mass will move in the first second of motion if they start from rest.

37. In the system shown in Figure P5.37, a horizontal force F_x acts on the 8.00-kg mass. The horizontal surface is frictionless. (a) For what values of F_x does the 2.00-kg mass accelerate upward? (b) For what values of F_x is the tension in the cord zero? (c) Plot the acceleration of the 8.00-kg mass versus F_x. Include values of F_x from − 100 N to + 100 N.

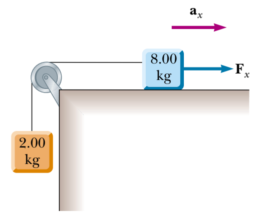


Figure P5.37

38. Mass m_1 on a frictionless horizontal table is connected to mass m_2 by means of a very light pulley P_1 and a light fixed pulley P_2 as shown in Figure P5.38. (a) If a_1 and a_2

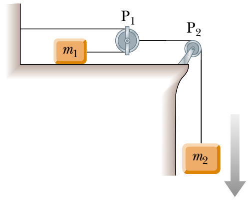


Figure P5.38

are the accelerations of m_1 and m_2, respectively, what is the relationship between these accelerations? Express (b) the tensions in the strings and (c) the accelerations a_1 and a_2 in terms of the masses m_1 and m_2 and g.

39. A 72.0-kg man stands on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.20 m/s in 0.800 s. It travels with this constant speed for the next 5.00 s. The elevator then undergoes a uniform acceleration in the negative y direction for 1.50 s and comes to rest. What does the spring scale register (a) before the elevator starts to move? (b) during the first 0.800 s? (c) while the elevator is traveling at constant speed? (d) during the time it is slowing down?

Section 5.8 Forces of Friction

40. The coefficient of static friction is 0.800 between the soles of a sprinter's running shoes and the level track surface on which she is running. Determine the maximum acceleration she can achieve. Do you need to know that her mass is 60.0 kg?

41. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion. After it is in motion, a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

42. A racing car accelerates uniformly from 0 to 80.0 mi/h in 8.00 s. The external force that accelerates the car is the frictional force between the tires and the road. If the tires do not slip, determine the minimum coefficient of friction between the tires and the road.

43. A car is traveling at 50.0 mi/h on a horizontal highway. (a) If the coefficient of friction between road and tires on a rainy day is 0.100, what is the minimum distance in which the car will stop? (b) What is the stopping distance when the surface is dry and $\mu_s = 0.600$?

44. A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle of θ above the horizontal (Fig. P5.44). She pulls on the strap with a 35.0-N force, and the frictional force on the suitcase is 20.0 N. Draw a free-body diagram for the suitcase. (a) What angle does the strap make with the horizontal? (b) What normal force does the ground exert on the suitcase?

WEB **45.** A 3.00-kg block starts from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the frictional force acting on the block, and (d) the speed of the block after it has slid 2.00 m.

46. To determine the coefficients of friction between rubber and various surfaces, a student uses a rubber eraser and an incline. In one experiment the eraser begins to slip down the incline when the angle of inclination is 36.0° and then moves down the incline with constant speed when the angle is reduced to 30.0°. From these data, determine the coefficients of static and kinetic friction for this experiment.

Figure P5.44

47. A boy drags his 60.0-N sled at constant speed up a 15.0° hill. He does so by pulling with a 25.0-N force on a rope attached to the sled. If the rope is inclined at 35.0° to the horizontal, (a) what is the coefficient of kinetic friction between sled and snow? (b) At the top of the hill, he jumps on the sled and slides down the hill. What is the magnitude of his acceleration down the slope?

48. Determine the stopping distance for a skier moving down a slope with friction with an initial speed of 20.0 m/s (Fig. P5.48). Assume $\mu_k = 0.180$ and $\theta = 5.00°$.

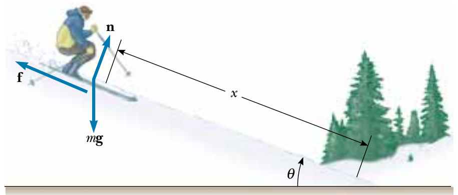


Figure P5.48

49. A 9.00-kg hanging weight is connected by a string over a pulley to a 5.00-kg block that is sliding on a flat table (Fig. P5.49). If the coefficient of kinetic friction is 0.200, find the tension in the string.

50. Three blocks are connected on a table as shown in Figure P5.50. The table is rough and has a coefficient of kinetic friction of 0.350. The three masses are 4.00 kg, 1.00 kg, and 2.00 kg, and the pulleys are frictionless. Draw a free-body diagram for each block. (a) Determine the magnitude and direction of the acceleration of each block. (b) Determine the tensions in the two cords.

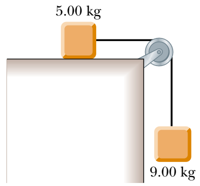


Figure P5.49

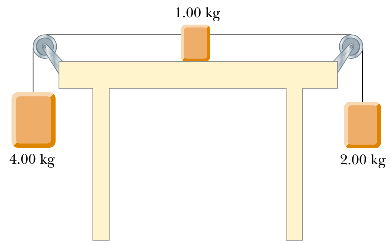


Figure P5.50

51. Two blocks connected by a rope of negligible mass are being dragged by a horizontal force **F** (see Fig. P5.35). Suppose that $F = 68.0$ N, $m_1 = 12.0$ kg, $m_2 = 18.0$ kg, and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block. (b) Determine the tension T and the magnitude of the acceleration of the system.

52. A block of mass 2.20 kg is accelerated across a rough surface by a rope passing over a pulley, as shown in Figure P5.52. The tension in the rope is 10.0 N, and the pulley is 10.0 cm above the top of the block. The coefficient of kinetic friction is 0.400. (a) Determine the acceleration of the block when $x = 0.400$ m. (b) Find the value of x at which the acceleration becomes zero.

53. A block of mass 3.00 kg is pushed up against a wall by a force **P** that makes a 50.0° angle with the horizontal as shown in Figure P5.53. The coefficient of static friction between the block and the wall is 0.250. Determine the possible values for the magnitude of **P** that allow the block to remain stationary.

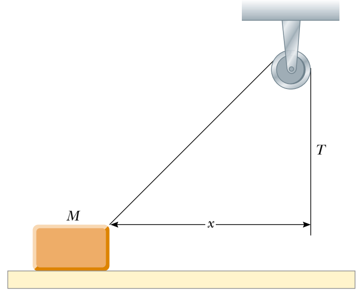


Figure P5.52

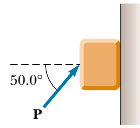


Figure P5.53

ADDITIONAL PROBLEMS

54. A time-dependent force $\mathbf{F} = (8.00\mathbf{i} - 4.00t\mathbf{j})$ N (where t is in seconds) is applied to a 2.00-kg object initially at rest. (a) At what time will the object be moving with a speed of 15.0 m/s? (b) How far is the object from its initial position when its speed is 15.0 m/s? (c) What is the object's displacement at the time calculated in (a)?

55. An inventive child named Pat wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley (Fig. P5.55), Pat pulls on the loose end of the rope with such a force that the spring scale reads 250 N. Pat's weight is 320 N, and the chair weighs 160 N. (a) Draw free-body diagrams for Pat and the chair considered as separate systems, and draw another diagram for Pat and the chair considered as one system. (b) Show that the acceleration of the system is *upward* and find its magnitude. (c) Find the force Pat exerts on the chair.

56. Three blocks are in contact with each other on a frictionless, horizontal surface, as in Figure P5.56. A horizontal force **F** is applied to m_1. If $m_1 = 2.00$ kg, $m_2 = 3.00$ kg, $m_3 = 4.00$ kg, and $F = 18.0$ N, draw a separate free-body diagram for each block and find (a) the acceleration of the blocks, (b) the *resultant* force on each block, and (c) the magnitudes of the contact forces between the blocks.

Figure P5.55

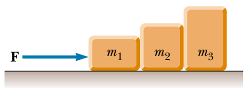


Figure P5.56

57. A high diver of mass 70.0 kg jumps off a board 10.0 m above the water. If his downward motion is stopped 2.00 s after he enters the water, what average upward force did the water exert on him?

58. Consider the three connected objects shown in Figure P5.58. If the inclined plane is frictionless and the system is in equilibrium, find (in terms of m, g, and θ) (a) the mass M and (b) the tensions T_1 and T_2. If the value of M is double the value found in part (a), find (c) the acceleration of each object, and (d) the tensions T_1 and T_2. If the coefficient of static friction between m and $2m$ and the inclined plane is μ_s, and the system is in equilibrium, find (e) the minimum value of M and (f) the maximum value of M. (g) Compare the values of T_2 when M has its minimum and maximum values.

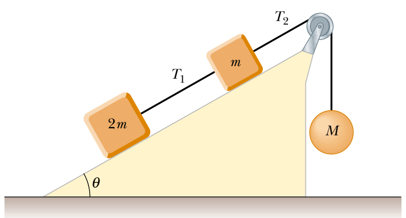


Figure P5.58

WEB **59.** A mass M is held in place by an applied force **F** and a pulley system as shown in Figure P5.59. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, T_1, T_2, T_3, T_4, and T_5 and (b) the magnitude of **F**. (*Hint:* Draw a free-body diagram for each pulley.)

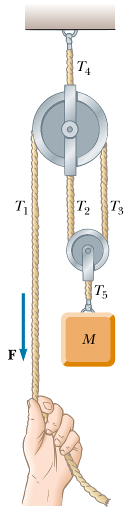


Figure P5.59

60. Two forces, given by $\mathbf{F}_1 = (-6.00\mathbf{i} - 4.00\mathbf{j})$ N and $\mathbf{F}_2 = (-3.00\mathbf{i} + 7.00\mathbf{j})$ N, act on a particle of mass 2.00 kg that is initially at rest at coordinates $(-2.00 \text{ m}, +4.00 \text{ m})$. (a) What are the components of the particle's velocity at $t = 10.0$ s? (b) In what direction is the particle moving at $t = 10.0$ s? (c) What displacement does the particle undergo during the first 10.0 s? (d) What are the coordinates of the particle at $t = 10.0$ s?

61. A crate of weight $\mathbf{F}_g$ is pushed by a force **P** on a horizontal floor. (a) If the coefficient of static friction is μ_s and **P** is directed at an angle θ below the horizontal, show that the minimum value of P that will move the crate is given by

$$P = \mu_s F_g \sec\theta(1 - \mu_s \tan\theta)^{-1}$$

(b) Find the minimum value of P that can produce motion when $\mu_s = 0.400$, $F_g = 100$ N, and $\theta = 0°$, 15.0°, 30.0°, 45.0°, and 60.0°.

62. Review Problem. A block of mass $m = 2.00$ kg is released from rest $h = 0.500$ m from the surface of a table, at the top of a $\theta = 30.0°$ incline as shown in Figure P5.62. The frictionless incline is fixed on a table of height $H = 2.00$ m. (a) Determine the acceleration of the block as it slides down the incline. (b) What is the velocity of the block as it leaves the incline? (c) How far from the table will the block hit the floor? (d) How much time has elapsed between when the block is released and when it hits the floor? (e) Does the mass of the block affect any of the above calculations?

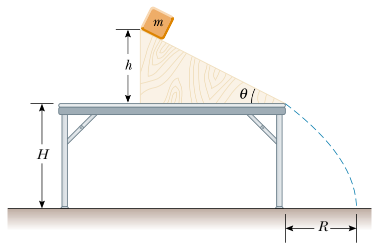


Figure P5.62

63. A 1.30-kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.350. To make the toaster start moving, you carelessly pull on its electric cord. (a) For the cord tension to be as small as possible, you should pull at what angle above the horizontal? (b) With this angle, how large must the tension be?

64. A 2.00-kg aluminum block and a 6.00-kg copper block are connected by a light string over a frictionless pulley. They sit on a steel surface, as shown in Figure P5.64, and $\theta = 30.0°$. Do they start to move once any holding mechanism is released? If so, determine (a) their acceleration and (b) the tension in the string. If not, determine the sum of the magnitudes of the forces of friction acting on the blocks.

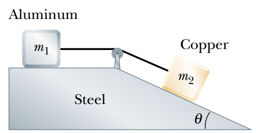


Figure P5.64

65. A block of mass $m = 2.00$ kg rests on the left edge of a block of larger mass $M = 8.00$ kg. The coefficient of kinetic friction between the two blocks is 0.300, and the surface on which the 8.00-kg block rests is frictionless. A constant horizontal force of magnitude $F = 10.0$ N is applied to the 2.00-kg block, setting it in motion as shown in Figure P5.65a. If the length L that the leading edge of the smaller block travels on the larger block is 3.00 m, (a) how long will it take before this block makes it to the right side of the 8.00-kg block, as shown in Figure P5.65b? (*Note:* Both blocks are set in motion when **F** is applied.) (b) How far does the 8.00-kg block move in the process?

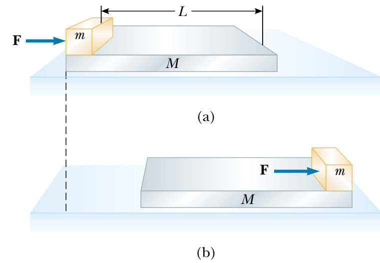


Figure P5.65

66. A student is asked to measure the acceleration of a cart on a "frictionless" inclined plane as seen in Figure P5.32, using an air track, a stopwatch, and a meter stick. The height of the incline is measured to be 1.774 cm, and the total length of the incline is measured to be $d = 127.1$ cm. Hence, the angle of inclination θ is determined from the relation $\sin\theta = 1.774/127.1$. The cart is released from rest at the top of the incline, and its displacement x along the incline is measured versus time, where $x = 0$ refers to the initial position of the cart. For x values of 10.0 cm, 20.0 cm, 35.0 cm, 50.0 cm, 75.0 cm, and 100 cm, the measured times to undergo these displacements (averaged over five runs) are 1.02 s, 1.53 s, 2.01 s, 2.64 s, 3.30 s, and 3.75 s, respectively. Construct a graph of x versus t^2, and perform a linear least-squares fit to the data. Determine the acceleration of the cart from the slope of this graph, and compare it with the value you would get using $a' = g\sin\theta$, where $g = 9.80$ m/s^2.

67. A 2.00-kg block is placed on top of a 5.00-kg block as in Figure P5.67. The coefficient of kinetic friction between the 5.00-kg block and the surface is 0.200. A horizontal force **F** is applied to the 5.00-kg block. (a) Draw a free-body diagram for each block. What force accelerates the 2.00-kg block? (b) Calculate the magnitude of the force necessary to pull both blocks to the right with an

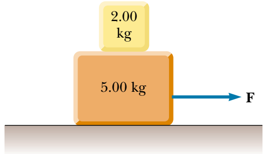


Figure P5.67

acceleration of 3.00 m/s^2. (c) Find the minimum coefficient of static friction between the blocks such that the 2.00-kg block does not slip under an acceleration of 3.00 m/s^2.

68. A 5.00-kg block is placed on top of a 10.0-kg block (Fig. P5.68). A horizontal force of 45.0 N is applied to the 10.0-kg block, and the 5.00-kg block is tied to the wall. The coefficient of kinetic friction between all surfaces is 0.200. (a) Draw a free-body diagram for each block and identify the action–reaction forces between the blocks. (b) Determine the tension in the string and the magnitude of the acceleration of the 10.0-kg block.

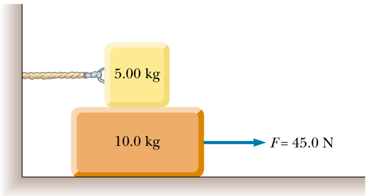


Figure P5.68

69. What horizontal force must be applied to the cart shown in Figure P5.69 so that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. (*Hint:* Note that the force exerted by the string accelerates m_1.)

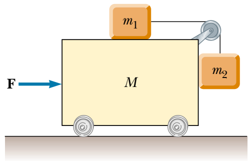


Figure P5.69 Problems 69 and 70.

70. Initially the system of masses shown in Figure P5.69 is held motionless. All surfaces, pulley, and wheels are frictionless. Let the force **F** be zero and assume that m_2 can move only vertically. At the instant after the system of masses is released, find (a) the tension T in the string, (b) the acceleration of m_2, (c) the acceleration of M, and (d) the acceleration of m_1. (*Note:* The pulley accelerates along with the cart.)

71. A block of mass 5.00 kg sits on top of a second block of mass 15.0 kg, which in turn sits on a horizontal table. The coefficients of friction between the two blocks are $\mu_s = 0.300$ and $\mu_k = 0.100$. The coefficients of friction between the lower block and the rough table are $\mu_s = 0.500$ and $\mu_k = 0.400$. You apply a constant horizontal force to the lower block, just large enough to make this block start sliding out from between the upper block and the table. (a) Draw a free-body diagram of each block, naming the forces acting on each. (b) Determine the magnitude of each force on each block at the instant when you have started pushing but motion has not yet started. (c) Determine the acceleration you measure for each block.

72. Two blocks of mass 3.50 kg and 8.00 kg are connected by a string of negligible mass that passes over a frictionless pulley (Fig. P5.72). The inclines are frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string.

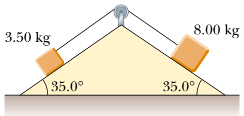


Figure P5.72 Problems 72 and 73.

73. The system shown in Figure P5.72 has an acceleration of magnitude 1.50 m/s^2. Assume the coefficients of kinetic friction between block and incline are the same for both inclines. Find (a) the coefficient of kinetic friction and (b) the tension in the string.

74. In Figure P5.74, a 500-kg horse pulls a sledge of mass 100 kg. The system (horse plus sledge) has a forward acceleration of 1.00 m/s^2 when the frictional force exerted on the sledge is 500 N. Find (a) the tension in the connecting rope and (b) the magnitude and direction of the force of friction exerted on the horse. (c) Verify that the total forces of friction the ground exerts on the system will give the system an acceleration of 1.00 m/s^2.

75. A van accelerates down a hill (Fig. P5.75), going from rest to 30.0 m/s in 6.00 s. During the acceleration, a toy (m = 0.100 kg) hangs by a string from the van's ceiling. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle θ and (b) the tension in the string.

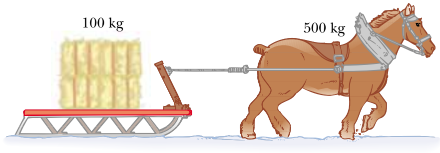


Figure P5.74

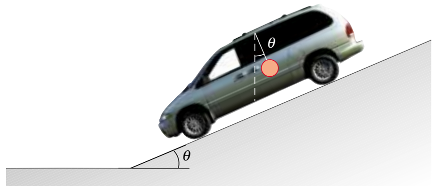


Figure P5.75

76. A mobile is formed by supporting four metal butterflies of equal mass m from a string of length L. The points of support are evenly spaced a distance ℓ apart as shown in Figure P5.76. The string forms an angle θ_1 with the ceiling at each end point. The center section of string is horizontal. (a) Find the tension in each section of string in terms of θ_1, m, and g. (b) Find the angle θ_2, in terms of θ_1, that the sections of string between the outside butterflies and the inside butterflies form with the horizontal. (c) Show that the distance D between the end points of the string is

$$D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\}$$

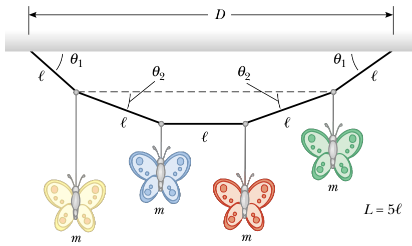


Figure P5.76

77. Before 1960 it was believed that the maximum attainable coefficient of static friction for an automobile tire was less than 1. Then about 1962, three companies independently developed racing tires with coefficients of 1.6. Since then, tires have improved, as illustrated in this problem. According to the 1990 Guinness Book of Records, the fastest time in which a piston-engine car initially at rest has covered a distance of one-quarter mile is 4.96 s. This record was set by Shirley Muldowney in September 1989 (Fig. P5.77). (a) Assuming that the rear wheels nearly lifted the front wheels off the pavement, what minimum value of μ_s is necessary to achieve the record time? (b) Suppose Muldowney were able to double her engine power, keeping other things equal. How would this change affect the elapsed time?

Figure P5.77

78. An 8.40-kg mass slides down a fixed, frictionless inclined plane. Use a computer to determine and tabulate the normal force exerted on the mass and its acceleration for a series of incline angles (measured from the horizontal) ranging from 0 to 90° in 5° increments. Plot a graph of the normal force and the acceleration as functions of the incline angle. In the limiting cases of 0 and 90°, are your results consistent with the known behavior?

ANSWERS TO QUICK QUIZZES

5.1 (a) True. Newton's first law tells us that motion requires no force: An object in motion continues to move at constant velocity in the absence of external forces. (b) True. A stationary object can have several forces acting on it, but if the vector sum of all these external forces is zero, there is no net force and the object remains stationary. It also is possible to have a net force and no motion, but only for an instant. A ball tossed vertically upward stops at the peak of its path for an infinitesimally short time, but the force of gravity is still acting on it. Thus, although $\mathbf{v} = 0$ at the peak, the net force acting on the ball is *not* zero.

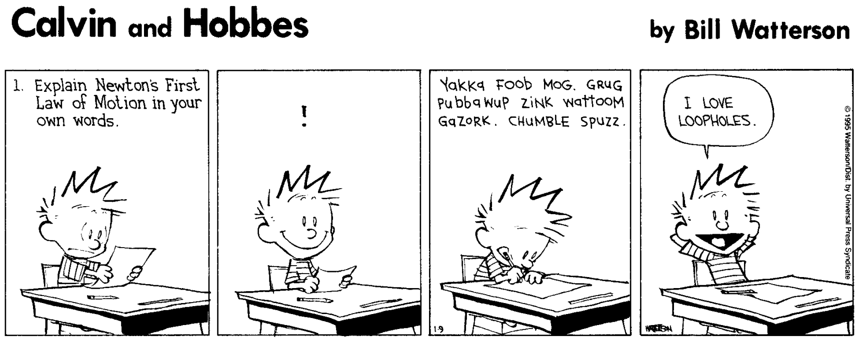
5.2 No. Direction of motion is part of an object's *velocity,* and force determines the direction of acceleration, not that of velocity.

5.3 (a) Force of gravity. (b) Force of gravity. The only external force acting on the ball at *all* points in its trajectory is the downward force of gravity.

5.4 As the person steps out of the boat, he pushes against it with his foot, expecting the boat to push back on him so that he accelerates toward the dock. However, because the boat is untied, the force exerted by the foot causes the boat to scoot away from the dock. As a result, the person is not able to exert a very large force on the boat before it moves out of reach. Therefore, the boat does not exert a very large reaction force on him, and he ends up not being accelerated sufficiently to make it to the dock. Consequently, he falls into the water instead. If a small dog were to jump from the untied boat toward the dock, the force exerted by the boat on the dog would probably be enough to ensure the dog's successful landing because of the dog's small mass.

5.5 (a) The same force is experienced by both. The fly and bus experience forces that are equal in magnitude but opposite in direction. (b) The fly. Because the fly has such a small mass, it undergoes a very large acceleration. The huge mass of the bus means that it more effectively resists any change in its motion.

5.6 (b) The crate accelerates to the right. Because the only horizontal force acting on it is the force of static friction between its bottom surface and the truck bed, that force must also be directed to the right.



PUZZLER

This sky diver is falling at more than 50 m/s (120 mi/h), but once her parachute opens, her downward velocity will be greatly reduced. Why does she slow down rapidly when her chute opens, enabling her to fall safely to the ground? If the chute does not function properly, the sky diver will almost certainly be seriously injured. What force exerted on her limits her maximum speed? *(Guy Savage/Photo Researchers, Inc.)*

chapter 6

Circular Motion and Other Applications of Newton's Laws

Chapter Outline

In the preceding chapter we introduced Newton's laws of motion and applied them to situations involving linear motion. Now we discuss motion that is slightly more complicated. For example, we shall apply Newton's laws to objects traveling in circular paths. Also, we shall discuss motion observed from an accelerating frame of reference and motion in a viscous medium. For the most part, this chapter is a series of examples selected to illustrate the application of Newton's laws to a wide variety of circumstances.

6.1 NEWTON'S SECOND LAW APPLIED TO UNIFORM CIRCULAR MOTION

In Section 4.4 we found that a particle moving with uniform speed v in a circular path of radius r experiences an acceleration $\mathbf{a}_r$ that has a magnitude

$$a_r = \frac{v^2}{r}$$

4.7 The acceleration is called the *centripetal acceleration* because $\mathbf{a}_r$ is directed toward the center of the circle. Furthermore, $\mathbf{a}_r$ is *always* perpendicular to $\mathbf{v}$. (If there were a component of acceleration parallel to $\mathbf{v}$, the particle's speed would be changing.)

Consider a ball of mass m that is tied to a string of length r and is being whirled at constant speed in a horizontal circular path, as illustrated in Figure 6.1. Its weight is supported by a low-friction table. Why does the ball move in a circle? Because of its inertia, the tendency of the ball is to move in a straight line; however, the string prevents motion along a straight line by exerting on the ball a force that makes it follow the circular path. This force is directed along the string toward the center of the circle, as shown in Figure 6.1. This force can be any one of our familiar forces causing an object to follow a circular path.

If we apply Newton's second law along the radial direction, we find that the value of the net force causing the centripetal acceleration can be evaluated:

Force causing centripetal acceleration

$$\sum F_r = ma_r = m\frac{v^2}{r} \tag{6.1}$$

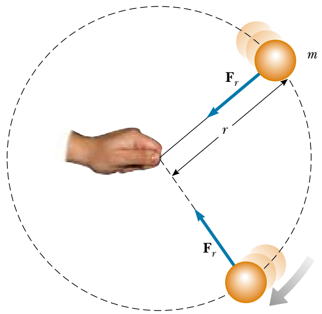


Figure 6.1 Overhead view of a ball moving in a circular path in a horizontal plane. A force $\mathbf{F}_r$ directed toward the center of the circle keeps the ball moving in its circular path.

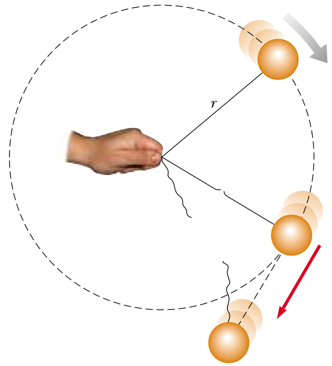
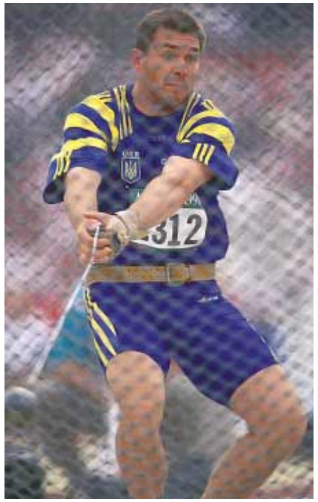


Figure 6.2 When the string breaks, the ball moves in the direction tangent to the circle.



An athlete in the process of throwing the hammer at the 1996 Olympic Games in Atlanta, Georgia. The force exerted by the chain is the force causing the circular motion. Only when the athlete releases the hammer will it move along a straight-line path tangent to the circle.

A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle. This idea is illustrated in Figure 6.2 for the ball whirling at the end of a string. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string broke.

Quick Quiz 6.1

Is it possible for a car to move in a circular path in such a way that it has a tangential acceleration but no centripetal acceleration?

CONCEPTUAL EXAMPLE 6.1 Forces That Cause Centripetal Acceleration

The force causing centripetal acceleration is sometimes called a *centripetal force.* We are familiar with a variety of forces in nature—friction, gravity, normal forces, tension, and so forth. Should we add *centripetal* force to this list?

Solution No; centripetal force *should not* be added to this list. This is a pitfall for many students. Giving the force causing circular motion a name—centripetal force—leads many students to consider it a new kind of force rather than a new *role* for force. A common mistake in force diagrams is to draw all the usual forces and then to add another vector for the centripetal force. But it is not a separate force—it is simply one of our familiar forces *acting in the role of a force that causes a circular motion.*

Consider some examples. For the motion of the Earth around the Sun, the centripetal force is *gravity.* For an object sitting on a rotating turntable, the centripetal force is *friction.* For a rock whirled on the end of a string, the centripetal force is the force of *tension* in the string. For an amusement-park patron pressed against the inner wall of a rapidly rotating circular room, the centripetal force is the *normal force* exerted by the wall. What's more, the centripetal force could be a combination of two or more forces. For example, as a Ferris-wheel rider passes through the lowest point, the centripetal force on her is the difference between the normal force exerted by the seat and her weight.

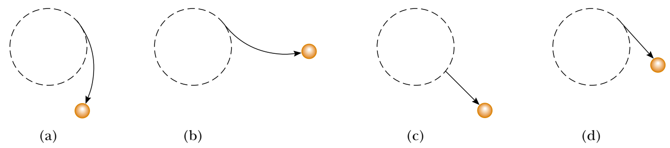


Figure 6.3 A ball that had been moving in a circular path is acted on by various external forces that change its path.

QuickLab

Tie a string to a tennis ball, swing it in a circle, and then, while it is swinging, let go of the string to verify your answer to the last part of Quick Quiz 6.2.

Quick Quiz 6.2

A ball is following the dotted circular path shown in Figure 6.3 under the influence of a force. At a certain instant of time, the force on the ball changes abruptly to a new force, and the ball follows the paths indicated by the solid line with an arrowhead in each of the four parts of the figure. For each part of the figure, describe the magnitude and direction of the force required to make the ball move in the solid path. If the dotted line represents the path of a ball being whirled on the end of a string, which path does the ball follow if the string breaks?

Let us consider some examples of uniform circular motion. In each case, be sure to recognize the external force (or forces) that causes the body to move in its circular path.

EXAMPLE 6.2 How Fast Can It Spin?

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as was shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks? Assume that the string remains horizontal during the motion.

Solution It is difficult to know what might be a reasonable value for the answer. Nonetheless, we know that it cannot be too large, say 100 m/s, because a person cannot make a ball move so quickly. It makes sense that the stronger the cord, the faster the ball can twirl before the cord breaks. Also, we expect a more massive ball to break the cord at a lower speed. (Imagine whirling a bowling ball!)

Because the force causing the centripetal acceleration in this case is the force $\mathbf{T}$ exerted by the cord on the ball, Equation 6.1 yields for $\Sigma F_r = ma_r$

$$T = m\frac{v^2}{r}$$

Solving for v, we have

$$v = \sqrt{\frac{Tr}{m}}$$

This shows that v increases with T and decreases with larger m, as we expect to see—for a given v, a large mass requires a large tension and a small mass needs only a small tension. The maximum speed the ball can have corresponds to the maximum tension. Hence, we find

$$v_{\max} = \sqrt{\frac{T_{\max}r}{m}} = \sqrt{\frac{(50.0\text{ N})(1.50\text{ m})}{0.500\text{ kg}}}$$

$$= \boxed{12.2\text{ m/s}}$$

Exercise Calculate the tension in the cord if the speed of the ball is 5.00 m/s.

Answer 8.33 N.

EXAMPLE 6.3 The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in Figure 6.4. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum.*) Find an expression for v.

Solution Let us choose θ to represent the angle between string and vertical. In the free-body diagram shown in Figure 6.4, the force $\mathbf{T}$ exerted by the string is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of revolution. Because the object does not accelerate in the vertical direction, $\Sigma F_y = ma_y = 0$, and the upward vertical component of **T** must balance the downward force of gravity. Therefore,

$$(1) \qquad T\cos\theta = mg$$

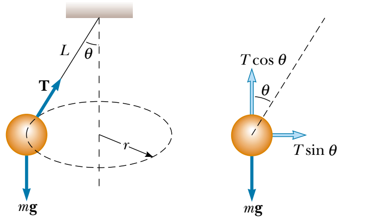


Figure 6.4 The conical pendulum and its free-body diagram.

Because the force providing the centripetal acceleration in this example is the component $T\sin\theta$, we can use Newton's second law and Equation 6.1 to obtain

$$(2) \qquad \sum F_r = T\sin\theta = ma_r = \frac{mv^2}{r}$$

Dividing (2) by (1) and remembering that $\sin\theta/\cos\theta = \tan\theta$, we eliminate T and find that

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

From the geometry in Figure 6.4, we note that $r = L\sin\theta$; therefore,

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

Note that the speed is independent of the mass of the object.

EXAMPLE 6.4 What Is the Maximum Speed of the Car?

A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as illustrated in Figure 6.5. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.

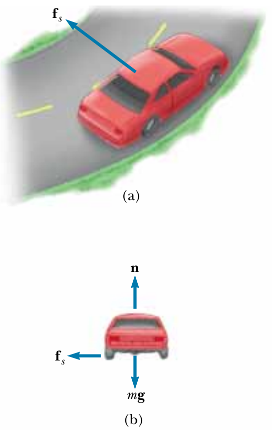


Figure 6.5 (a) The force of static friction directed toward the center of the curve keeps the car moving in a circular path. (b) The free-body diagram for the car.

Solution From experience, we should expect a maximum speed less than 50 m/s. (A convenient mental conversion is that 1 m/s is roughly 2 mi/h.) In this case, the force that enables the car to remain in its circular path is the force of static friction. (Because no slipping occurs at the point of contact between road and tires, the acting force is a force of static friction directed toward the center of the curve. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the road.) Hence, from Equation 6.1 we have

$$(1) \qquad f_s = m\frac{v^2}{r}$$

The maximum speed the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\max} = \mu_s n$. Because the car is on a horizontal road, the magnitude of the normal force equals the weight ($n = mg$) and thus $f_{s,\max} = \mu_s mg$. Substituting this value for f_s into (1), we find that the maximum speed is

$$v_{\max} = \sqrt{\frac{f_{s,\max}r}{m}} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr}$$

$$= \sqrt{(0.500)(9.80\ \text{m/s}^2)(35.0\ \text{m})} = \boxed{13.1\ \text{m/s}}$$

Note that the maximum speed does not depend on the mass of the car. That is why curved highways do not need multiple speed limit signs to cover the various masses of vehicles using the road.

Exercise On a wet day, the car begins to skid on the curve when its speed reaches 8.00 m/s. What is the coefficient of static friction in this case?

Answer 0.187.

EXAMPLE 6.5 The Banked Exit Ramp

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked;* this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 50.0 m. At what angle should the curve be banked?

Solution On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between car and road, as we saw in the previous example. However, if the road is banked at an angle θ, as shown in Figure 6.6, the normal force **n** has a horizontal component

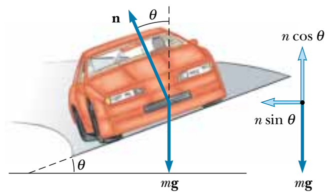


Figure 6.6 Car rounding a curve on a road banked at an angle θ to the horizontal. When friction is neglected, the force that causes the centripetal acceleration and keeps the car moving in its circular path is the horizontal component of the normal force. Note that **n** is the *sum* of the forces exerted by the road on the wheels.

$n\sin\theta$ pointing toward the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $n\sin\theta$ causes the centripetal acceleration. Hence, Newton's second law written for the radial direction gives

$$(1) \qquad \sum F_r = n\sin\theta = \frac{mv^2}{r}$$

The car is in equilibrium in the vertical direction. Thus, from $\Sigma F_y = 0$, we have

$$(2) \qquad n\cos\theta = mg$$

Dividing (1) by (2) gives

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left[\frac{(13.4\ \text{m/s})^2}{(50.0\ \text{m})(9.80\ \text{m/s}^2)}\right] = \boxed{20.1^\circ}$$

If a car rounds the curve at a speed less than 13.4 m/s, friction is needed to keep it from sliding down the bank (to the left in Fig. 6.6). A driver who attempts to negotiate the curve at a speed greater than 13.4 m/s has to depend on friction to keep from sliding up the bank (to the right in Fig. 6.6). The banking angle is independent of the mass of the vehicle negotiating the curve.

Exercise Write Newton's second law applied to the radial direction when a frictional force $\mathbf{f}_s$ is directed down the bank, toward the center of the curve.

Answer $n\sin\theta + f_s\cos\theta = \dfrac{mv^2}{r}$

EXAMPLE 6.6 Satellite Motion

This example treats a satellite moving in a circular orbit around the Earth. To understand this situation, you must know that the gravitational force between spherical objects and small objects that can be modeled as particles having masses m_1 and m_2 and separated by a distance r is attractive and has a magnitude

$$F_g = G\frac{m_1 m_2}{r^2}$$

where $G = 6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. This is Newton's law of gravitation, which we study in Chapter 14.

Consider a satellite of mass m moving in a circular orbit around the Earth at a constant speed v and at an altitude h above the Earth's surface, as illustrated in Figure 6.7. Determine the speed of the satellite in terms of G, h, R_E (the radius of the Earth), and M_E (the mass of the Earth).

Solution The only external force acting on the satellite is the force of gravity, which acts toward the center of the Earth

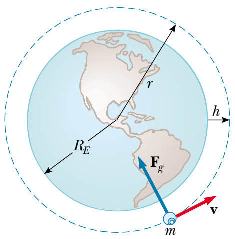


Figure 6.7 A satellite of mass m moving around the Earth at a constant speed v in a circular orbit of radius $r = R_E + h$. The force $\mathbf{F}_g$ acting on the satellite that causes the centripetal acceleration is the gravitational force exerted by the Earth on the satellite.

and keeps the satellite in its circular orbit. Therefore,

$$F_r = F_g = G\frac{M_E m}{r^2}$$

From Newton's second law and Equation 6.1 we obtain

$$G\frac{M_E m}{r^2} = m\frac{v^2}{r}$$

Solving for v and remembering that the distance r from the center of the Earth to the satellite is $r = R_E + h$, we obtain

$$(1) \qquad v = \sqrt{\frac{GM_E}{r}} = \boxed{\sqrt{\frac{GM_E}{R_E + h}}}$$

If the satellite were orbiting a different planet, its velocity would increase with the mass of the planet and decrease as the satellite's distance from the center of the planet increased.

Exercise A satellite is in a circular orbit around the Earth at an altitude of 1 000 km. The radius of the Earth is equal to 6.37×10^6 m, and its mass is 5.98×10^{24} kg. Find the speed of the satellite, and then find the *period*, which is the time it needs to make one complete revolution.

Answer 7.36×10^3 m/s; 6.29×10^3 s = 105 min.

EXAMPLE 6.7 Let's Go Loop-the-Loop!

A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in Figure 6.8a. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot (a) at the bottom of the loop and (b) at the top of the loop. Express your answers in terms of the weight of the pilot mg.

Solution We expect the answer for (a) to be greater than that for (b) because at the bottom of the loop the normal and gravitational forces act in opposite directions, whereas at the top of the loop these two forces act in the same direction. It is the vector sum of these two forces that gives the force of constant magnitude that keeps the pilot moving in a circular path. To yield net force vectors with the same magnitude, the normal force at the bottom (where the normal and gravitational forces are in opposite directions) must be greater than that at the top (where the normal and gravitational forces are in the same direction). (a) The free-body diagram for the pilot at the bottom of the loop is shown in Figure 6.8b. The only forces acting on him are the downward force of gravity $\mathbf{F}_g = m\mathbf{g}$ and the upward force $\mathbf{n}_{\text{bot}}$ exerted by the seat. Because the net upward force that provides the centripetal acceleration has a magnitude $n_{\text{bot}} - mg$, Newton's second law for the radial direction combined with Equation 6.1 gives

$$\sum F_r = n_{\text{bot}} - mg = m\frac{v^2}{r}$$

$$n_{\text{bot}} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

Substituting the values given for the speed and radius gives

$$n_{\text{bot}} = mg\left[1 + \frac{(225\ \text{m/s})^2}{(2.70 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)}\right] = \boxed{2.91mg}$$

Hence, the magnitude of the force $\mathbf{n}_{\text{bot}}$ exerted by the seat on the pilot is *greater* than the weight of the pilot by a factor of 2.91. This means that the pilot experiences an apparent weight that is greater than his true weight by a factor of 2.91.

(b) The free-body diagram for the pilot at the top of the loop is shown in Figure 6.8c. As we noted earlier, both the gravitational force exerted by the Earth and the force $\mathbf{n}_{\text{top}}$ exerted by the seat on the pilot act downward, and so the net downward force that provides the centripetal acceleration has

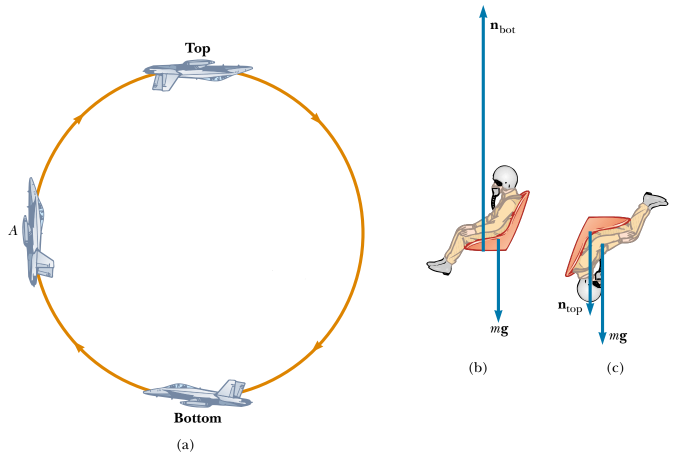


Figure 6.8 (a) An aircraft executes a loop-the-loop maneuver as it moves in a vertical circle at constant speed. (b) Free-body diagram for the pilot at the bottom of the loop. In this position the pilot experiences an apparent weight greater than his true weight. (c) Free-body diagram for the pilot at the top of the loop.

a magnitude $n_{\text{top}} + mg$. Applying Newton's second law yields

$$\sum F_r = n_{\text{top}} + mg = m\frac{v^2}{r}$$

$$n_{\text{top}} = m\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

$$n_{\text{top}} = mg\left[\frac{(225\ \text{m/s})^2}{(2.70 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)} - 1\right] = \boxed{0.913mg}$$

In this case, the magnitude of the force exerted by the seat on the pilot is *less* than his true weight by a factor of 0.913, and the pilot feels lighter.

Exercise Determine the magnitude of the radially directed force exerted on the pilot by the seat when the aircraft is at point A in Figure 6.8a, midway up the loop.

Answer $n_A = 1.913mg$ directed to the right.

Quick Quiz 6.3

A bead slides freely along a curved wire at constant speed, as shown in the overhead view of Figure 6.9. At each of the points Ⓐ, Ⓑ, and Ⓒ, draw the vector representing the force that the wire exerts on the bead in order to cause it to follow the path of the wire at that point.

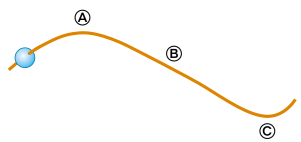


Figure 6.9

QuickLab

Hold a shoe by the end of its lace and spin it in a vertical circle. Can you feel the difference in the tension in the lace when the shoe is at top of the circle compared with when the shoe is at the bottom?

6.2 NONUNIFORM CIRCULAR MOTION

In Chapter 4 we found that if a particle moves with varying speed in a circular path, there is, in addition to the centripetal (radial) component of acceleration, a tangential component having magnitude dv/dt. Therefore, the force acting on the

Some examples of forces acting during circular motion. *(Left)* As these speed skaters round a curve, the force exerted by the ice on their skates provides the centripetal acceleration. *(Right)* Passengers on a "corkscrew" roller coaster. What are the origins of the forces in this example?

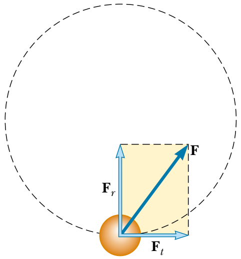


Figure 6.10 When the force acting on a particle moving in a circular path has a tangential component F_t, the particle's speed changes. The total force exerted on the particle in this case is the vector sum of the radial force and the tangential force. That is, $\mathbf{F} = \mathbf{F}_r + \mathbf{F}_t$.

particle must also have a tangential and a radial component. Because the total acceleration is $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the total force exerted on the particle is $\mathbf{F} = \mathbf{F}_r + \mathbf{F}_t$, as shown in Figure 6.10. The vector $\mathbf{F}_r$ is directed toward the center of the circle and is responsible for the centripetal acceleration. The vector $\mathbf{F}_t$ tangent to the circle is responsible for the tangential acceleration, which represents a change in the speed of the particle with time. The following example demonstrates this type of motion.

EXAMPLE 6.8 Keep Your Eye on the Ball

A small sphere of mass m is attached to the end of a cord of length R and whirls in a *vertical* circle about a fixed point O, as illustrated in Figure 6.11a. Determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

Solution Unlike the situation in Example 6.7, the speed is *not* uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere. From the free-body diagram in Figure 6.11b, we see that the only forces acting on

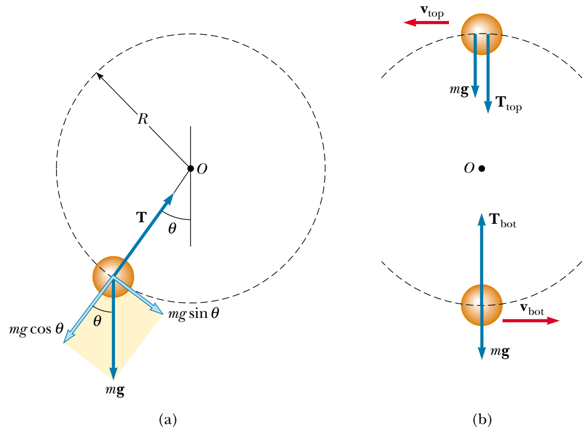


Figure 6.11 (a) Forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O. (b) Forces acting on the sphere at the top and bottom of the circle. The tension is a maximum at the bottom and a minimum at the top.

the sphere are the gravitational force $\mathbf{F}_g = m\mathbf{g}$ exerted by the Earth and the force $\mathbf{T}$ exerted by the cord. Now we resolve $\mathbf{F}_g$ into a tangential component $mg \sin\theta$ and a radial component $mg\cos\theta$. Applying Newton's second law to the forces acting on the sphere in the tangential direction yields

$$\sum F_t = mg\sin\theta = ma_t$$

$$a_t = g\sin\theta$$

This tangential component of the acceleration causes v to change in time because $a_t = dv/dt$.

Applying Newton's second law to the forces acting on the sphere in the radial direction and noting that both $\mathbf{T}$ and $\mathbf{a}_r$ are directed toward O, we obtain

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$

Special Cases At the top of the path, where $\theta = 180°$, we have $\cos 180° = -1$, and the tension equation becomes

$$T_{top} = m\left(\frac{v_{top}^2}{R} - g\right)$$

This is the minimum value of T. Note that at this point $a_t = 0$ and therefore the acceleration is purely radial and directed downward.

At the bottom of the path, where $\theta = 0$, we see that, because $\cos 0 = 1$,

$$T_{bot} = m\left(\frac{v_{bot}^2}{R} + g\right)$$

This is the maximum value of T. At this point, a_t is again 0 and the acceleration is now purely radial and directed upward.

Exercise At what position of the sphere would the cord most likely break if the average speed were to increase?

Answer At the bottom, where T has its maximum value.

Optional Section

6.3 MOTION IN ACCELERATED FRAMES

When Newton's laws of motion were introduced in Chapter 5, we emphasized that they are valid only when observations are made in an inertial frame of reference. In this section, we analyze how an observer in a noninertial frame of reference (one that is accelerating) applies Newton's second law.

To understand the motion of a system that is noninertial because an object is moving along a curved path, consider a car traveling along a highway at a high speed and approaching a curved exit ramp, as shown in Figure 6.12a. As the car takes the sharp left turn onto the ramp, a person sitting in the passenger seat slides to the right and hits the door. At that point, the force exerted on her by the door keeps her from being ejected from the car. What causes her to move toward the door? A popular, but improper, explanation is that some mysterious force acting from left to right pushes her outward. (This is often called the "centrifugal" force, but we shall not use this term because it often creates confusion.) The passenger invents this **fictitious force** to explain what is going on in her accelerated frame of reference, as shown in Figure 6.12b. (The driver also experiences this effect but holds on to the steering wheel to keep from sliding to the right.)

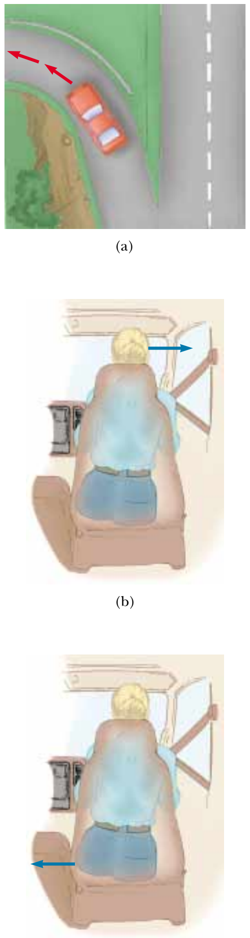
The phenomenon is correctly explained as follows. Before the car enters the ramp, the passenger is moving in a straight-line path. As the car enters the ramp and travels a curved path, the passenger tends to move along the original straight-line path. This is in accordance with Newton's first law: The natural tendency of a body is to continue moving in a straight line. However, if a sufficiently large force (toward the center of curvature) acts on the passenger, as in Figure 6.12c, she will move in a curved path along with the car. The origin of this force is the force of friction between her and the car seat. If this frictional force is not large enough, she will slide to the right as the car turns to the left under her. Eventually, she encounters the door, which provides a force large enough to enable her to follow the same curved path as the car. She slides toward the door not because of some mysterious outward force but because **the force of friction is not sufficiently great to allow her to travel along the circular path followed by the car.**

In general, if a particle moves with an acceleration **a** relative to an observer in an inertial frame, that observer may use Newton's second law and correctly claim that $\Sigma \mathbf{F} = m\mathbf{a}$. If another observer in an accelerated frame tries to apply Newton's second law to the motion of the particle, the person must introduce fictitious forces to make Newton's second law work. These forces "invented" by the observer in the accelerating frame appear to be real. However, we emphasize that **these fictitious forces do not exist when the motion is observed in an inertial frame.** Fictitious forces are used only in an accelerating frame and do not represent "real" forces acting on the particle. (By real forces, we mean the interaction of the particle with its environment.) If the fictitious forces are properly defined in the accelerating frame, the description of motion in this frame is equivalent to the description given by an inertial observer who considers only real forces. Usually, we analyze motions using inertial reference frames, but there are cases in which it is more convenient to use an accelerating frame.

QuickLab

Use a string, a small weight, and a protractor to measure your acceleration as you start sprinting from a standing position.

Fictitious forces



(a) (b) (c)

Figure 6.12 (a) A car approaching a curved exit ramp. What causes a front-seat passenger to move toward the right-hand door? (b) From the frame of reference of the passenger, a (fictitious) force pushes her toward the right door. (c) Relative to the reference frame of the Earth, the car seat applies a leftward force to the passenger, causing her to change direction along with the rest of the car.

EXAMPLE 6.9 Fictitious Forces in Linear Motion

A small sphere of mass m is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown in Figure 6.13. According to the inertial observer at rest (Fig. 6.13a), the forces on the sphere are the force **T** exerted by the cord and the force of gravity. The inertial observer concludes that the acceleration of the sphere is the same as that of the boxcar and that this acceleration is provided by the horizontal component of **T**. Also, the vertical component of **T** balances the force of gravity. Therefore, she writes Newton's second law as $\Sigma \mathbf{F} = \mathbf{T} + m\mathbf{g} = m\mathbf{a}$, which in component form becomes

$$\text{Inertial observer} \begin{cases} (1) \quad \sum F_x = T\sin\theta = ma \\ (2) \quad \sum F_y = T\cos\theta - mg = 0 \end{cases}$$

Thus, by solving (1) and (2) simultaneously for a, the inertial observer can determine the magnitude of the car's acceleration through the relationship

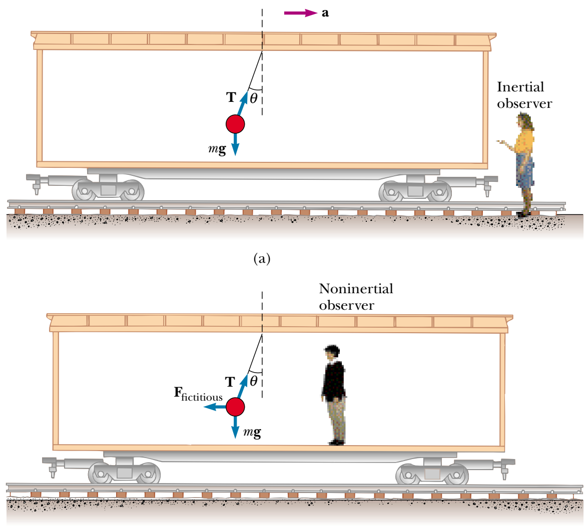
$$a = g\tan\theta$$

Because the deflection of the cord from the vertical serves as a measure of acceleration, *a simple pendulum can be used as an accelerometer.*

According to the noninertial observer riding in the car (Fig. 6.13b), the cord still makes an angle θ with the vertical; however, to her the sphere is at rest and so its acceleration is zero. Therefore, she introduces a fictitious force to balance the horizontal component of **T** and claims that the net force on the sphere is *zero!* In this noninertial frame of reference, Newton's second law in component form yields

$$\text{Noninertial observer} \begin{cases} \sum F'_x = T\sin\theta - F_{\text{fictitious}} = 0 \\ \sum F'_y = T\cos\theta - mg = 0 \end{cases}$$

If we recognize that $F_{\text{fictitious}} = ma_{\text{inertial}} = ma$, then these expressions are equivalent to (1) and (2); therefore, the noninertial observer obtains the same mathematical results as the inertial observer does. However, the physical interpretation of the deflection of the cord differs in the two frames of reference.



(a) (b)

Figure 6.13 A small sphere suspended from the ceiling of a boxcar accelerating to the right is deflected as shown. (a) An inertial observer at rest outside the car claims that the acceleration of the sphere is provided by the horizontal component of **T**. (b) A noninertial observer riding in the car says that the net force on the sphere is zero and that the deflection of the cord from the vertical is due to a fictitious force $\mathbf{F}_{\text{fictitious}}$ that balances the horizontal component of **T**.

EXAMPLE 6.10 Fictitious Force in a Rotating System

Suppose a block of mass m lying on a horizontal, frictionless turntable is connected to a string attached to the center of the turntable, as shown in Figure 6.14. According to an inertial observer, if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r, where v is its linear speed. The inertial observer concludes that this centripetal acceleration is provided by the force $\mathbf{T}$ exerted by the string and writes Newton's second law as $T = mv^2/r$.

According to a noninertial observer attached to the turntable, the block is at rest and its acceleration is zero. Therefore, she must introduce a fictitious outward force of magnitude mv^2/r to balance the inward force exerted by the string. According to her, the net force on the block is zero, and she writes Newton's second law as $T - mv^2/r = 0$.

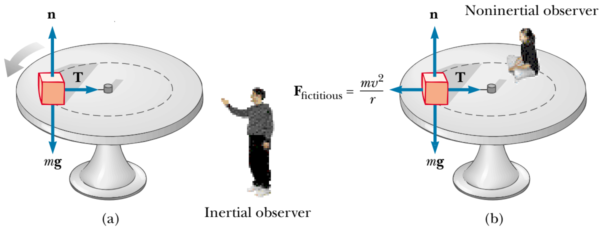


Figure 6.14 A block of mass m connected to a string tied to the center of a rotating turntable. (a) The inertial observer claims that the force causing the circular motion is provided by the force $\mathbf{T}$ exerted by the string on the block. (b) The noninertial observer claims that the block is not accelerating, and therefore she introduces a fictitious force of magnitude mv^2/r that acts outward and balances the force $\mathbf{T}$.

Optional Section

6.4 MOTION IN THE PRESENCE OF RESISTIVE FORCES

4.9 In the preceding chapter we described the force of kinetic friction exerted on an object moving on some surface. We completely ignored any interaction between the object and the medium through which it moves. Now let us consider the effect of that medium, which can be either a liquid or a gas. The medium exerts a **resistive force R** on the object moving through it. Some examples are the air resistance associated with moving vehicles (sometimes called *air drag*) and the viscous forces that act on objects moving through a liquid. The magnitude of $\mathbf{R}$ depends on such factors as the speed of the object, and the direction of $\mathbf{R}$ is always opposite the direction of motion of the object relative to the medium. The magnitude of $\mathbf{R}$ nearly always increases with increasing speed.

The magnitude of the resistive force can depend on speed in a complex way, and here we consider only two situations. In the first situation, we assume the resistive force is proportional to the speed of the moving object; this assumption is valid for objects falling slowly through a liquid and for very small objects, such as dust particles, moving through air. In the second situation, we assume a resistive force that is proportional to the square of the speed of the moving object; large objects, such as a skydiver moving through air in free fall, experience such a force.

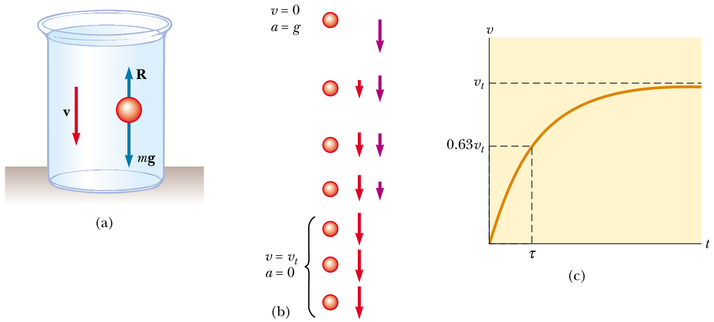


Figure 6.15 (a) A small sphere falling through a liquid. (b) Motion diagram of the sphere as it falls. (c) Speed–time graph for the sphere. The sphere reaches a maximum, or terminal, speed v_t, and the time constant τ is the time it takes to reach $0.63v_t$.

Resistive Force Proportional to Object Speed

If we assume that the resistive force acting on an object moving through a liquid or gas is proportional to the object's speed, then the magnitude of the resistive force can be expressed as

$$R = bv \tag{6.2}$$

where v is the speed of the object and b is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object. If the object is a sphere of radius r, then b is proportional to r.

Consider a small sphere of mass m released from rest in a liquid, as in Figure 6.15a. Assuming that the only forces acting on the sphere are the resistive force bv and the force of gravity F_g, let us describe its motion.[1] Applying Newton's second law to the vertical motion, choosing the downward direction to be positive, and noting that $\Sigma F_y = mg - bv$, we obtain

$$mg - bv = ma = m\frac{dv}{dt} \tag{6.3}$$

where the acceleration dv/dt is downward. Solving this expression for the acceleration gives

$$\frac{dv}{dt} = g - \frac{b}{m}v \tag{6.4}$$

This equation is called a *differential equation,* and the methods of solving it may not be familiar to you as yet. However, note that initially, when $v = 0$, the resistive force $-bv$ is also zero and the acceleration dv/dt is simply g. As t increases, the resistive force increases and the acceleration decreases. Eventually, the acceleration becomes zero when the magnitude of the resistive force equals the sphere's weight. At this point, the sphere reaches its **terminal speed** v_t, and from then on

Terminal speed

[1] There is also a *buoyant force* acting on the submerged object. This force is constant, and its magnitude is equal to the weight of the displaced liquid. This force changes the apparent weight of the sphere by a constant factor, so we will ignore the force here. We discuss buoyant forces in Chapter 15.

it continues to move at this speed with zero acceleration, as shown in Figure 6.15b. We can obtain the terminal speed from Equation 6.3 by setting $a = dv/dt = 0$. This gives

$$mg - bv_t = 0 \qquad \text{or} \qquad v_t = mg/b$$

The expression for v that satisfies Equation 6.4 with $v = 0$ at $t = 0$ is

$$v = \frac{mg}{b}(1 - e^{-bt/m}) = v_t(1 - e^{-t/\tau}) \tag{6.5}$$

This function is plotted in Figure 6.15c. The **time constant** $\tau = m/b$ (Greek letter tau) is the time it takes the sphere to reach 63.2% ($= 1 - 1/e$) of its terminal speed. This can be seen by noting that when $t = \tau$, Equation 6.5 yields $v = 0.632v_t$.

We can check that Equation 6.5 is a solution to Equation 6.4 by direct differentiation:

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{mg}{b} - \frac{mg}{b}e^{-bt/m}\right) = -\frac{mg}{b}\frac{d}{dt}e^{-bt/m} = ge^{-bt/m}$$

(See Appendix Table B.4 for the derivative of e raised to some power.) Substituting into Equation 6.4 both this expression for dv/dt and the expression for v given by Equation 6.5 shows that our solution satisfies the differential equation.

Aerodynamic car. A streamlined body reduces air drag and increases fuel efficiency.

EXAMPLE 6.11 Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time it takes the sphere to reach 90% of its terminal speed.

Solution Because the terminal speed is given by $v_t = mg/b$, the coefficient b is

$$b = \frac{mg}{v_t} = \frac{(2.00\text{ g})(980\text{ cm/s}^2)}{5.00\text{ cm/s}} = 392\text{ g/s}$$

Therefore, the time constant τ is

$$\tau = \frac{m}{b} = \frac{2.00\text{ g}}{392\text{ g/s}} = \boxed{5.10 \times 10^{-3}\text{ s}}$$

The speed of the sphere as a function of time is given by Equation 6.5. To find the time t it takes the sphere to reach a speed of $0.900v_t$, we set $v = 0.900v_t$ in Equation 6.5 and solve for t:

$$0.900v_t = v_t(1 - e^{-t/\tau})$$
$$1 - e^{-t/\tau} = 0.900$$
$$e^{-t/\tau} = 0.100$$
$$-\frac{t}{\tau} = \ln(0.100) = -2.30$$
$$t = 2.30\tau = 2.30(5.10 \times 10^{-3}\text{ s}) = 11.7 \times 10^{-3}\text{ s}$$
$$= \boxed{11.7\text{ ms}}$$

Thus, the sphere reaches 90% of its terminal (maximum) speed in a very short time.

Exercise What is the sphere's speed through the oil at $t = 11.7$ ms? Compare this value with the speed the sphere would have if it were falling in a vacuum and so were influenced only by gravity.

Answer 4.50 cm/s in oil versus 11.5 cm/s in free fall.

Air Drag at High Speeds

For objects moving at high speeds through air, such as airplanes, sky divers, cars, and baseballs, the resistive force is approximately proportional to the square of the speed. In these situations, the magnitude of the resistive force can be expressed as

$$R = \tfrac{1}{2}D\rho Av^2 \tag{6.6}$$

where ρ is the density of air, A is the cross-sectional area of the falling object measured in a plane perpendicular to its motion, and D is a dimensionless empirical quantity called the *drag coefficient.* The drag coefficient has a value of about 0.5 for spherical objects but can have a value as great as 2 for irregularly shaped objects.

Let us analyze the motion of an object in free fall subject to an upward air resistive force of magnitude $R = \frac{1}{2}D\rho Av^2$. Suppose an object of mass m is released from rest. As Figure 6.16 shows, the object experiences two external forces: the downward force of gravity $\mathbf{F}_g = m\mathbf{g}$ and the upward resistive force $\mathbf{R}$. (There is also an upward buoyant force that we neglect.) Hence, the magnitude of the net force is

$$\sum F = mg - \tfrac{1}{2}D\rho Av^2 \tag{6.7}$$

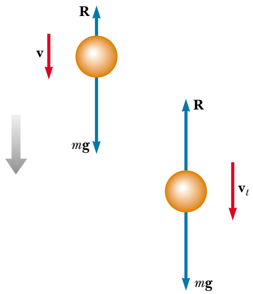


Figure 6.16 An object falling through air experiences a resistive force $\mathbf{R}$ and a gravitational force $\mathbf{F}_g = m\mathbf{g}$. The object reaches terminal speed (on the right) when the net force acting on it is zero, that is, when $\mathbf{R} = -\mathbf{F}_g$ or $R = mg$. Before this occurs, the acceleration varies with speed according to Equation 6.8.

where we have taken downward to be the positive vertical direction. Substituting $\Sigma F = ma$ into Equation 6.7, we find that the object has a downward acceleration of magnitude

$$a = g - \left(\frac{D\rho A}{2m}\right)v^2 \tag{6.8}$$

We can calculate the terminal speed v_t by using the fact that when the force of gravity is balanced by the resistive force, the net force on the object is zero and therefore its acceleration is zero. Setting $a = 0$ in Equation 6.8 gives

$$g - \left(\frac{D\rho A}{2m}\right)v_t^2 = 0$$

$$v_t = \sqrt{\frac{2mg}{D\rho A}} \tag{6.9}$$

Using this expression, we can determine how the terminal speed depends on the dimensions of the object. Suppose the object is a sphere of radius r. In this case, $A \propto r^2$ (from $A = \pi r^2$) and $m \propto r^3$ (because the mass is proportional to the volume of the sphere, which is $V = \frac{4}{3}\pi r^3$). Therefore, $v_t \propto \sqrt{r}$.

Table 6.1 lists the terminal speeds for several objects falling through air.

The high cost of fuel has prompted many truck owners to install wind deflectors on their cabs to reduce drag.

TABLE 6.1 Terminal Speed for Various Objects Falling Through Air

Object	Mass (kg)	Cross-Sectional Area (m^2)	v_t (m/s)
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0

CONCEPTUAL EXAMPLE 6.12

Consider a sky surfer who jumps from a plane with her feet attached firmly to her surfboard, does some tricks, and then opens her parachute. Describe the forces acting on her during these maneuvers.

Solution When the surfer first steps out of the plane, she has no vertical velocity. The downward force of gravity causes her to accelerate toward the ground. As her downward speed increases, so does the upward resistive force exerted by the air on her body and the board. This upward force reduces their acceleration, and so their speed increases more slowly. Eventually, they are going so fast that the upward resistive force matches the downward force of gravity. Now the net force is zero and they no longer accelerate, but reach their terminal speed. At some point after reaching terminal speed, she opens her parachute, resulting in a drastic increase in the upward resistive force. The net force (and thus the acceleration) is now upward, in the direction opposite the direction of the velocity. This causes the downward velocity to decrease rapidly; this means the resistive force on the chute also decreases. Eventually the upward resistive force and the downward force of gravity balance each other and a much smaller terminal speed is reached, permitting a safe landing.

(Contrary to popular belief, the velocity vector of a sky diver never points upward. You may have seen a videotape in which a sky diver appeared to "rocket" upward once the chute opened. In fact, what happened is that the diver slowed down while the person holding the camera continued falling at high speed.)

A sky surfer takes advantage of the upward force of the air on her board. (

EXAMPLE 6.13 Falling Coffee Filters

The dependence of resistive force on speed is an empirical relationship. In other words, it is based on observation rather than on a theoretical model. A series of stacked filters is dropped, and the terminal speeds are measured. Table 6.2 presents data for these coffee filters as they fall through the air. The time constant τ is small, so that a dropped filter quickly reaches terminal speed. Each filter has a mass of 1.64 g. When the filters are nested together, they stack in

such a way that the front-facing surface area does not increase. Determine the relationship between the resistive force exerted by the air and the speed of the falling filters.

Solution At terminal speed, the upward resistive force balances the downward force of gravity. So, a single filter falling at its terminal speed experiences a resistive force of

$$R = mg = \left(\frac{1.64 \text{ g}}{1000 \text{ g/kg}}\right)(9.80 \text{ m/s}^2) = 0.016\,1 \text{ N}$$

Two filters nested together experience 0.032 2 N of resistive force, and so forth. A graph of the resistive force on the filters as a function of terminal speed is shown in Figure 6.17a. A straight line would not be a good fit, indicating that the resistive force is not proportional to the speed. The curved line is for a second-order polynomial, indicating a proportionality of the resistive force to the square of the speed. This proportionality is more clearly seen in Figure 6.17b, in which the resistive force is plotted as a function of the square of the terminal speed.

TABLE 6.2 Terminal Speed for Stacked Coffee Filters

Number of Filters	v_t (m/s)[a]
1	1.01
2	1.40
3	1.63
4	2.00
5	2.25
6	2.40
7	2.57
8	2.80
9	3.05
10	3.22

[a] All values of v_t are approximate.

Pleated coffee filters can be nested together so that the force of air resistance can be studied. (

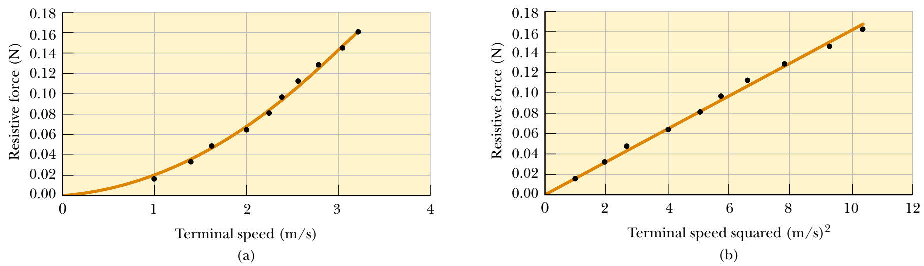


Figure 6.17 (a) Relationship between the resistive force acting on falling coffee filters and their terminal speed. The curved line is a second-order polynomial fit. (b) Graph relating the resistive force to the square of the terminal speed. The fit of the straight line to the data points indicates that the resistive force is proportional to the terminal speed squared. Can you find the proportionality constant?

EXAMPLE 6.14 Resistive Force Exerted on a Baseball

A pitcher hurls a 0.145-kg baseball past a batter at 40.2 m/s (=90 mi/h). Find the resistive force acting on the ball at this speed.

Solution We do not expect the air to exert a huge force on the ball, and so the resistive force we calculate from Equation 6.6 should not be more than a few newtons. First, we must determine the drag coefficient D. We do this by imagining that we drop the baseball and allow it to reach terminal speed. We solve Equation 6.9 for D and substitute the appropriate values for m, v_t, and A from Table 6.1. Taking the density of air as 1.29 kg/m^3, we obtain

$$D = \frac{2\,mg}{v_t^2\,\rho A} = \frac{2(0.145\text{ kg})(9.80\text{ m/s}^2)}{(43\text{ m/s})^2\,(1.29\text{ kg/m}^3)(4.2\times 10^{-3}\text{ m}^2)}$$
$$= 0.284$$

This number has no dimensions. We have kept an extra digit beyond the two that are significant and will drop it at the end of our calculation.

We can now use this value for D in Equation 6.6 to find the magnitude of the resistive force:

$$R = \tfrac{1}{2} D\rho A v^2$$
$$= \tfrac{1}{2}(0.284)(1.29\text{ kg/m}^3)(4.2\times 10^{-3}\text{ m}^2)(40.2\text{ m/s})^2$$
$$= \boxed{1.2\text{ N}}$$

Optional Section

6.5 NUMERICAL MODELING IN PARTICLE DYNAMICS[2]

As we have seen in this and the preceding chapter, the study of the dynamics of a particle focuses on describing the position, velocity, and acceleration as functions of time. Cause-and-effect relationships exist among these quantities: Velocity causes position to change, and acceleration causes velocity to change. Because acceleration is the direct result of applied forces, any analysis of the dynamics of a particle usually begins with an evaluation of the net force being exerted on the particle.

Up till now, we have used what is called the *analytical method* to investigate the position, velocity, and acceleration of a moving particle. Let us review this method briefly before learning about a second way of approaching problems in dynamics. (Because we confine our discussion to one-dimensional motion in this section, boldface notation will not be used for vector quantities.)

If a particle of mass m moves under the influence of a net force ΣF, Newton's second law tells us that the acceleration of the particle is $a = \Sigma F/m$. In general, we apply the analytical method to a dynamics problem using the following procedure:

1. Sum all the forces acting on the particle to get the net force ΣF.
2. Use this net force to determine the acceleration from the relationship $a = \Sigma F/m$.
3. Use this acceleration to determine the velocity from the relationship $dv/dt = a$.
4. Use this velocity to determine the position from the relationship $dx/dt = v$.

The following straightforward example illustrates this method.

EXAMPLE 6.15 An Object Falling in a Vacuum—Analytical Method

Consider a particle falling in a vacuum under the influence of the force of gravity, as shown in Figure 6.18. Use the analytical method to find the acceleration, velocity, and position of the particle.

Solution The only force acting on the particle is the downward force of gravity of magnitude F_g, which is also the net force. Applying Newton's second law, we set the net force acting on the particle equal to the mass of the particle times its acceleration (taking upward to be the positive y direction):

$$F_g = ma_y = -mg$$

Thus, $a_y = -g$, which means the acceleration is constant. Because $dv_y/dt = a_y$, we see that $dv_y/dt = -g$, which may be integrated to yield

$$v_y(t) = v_{yi} - gt$$

Then, because $v_y = dy/dt$, the position of the particle is obtained from another integration, which yields the well-known result

$$y(t) = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

In these expressions, y_i and v_{yi} represent the position and speed of the particle at $t_i = 0$.

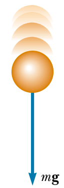


Figure 6.18 An object falling in vacuum under the influence of gravity.

[2] The authors are most grateful to Colonel James Head of the U.S. Air Force Academy for preparing this section. See the *Student Tools CD-ROM* for some assistance with numerical modeling.

The analytical method is straightforward for many physical situations. In the "real world," however, complications often arise that make analytical solutions difficult and perhaps beyond the mathematical abilities of most students taking introductory physics. For example, the net force acting on a particle may depend on the particle's position, as in cases where the gravitational acceleration varies with height. Or the force may vary with velocity, as in cases of resistive forces caused by motion through a liquid or gas.

Another complication arises because the expressions relating acceleration, velocity, position, and time are differential equations rather than algebraic ones. Differential equations are usually solved using integral calculus and other special techniques that introductory students may not have mastered.

When such situations arise, scientists often use a procedure called *numerical modeling* to study motion. The simplest numerical model is called the Euler method, after the Swiss mathematician Leonhard Euler (1707–1783).

The Euler Method

In the **Euler method** for solving differential equations, derivatives are approximated as ratios of finite differences. Considering a small increment of time Δt, we can approximate the relationship between a particle's speed and the magnitude of its acceleration as

$$a(t) \approx \frac{\Delta v}{\Delta t} = \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

Then the speed $v(t+\Delta t)$ of the particle at the end of the time interval Δt is approximately equal to the speed $v(t)$ at the beginning of the time interval plus the magnitude of the acceleration during the interval multiplied by Δt:

$$v(t+\Delta t) \approx v(t) + a(t)\Delta t \qquad \textbf{(6.10)}$$

Because the acceleration is a function of time, this estimate of $v(t+\Delta t)$ is accurate only if the time interval Δt is short enough that the change in acceleration during it is very small (as is discussed later). Of course, Equation 6.10 is exact if the acceleration is constant.

The position $x(t + \Delta t)$ of the particle at the end of the interval Δt can be found in the same manner:

$$v(t) \approx \frac{\Delta x}{\Delta t} = \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$x(t + \Delta t) \approx x(t) + v(t)\Delta t \qquad \textbf{(6.11)}$$

You may be tempted to add the term $\frac{1}{2}\,a(\Delta t)^2$ to this result to make it look like the familiar kinematics equation, but this term is not included in the Euler method because Δt is assumed to be so small that Δt^2 is nearly zero.

If the acceleration at any instant t is known, the particle's velocity and position at a time $t + \Delta t$ can be calculated from Equations 6.10 and 6.11. The calculation then proceeds in a series of finite steps to determine the velocity and position at any later time. The acceleration is determined from the net force acting on the particle, and this force may depend on position, velocity, or time:

$$a(x, v, t) = \frac{\sum F(x, v, t)}{m} \qquad \textbf{(6.12)}$$

See the spreadsheet file "Baseball with Drag" on the Student Web site (address below) for an example of how this technique can be applied to find the initial speed of the baseball described in Example 6.14. We cannot use our regular approach because our kinematics equations assume constant acceleration. Euler's method provides a way to circumvent this difficulty.

It is convenient to set up the numerical solution to this kind of problem by numbering the steps and entering the calculations in a table, a procedure that is illustrated in Table 6.3.

The equations in the table can be entered into a spreadsheet and the calculations performed row by row to determine the velocity, position, and acceleration as functions of time. The calculations can also be carried out by using a program written in either BASIC, C++, or FORTRAN or by using commercially available mathematics packages for personal computers. Many small increments can be taken, and accurate results can usually be obtained with the help of a computer. Graphs of velocity versus time or position versus time can be displayed to help you visualize the motion.

One advantage of the Euler method is that the dynamics is not obscured—the fundamental relationships between acceleration and force, velocity and acceleration, and position and velocity are clearly evident. Indeed, these relationships form the heart of the calculations. There is no need to use advanced mathematics, and the basic physics governs the dynamics.

The Euler method is completely reliable for infinitesimally small time increments, but for practical reasons a finite increment size must be chosen. For the finite difference approximation of Equation 6.10 to be valid, the time increment must be small enough that the acceleration can be approximated as being constant during the increment. We can determine an appropriate size for the time in-

A detailed solution to Problem 41 involving iterative integration appears in the *Student Solutions Manual and Study Guide* and is posted on the Web at **http://www.saunderscollege.com/physics**

TABLE 6.3 The Euler Method for Solving Dynamics Problems

Step	Time	Position	Velocity	Acceleration
0	t_0	x_0	v_0	$a_0 = F(x_0, v_0, t_0)/m$
1	$t_1 = t_0 + \Delta t$	$x_1 = x_0 + v_0\,\Delta t$	$v_1 = v_0 + a_0\,\Delta t$	$a_1 = F(x_1, v_1, t_1)/m$
2	$t_2 = t_1 + \Delta t$	$x_2 = x_1 + v_1\,\Delta t$	$v_2 = v_1 + a_1\,\Delta t$	$a_2 = F(x_2, v_2, t_2)/m$
3	$t_3 = t_2 + \Delta t$	$x_3 = x_2 + v_2\,\Delta t$	$v_3 = v_2 + a_2\,\Delta t$	$a_3 = F(x_3, v_3, t_3)/m$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	t_n	x_n	v_n	a_n

crement by examining the particular problem being investigated. The criterion for the size of the time increment may need to be changed during the course of the motion. In practice, however, we usually choose a time increment appropriate to the initial conditions and use the same value throughout the calculations.

The size of the time increment influences the accuracy of the result, but unfortunately it is not easy to determine the accuracy of an Euler-method solution without a knowledge of the correct analytical solution. One method of determining the accuracy of the numerical solution is to repeat the calculations with a smaller time increment and compare results. If the two calculations agree to a certain number of significant figures, you can assume that the results are correct to that precision.

SUMMARY

Newton's second law applied to a particle moving in uniform circular motion states that the net force causing the particle to undergo a centripetal acceleration is

$$\sum F_r = ma_r = \frac{mv^2}{r} \qquad \textbf{(6.1)}$$

You should be able to use this formula in situations where the force providing the centripetal acceleration could be the force of gravity, a force of friction, a force of string tension, or a normal force.

A particle moving in nonuniform circular motion has both a centripetal component of acceleration and a nonzero tangential component of acceleration. In the case of a particle rotating in a vertical circle, the force of gravity provides the tangential component of acceleration and part or all of the centripetal component of acceleration. Be sure you understand the directions and magnitudes of the velocity and acceleration vectors for nonuniform circular motion.

An observer in a noninertial (accelerating) frame of reference must introduce **fictitious forces** when applying Newton's second law in that frame. If these fictitious forces are properly defined, the description of motion in the noninertial frame is equivalent to that made by an observer in an inertial frame. However, the observers in the two frames do not agree on the causes of the motion. You should be able to distinguish between inertial and noninertial frames and identify the fictitious forces acting in a noninertial frame.

A body moving through a liquid or gas experiences a **resistive force** that is speed-dependent. This resistive force, which opposes the motion, generally increases with speed. The magnitude of the resistive force depends on the shape of the body and on the properties of the medium through which the body is moving. In the limiting case for a falling body, when the magnitude of the resistive force equals the body's weight, the body reaches its **terminal speed.** You should be able to apply Newton's laws to analyze the motion of objects moving under the influence of resistive forces. You may need to apply **Euler's method** if the force depends on velocity, as it does for air drag.

QUESTIONS

1. Because the Earth rotates about its axis and revolves around the Sun, it is a noninertial frame of reference. Assuming the Earth is a uniform sphere, why would the apparent weight of an object be greater at the poles than at the equator?
2. Explain why the Earth bulges at the equator.

3. Why is it that an astronaut in a space capsule orbiting the Earth experiences a feeling of weightlessness?
4. Why does mud fly off a rapidly turning automobile tire?
5. Imagine that you attach a heavy object to one end of a spring and then whirl the spring and object in a horizontal circle (by holding the free end of the spring). Does the spring stretch? If so, why? Discuss this in terms of the force causing the circular motion.
6. It has been suggested that rotating cylinders about 10 mi in length and 5 mi in diameter be placed in space and used as colonies. The purpose of the rotation is to simulate gravity for the inhabitants. Explain this concept for producing an effective gravity.
7. Why does a pilot tend to black out when pulling out of a steep dive?
8. Describe a situation in which a car driver can have a centripetal acceleration but no tangential acceleration.
9. Describe the path of a moving object if its acceleration is constant in magnitude at all times and (a) perpendicular to the velocity; (b) parallel to the velocity.
10. Analyze the motion of a rock falling through water in terms of its speed and acceleration as it falls. Assume that the resistive force acting on the rock increases as the speed increases.
11. Consider a small raindrop and a large raindrop falling through the atmosphere. Compare their terminal speeds. What are their accelerations when they reach terminal speed?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 6.1 Newton's Second Law Applied to Uniform Circular Motion

1. A toy car moving at constant speed completes one lap around a circular track (a distance of 200 m) in 25.0 s. (a) What is its average speed? (b) If the mass of the car is 1.50 kg, what is the magnitude of the force that keeps it in a circle?
2. A 55.0-kg ice skater is moving at 4.00 m/s when she grabs the loose end of a rope, the opposite end of which is tied to a pole. She then moves in a circle of radius 0.800 m around the pole. (a) Determine the force exerted by the rope on her arms. (b) Compare this force with her weight.
3. A light string can support a stationary hanging load of 25.0 kg before breaking. A 3.00-kg mass attached to the string rotates on a horizontal, frictionless table in a circle of radius 0.800 m. What range of speeds can the mass have before the string breaks?
4. In the Bohr model of the hydrogen atom, the speed of the electron is approximately 2.20×10^6 m/s. Find (a) the force acting on the electron as it revolves in a circular orbit of radius 0.530×10^{-10} m and (b) the centripetal acceleration of the electron.
5. In a cyclotron (one type of particle accelerator), a deuteron (of atomic mass 2.00 u) reaches a final speed of 10.0% of the speed of light while moving in a circular path of radius 0.480 m. The deuteron is maintained in the circular path by a magnetic force. What magnitude of force is required?
6. A satellite of mass 300 kg is in a circular orbit around the Earth at an altitude equal to the Earth's mean radius (see Example 6.6). Find (a) the satellite's orbital speed, (b) the period of its revolution, and (c) the gravitational force acting on it.
7. Whenever two Apollo astronauts were on the surface of the Moon, a third astronaut orbited the Moon. Assume the orbit to be circular and 100 km above the surface of the Moon. If the mass of the Moon is 7.40×10^{22} kg and its radius is 1.70×10^6 m, determine (a) the orbiting astronaut's acceleration, (b) his orbital speed, and (c) the period of the orbit.
8. The speed of the tip of the minute hand on a town clock is 1.75×10^{-3} m/s. (a) What is the speed of the tip of the second hand of the same length? (b) What is the centripetal acceleration of the tip of the second hand?
9. A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm/s. (a) What provides the force in the radial direction when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between coin and turntable?
10. The cornering performance of an automobile is evaluated on a skid pad, where the maximum speed that a car can maintain around a circular path on a dry, flat surface is measured. The centripetal acceleration, also called the lateral acceleration, is then calculated as a multiple of the free-fall acceleration *g*. The main factors affecting the performance are the tire characteristics and the suspension system of the car. A Dodge Viper GTS can negotiate a skid pad of radius 61.0 m at 86.5 km/h. Calculate its maximum lateral acceleration.
11. A crate of eggs is located in the middle of the flatbed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, how fast can the truck be moving without the crate sliding?

12. A car initially traveling eastward turns north by traveling in a circular path at uniform speed as in Figure P6.12. The length of the arc *ABC* is 235 m, and the car completes the turn in 36.0 s. (a) What is the acceleration when the car is at *B* located at an angle of 35.0°? Express your answer in terms of the unit vectors **i** and **j**. Determine (b) the car's average speed and (c) its average acceleration during the 36.0-s interval.

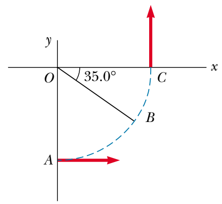


Figure P6.12

13. Consider a conical pendulum with an 80.0-kg bob on a 10.0-m wire making an angle of $\theta = 5.00°$ with the vertical (Fig. P6.13). Determine (a) the horizontal and vertical components of the force exerted by the wire on the pendulum and (b) the radial acceleration of the bob.

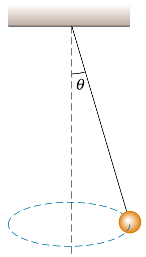


Figure P6.13

Section 6.2 Nonuniform Circular Motion

14. A car traveling on a straight road at 9.00 m/s goes over a hump in the road. The hump may be regarded as an arc of a circle of radius 11.0 m. (a) What is the apparent weight of a 600-N woman in the car as she rides over the hump? (b) What must be the speed of the car over the hump if she is to experience weightlessness? (That is, if her apparent weight is zero.)
15. WEB Tarzan (m = 85.0 kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) is 8.00 m/s. Tarzan doesn't know that the vine has a breaking strength of 1 000 N. Does he make it safely across the river?
16. A hawk flies in a horizontal arc of radius 12.0 m at a constant speed of 4.00 m/s. (a) Find its centripetal acceleration. (b) It continues to fly along the same horizontal arc but steadily increases its speed at the rate of 1.20 m/s^2. Find the acceleration (magnitude and direction) under these conditions.
17. A 40.0-kg child sits in a swing supported by two chains, each 3.00 m long. If the tension in each chain at the lowest point is 350 N, find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.)
18. A child of mass *m* sits in a swing supported by two chains, each of length *R*. If the tension in each chain at the lowest point is *T*, find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.)
19. WEB A pail of water is rotated in a vertical circle of radius 1.00 m. What must be the minimum speed of the pail at the top of the circle if no water is to spill out?
20. A 0.400-kg object is swung in a vertical circular path on a string 0.500 m long. If its speed is 4.00 m/s at the top of the circle, what is the tension in the string there?
21. A roller-coaster car has a mass of 500 kg when fully loaded with passengers (Fig. P6.21). (a) If the car has a speed of 20.0 m/s at point *A*, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the car can have at *B* and still remain on the track?

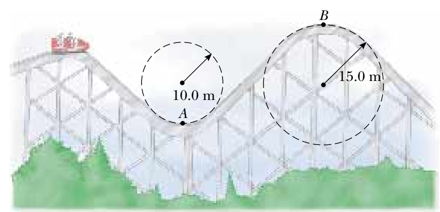


Figure P6.21

22. A roller coaster at the Six Flags Great America amusement park in Gurnee, Illinois, incorporates some of the latest design technology and some basic physics. Each vertical loop, instead of being circular, is shaped like a teardrop (Fig. P6.22). The cars ride on the inside of the loop at the top, and the speeds are high enough to ensure that the cars remain on the track. The biggest loop is 40.0 m high, with a maximum speed of 31.0 m/s (nearly 70 mi/h) at the bottom. Suppose the speed at the top is 13.0 m/s and the corresponding centripetal acceleration is $2g$. (a) What is the radius of the arc of the teardrop at the top? (b) If the total mass of the cars plus people is M, what force does the rail exert on this total mass at the top? (c) Suppose the roller coaster had a loop of radius 20.0 m. If the cars have the same speed, 13.0 m/s at the top, what is the centripetal acceleration at the top? Comment on the normal force at the top in this situation.

Figure P6.22 *(Frank Cezus/FPG International)*

(Optional)
Section 6.3 Motion in Accelerated Frames

23. A merry-go-round makes one complete revolution in 12.0 s. If a 45.0-kg child sits on the horizontal floor of the merry-go-round 3.00 m from the center, find (a) the child's acceleration and (b) the horizontal force of friction that acts on the child. (c) What minimum coefficient of static friction is necessary to keep the child from slipping?

24. A 5.00-kg mass attached to a spring scale rests on a frictionless, horizontal surface as in Figure P6.24. The spring scale, attached to the front end of a boxcar, reads 18.0 N when the car is in motion. (a) If the spring scale reads zero when the car is at rest, determine the acceleration of the car. (b) What will the spring scale read if the car moves with constant velocity? (c) Describe the forces acting on the mass as observed by someone in the car and by someone at rest outside the car.

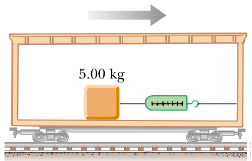


Figure P6.24

25. A 0.500-kg object is suspended from the ceiling of an accelerating boxcar as was seen in Figure 6.13. If $a = 3.00\ m/s^2$, find (a) the angle that the string makes with the vertical and (b) the tension in the string.

26. The Earth rotates about its axis with a period of 24.0 h. Imagine that the rotational speed can be increased. If an object at the equator is to have zero apparent weight, (a) what must the new period be? (b) By what factor would the speed of the object be increased when the planet is rotating at the higher speed? (*Hint:* See Problem 53 and note that the apparent weight of the object becomes zero when the normal force exerted on it is zero. Also, the distance traveled during one period is $2\pi R$, where R is the Earth's radius.)

27. A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of 591 N. As the elevator later stops, the scale reading is 391 N. Assume the magnitude of the acceleration is the same during starting and stopping, and determine (a) the weight of the person, (b) the person's mass, and (c) the acceleration of the elevator.

28. A child on vacation wakes up. She is lying on her back. The tension in the muscles on both sides of her neck is 55.0 N as she raises her head to look past her toes and out the motel window. Finally, it is not raining! Ten minutes later she is screaming and sliding feet first down a water slide at a constant speed of 5.70 m/s, riding high on the outside wall of a horizontal curve of radius 2.40 m (Fig. P6.28). She raises her head to look forward past her toes; find the tension in the muscles on both sides of her neck.

Figure P6.28

29. A plumb bob does not hang exactly along a line directed to the center of the Earth, because of the Earth's rotation. How much does the plumb bob deviate from a radial line at 35.0° north latitude? Assume that the Earth is spherical.

(Optional)
Section 6.4 Motion in the Presence of Resistive Forces

30. A sky diver of mass 80.0 kg jumps from a slow-moving aircraft and reaches a terminal speed of 50.0 m/s. (a) What is the acceleration of the sky diver when her speed is 30.0 m/s? What is the drag force exerted on the diver when her speed is (b) 50.0 m/s? (c) 30.0 m/s?

31. A small piece of Styrofoam packing material is dropped from a height of 2.00 m above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by $a = g - bv$. After falling 0.500 m, the Styrofoam effectively reaches its terminal speed, and then takes 5.00 s more to reach the ground. (a) What is the value of the constant b? (b) What is the acceleration at $t = 0$? (c) What is the acceleration when the speed is 0.150 m/s?

32. (a) Estimate the terminal speed of a wooden sphere (density $0.830\ g/cm^3$) falling through the air if its radius is 8.00 cm. (b) From what height would a freely falling object reach this speed in the absence of air resistance?

33. Calculate the force required to pull a copper ball of radius 2.00 cm upward through a fluid at the constant speed 9.00 cm/s. Take the drag force to be proportional to the speed, with proportionality constant 0.950 kg/s. Ignore the buoyant force.

34. A fire helicopter carries a 620-kg bucket at the end of a cable 20.0 m long as in Figure P6.34. As the helicopter flies to a fire at a constant speed of 40.0 m/s, the cable makes an angle of 40.0° with respect to the vertical. The bucket presents a cross-sectional area of $3.80\ m^2$ in a plane perpendicular to the air moving past it. Determine the drag coefficient assuming that the resistive force is proportional to the square of the bucket's speed.

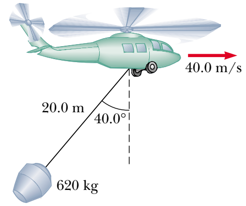


Figure P6.34

35. A small, spherical bead of mass 3.00 g is released from rest at $t = 0$ in a bottle of liquid shampoo. The terminal speed is observed to be $v_t = 2.00$ cm/s. Find (a) the value of the constant b in Equation 6.4, (b) the time τ the bead takes to reach $0.632v_t$, and (c) the value of the resistive force when the bead reaches terminal speed.

36. The mass of a sports car is 1 200 kg. The shape of the car is such that the aerodynamic drag coefficient is 0.250 and the frontal area is $2.20\ m^2$. Neglecting all other sources of friction, calculate the initial acceleration of the car if, after traveling at 100 km/h, it is shifted into neutral and is allowed to coast.

WEB **37.** A motorboat cuts its engine when its speed is 10.0 m/s and coasts to rest. The equation governing the motion of the motorboat during this period is $v = v_i e^{-ct}$, where v is the speed at time t, v_i is the initial speed, and c is a constant. At $t = 20.0$ s, the speed is 5.00 m/s. (a) Find the constant c. (b) What is the speed at $t = 40.0$ s? (c) Differentiate the expression for $v(t)$ and thus show that the acceleration of the boat is proportional to the speed at any time.

38. Assume that the resistive force acting on a speed skater is $f = -kmv^2$, where k is a constant and m is the skater's mass. The skater crosses the finish line of a straight-line race with speed v_f and then slows down by coasting on his skates. Show that the skater's speed at any time t after crossing the finish line is $v(t) = v_f/(1 + ktv_f)$.

39. You can feel a force of air drag on your hand if you stretch your arm out of the open window of a speeding car. (*Note:* Do not get hurt.) What is the order of magnitude of this force? In your solution, state the quantities you measure or estimate and their values.

(Optional)
6.5 Numerical Modeling in Particle Dynamics

40. A 3.00-g leaf is dropped from a height of 2.00 m above the ground. Assume the net downward force exerted on the leaf is $F = mg - bv$, where the drag factor is $b = 0.030\ 0$ kg/s. (a) Calculate the terminal speed of the leaf. (b) Use Euler's method of numerical analysis to find the speed and position of the leaf as functions of time, from the instant it is released until 99% of terminal speed is reached. (*Hint:* Try $\Delta t = 0.005$ s.)

WEB 41. A hailstone of mass 4.80×10^{-4} kg falls through the air and experiences a net force given by

$$F = -mg + Cv^2$$

where $C = 2.50 \times 10^{-5}$ kg/m. (a) Calculate the terminal speed of the hailstone. (b) Use Euler's method of numerical analysis to find the speed and position of the hailstone at 0.2-s intervals, taking the initial speed to be zero. Continue the calculation until the hailstone reaches 99% of terminal speed.

42. A 0.142-kg baseball has a terminal speed of 42.5 m/s (95 mi/h). (a) If a baseball experiences a drag force of magnitude $R = Cv^2$, what is the value of the constant C? (b) What is the magnitude of the drag force when the speed of the baseball is 36.0 m/s? (c) Use a computer to determine the motion of a baseball thrown vertically upward at an initial speed of 36.0 m/s. What maximum height does the ball reach? How long is it in the air? What is its speed just before it hits the ground?

43. A 50.0-kg parachutist jumps from an airplane and falls with a drag force proportional to the square of the speed $R = Cv^2$. Take $C = 0.200$ kg/m with the parachute closed and $C = 20.0$ kg/m with the chute open. (a) Determine the terminal speed of the parachutist in both configurations, before and after the chute is opened. (b) Set up a numerical analysis of the motion and compute the speed and position as functions of time, assuming the jumper begins the descent at 1 000 m above the ground and is in free fall for 10.0 s before opening the parachute. (*Hint:* When the parachute opens, a sudden large acceleration takes place; a smaller time step may be necessary in this region.)

44. Consider a 10.0-kg projectile launched with an initial speed of 100 m/s, at an angle of 35.0° elevation. The resistive force is $\mathbf{R} = -b\mathbf{v}$, where $b = 10.0$ kg/s. (a) Use a numerical method to determine the horizontal and vertical positions of the projectile as functions of time. (b) What is the range of this projectile? (c) Determine the elevation angle that gives the maximum range for the projectile. (*Hint:* Adjust the elevation angle by trial and error to find the greatest range.)

45. A professional golfer hits a golf ball of mass 46.0 g with her 5-iron, and the ball first strikes the ground 155 m (170 yards) away. The ball experiences a drag force of magnitude $R = Cv^2$ and has a terminal speed of 44.0 m/s. (a) Calculate the drag constant C for the golf ball. (b) Use a numerical method to analyze the trajectory of this shot. If the initial velocity of the ball makes an angle of 31.0° (the loft angle) with the horizontal, what initial speed must the ball have to reach the 155-m distance? (c) If the same golfer hits the ball with her 9-iron (47.0° loft) and it first strikes the ground 119 m away, what is the initial speed of the ball? Discuss the differences in trajectories between the two shots.

ADDITIONAL PROBLEMS

46. An 1 800-kg car passes over a bump in a road that follows the arc of a circle of radius 42.0 m as in Figure P6.46. (a) What force does the road exert on the car as the car passes the highest point of the bump if the car travels at 16.0 m/s? (b) What is the maximum speed the car can have as it passes this highest point before losing contact with the road?

47. A car of mass m passes over a bump in a road that follows the arc of a circle of radius R as in Figure P6.46. (a) What force does the road exert on the car as the car passes the highest point of the bump if the car travels at a speed v? (b) What is the maximum speed the car can have as it passes this highest point before losing contact with the road?

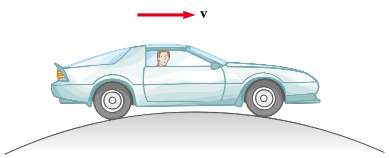


Figure P6.46 Problems 46 and 47.

48. In one model of a hydrogen atom, the electron in orbit around the proton experiences an attractive force of about 8.20×10^{-8} N. If the radius of the orbit is 5.30×10^{-11} m, how many revolutions does the electron make each second? (This number of revolutions per unit time is called the *frequency* of the motion.) See the inside front cover for additional data.

49. A student builds and calibrates an accelerometer, which she uses to determine the speed of her car around a certain unbanked highway curve. The accelerometer is a plumb bob with a protractor that she attaches to the roof of her car. A friend riding in the car with her observes that the plumb bob hangs at an angle of 15.0° from the vertical when the car has a speed of 23.0 m/s. (a) What is the centripetal acceleration of the car rounding the curve? (b) What is the radius of the curve? (c) What is the speed of the car if the plumb bob deflection is 9.00° while the car is rounding the same curve?

50. Suppose the boxcar shown in Figure 6.13 is moving with constant acceleration a up a hill that makes an angle ϕ with the horizontal. If the hanging pendulum makes a constant angle θ with the perpendicular to the ceiling, what is a?

51. An air puck of mass 0.250 kg is tied to a string and allowed to revolve in a circle of radius 1.00 m on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a mass of 1.00 kg is tied to it (Fig. P6.51). The suspended mass remains in equilibrium while the puck on the tabletop revolves. What are (a) the tension in the string, (b) the force exerted by the string on the puck, and (c) the speed of the puck?

52. An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a mass m_2 is tied to it (Fig. P6.51). The suspended mass remains in equilibrium while the puck on the tabletop revolves. What are (a) the tension in the string? (b) the central force exerted on the puck? (c) the speed of the puck?

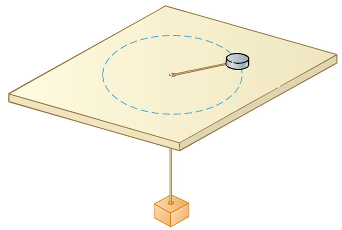


Figure P6.51 Problems 51 and 52.

WEB 53. Because the Earth rotates about its axis, a point on the equator experiences a centripetal acceleration of 0.033 7 m/s^2, while a point at one of the poles experiences no centripetal acceleration. (a) Show that at the equator the gravitational force acting on an object (the true weight) must exceed the object's apparent weight. (b) What is the apparent weight at the equator and at the poles of a person having a mass of 75.0 kg? (Assume the Earth is a uniform sphere and take $g = 9.800$ m/s^2.)

54. A string under a tension of 50.0 N is used to whirl a rock in a horizontal circle of radius 2.50 m at a speed of 20.4 m/s. The string is pulled in and the speed of the rock increases. When the string is 1.00 m long and the speed of the rock is 51.0 m/s, the string breaks. What is the breaking strength (in newtons) of the string?

55. A child's toy consists of a small wedge that has an acute angle θ (Fig. P6.55). The sloping side of the wedge is frictionless, and a mass m on it remains at constant height if the wedge is spun at a certain constant speed. The wedge is spun by rotating a vertical rod that is firmly attached to the wedge at the bottom end. Show that, when the mass sits a distance L up along the sloping side, the speed of the mass must be

$$v = (gL \sin \theta)^{1/2}$$

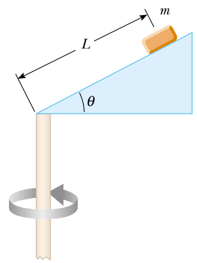


Figure P6.55

56. The pilot of an airplane executes a constant-speed loop-the-loop maneuver. His path is a vertical circle. The speed of the airplane is 300 mi/h, and the radius of the circle is 1 200 ft. (a) What is the pilot's apparent weight at the lowest point if his true weight is 160 lb? (b) What is his apparent weight at the highest point? (c) Describe how the pilot could experience apparent weightlessness if both the radius and the speed can be varied. (*Note:* His apparent weight is equal to the force that the seat exerts on his body.)

57. For a satellite to move in a stable circular orbit at a constant speed, its centripetal acceleration must be inversely proportional to the square of the radius r of the orbit. (a) Show that the tangential speed of a satellite is proportional to $r^{-1/2}$. (b) Show that the time required to complete one orbit is proportional to $r^{3/2}$.

58. A penny of mass 3.10 g rests on a small 20.0-g block supported by a spinning disk (Fig. P6.58). If the coeffi-

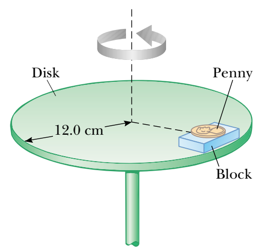


Figure P6.58

cients of friction between block and disk are 0.750 (static) and 0.640 (kinetic) while those for the penny and block are 0.450 (kinetic) and 0.520 (static), what is the maximum rate of rotation (in revolutions per minute) that the disk can have before either the block or the penny starts to slip?

59. Figure P6.59 shows a Ferris wheel that rotates four times each minute and has a diameter of 18.0 m. (a) What is the centripetal acceleration of a rider? What force does the seat exert on a 40.0-kg rider (b) at the lowest point of the ride and (c) at the highest point of the ride? (d) What force (magnitude and direction) does the seat exert on a rider when the rider is halfway between top and bottom?

Figure P6.59 *(Color Box/FPG)*

60. A space station, in the form of a large wheel 120 m in diameter, rotates to provide an "artificial gravity" of 3.00 m/s^2 for persons situated at the outer rim. Find the rotational frequency of the wheel (in revolutions per minute) that will produce this effect.

61. An amusement park ride consists of a rotating circular platform 8.00 m in diameter from which 10.0-kg seats are suspended at the end of 2.50-m massless chains (Fig. P6.61). When the system rotates, the chains make an angle $\theta = 28.0°$ with the vertical. (a) What is the speed of each seat? (b) Draw a free-body diagram of a 40.0-kg child riding in a seat and find the tension in the chain.

62. A piece of putty is initially located at point A on the rim of a grinding wheel rotating about a horizontal axis. The putty is dislodged from point A when the diameter through A is horizontal. The putty then rises vertically and returns to A the instant the wheel completes one revolution. (a) Find the speed of a point on the rim of the wheel in terms of the acceleration due to gravity and the radius R of the wheel. (b) If the mass of the putty is m, what is the magnitude of the force that held it to the wheel?

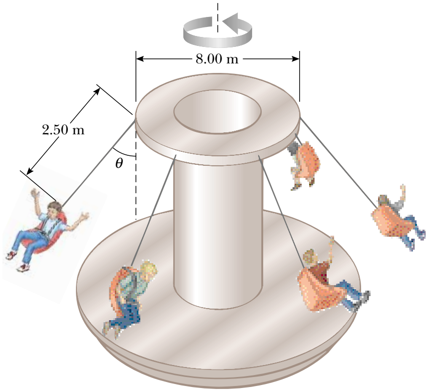


Figure P6.61

63. An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P6.63). The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R. (a) Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R\mu_s/g)^{1/2}$. (b) Obtain a numerical value for T

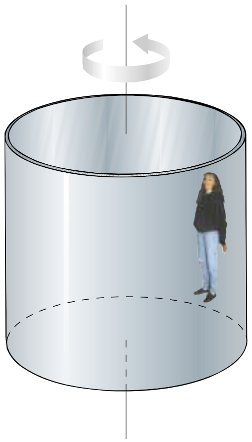


Figure P6.63

if $R = 4.00$ m and $\mu_s = 0.400$. How many revolutions per minute does the cylinder make?

64. *An example of the Coriolis effect.* Suppose air resistance is negligible for a golf ball. A golfer tees off from a location precisely at $\phi_i = 35.0°$ north latitude. He hits the ball due south, with range 285 m. The ball's initial velocity is at 48.0° above the horizontal. (a) For what length of time is the ball in flight? The cup is due south of the golfer's location, and he would have a hole-in-one if the Earth were not rotating. As shown in Figure P6.64, the Earth's rotation makes the tee move in a circle of radius $R_E \cos \phi_i = (6.37 \times 10^6 \text{ m}) \cos 35.0°$, completing one revolution each day. (b) Find the eastward speed of the tee, relative to the stars. The hole is also moving eastward, but it is 285 m farther south and thus at a slightly lower latitude ϕ_f. Because the hole moves eastward in a slightly larger circle, its speed must be greater than that of the tee. (c) By how much does the hole's speed exceed that of the tee? During the time the ball is in flight, it moves both upward and downward, as well as southward with the projectile motion you studied in Chapter 4, but it also moves eastward with the speed you found in part (b). The hole moves to the east at a faster speed, however, pulling ahead of the ball with the relative speed you found in part (c). (d) How far to the west of the hole does the ball land?

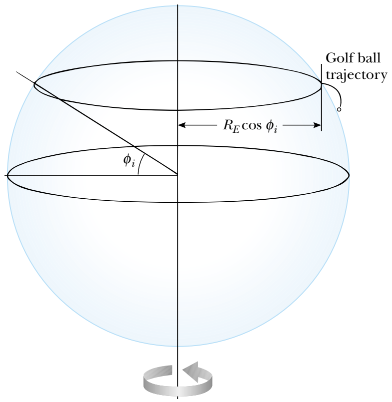


Figure P6.64

65. A curve in a road forms part of a horizontal circle. As a car goes around it at constant speed 14.0 m/s, the total force exerted on the driver has magnitude 130 N. What are the magnitude and direction of the total force exerted on the driver if the speed is 18.0 m/s instead?

66. A car rounds a banked curve as shown in Figure 6.6. The radius of curvature of the road is R, the banking angle is θ, and the coefficient of static friction is μ_s. (a) Determine the range of speeds the car can have without slipping up or down the banked surface. (b) Find the minimum value for μ_s such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100$ m, $\theta = 10.0°$, and $\mu_s = 0.100$ (slippery conditions)?

67. A single bead can slide with negligible friction on a wire that is bent into a circle of radius 15.0 cm, as in Figure P6.67. The circle is always in a vertical plane and rotates steadily about its vertical diameter with a period of 0.450 s. The position of the bead is described by the angle θ that the radial line from the center of the loop to the bead makes with the vertical. (a) At what angle up from the lowest point can the bead stay motionless relative to the turning circle? (b) Repeat the problem if the period of the circle's rotation is 0.850 s.

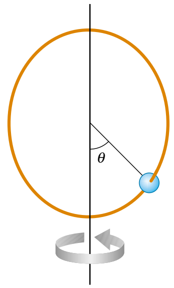


Figure P6.67

68. The expression $F = arv + br^2v^2$ gives the magnitude of the resistive force (in newtons) exerted on a sphere of radius r (in meters) by a stream of air moving at speed v (in meters per second), where a and b are constants with appropriate SI units. Their numerical values are $a = 3.10 \times 10^{-4}$ and $b = 0.870$. Using this formula, find the terminal speed for water droplets falling under their own weight in air, taking the following values for the drop radii: (a) 10.0 μm, (b) 100 μm, (c) 1.00 mm. Note that for (a) and (c) you can obtain accurate answers without solving a quadratic equation, by considering which of the two contributions to the air resistance is dominant and ignoring the lesser contribution.

69. A model airplane of mass 0.750 kg flies in a horizontal circle at the end of a 60.0-m control wire, with a speed of 35.0 m/s. Compute the tension in the wire if it makes a constant angle of 20.0° with the horizontal. The forces exerted on the airplane are the pull of the control wire, its own weight, and aerodynamic lift, which acts at 20.0° inward from the vertical as shown in Figure P6.69.

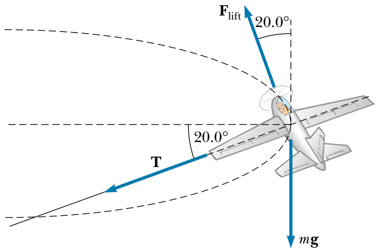


Figure P6.69

70. A 9.00-kg object starting from rest falls through a viscous medium and experiences a resistive force $\mathbf{R} = -b\mathbf{v}$, where $\mathbf{v}$ is the velocity of the object. If the object's speed reaches one-half its terminal speed in 5.54 s, (a) determine the terminal speed. (b) At what time is the speed of the object three-fourths the terminal speed? (c) How far has the object traveled in the first 5.54 s of motion?

71. Members of a skydiving club were given the following data to use in planning their jumps. In the table, d is the distance fallen from rest by a sky diver in a "free-fall stable spread position" versus the time of fall t. (a) Convert the distances in feet into meters. (b) Graph d (in meters) versus t. (c) Determine the value of the terminal speed v_t by finding the slope of the straight portion of the curve. Use a least-squares fit to determine this slope.

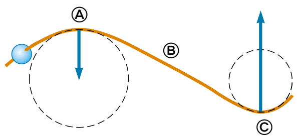
t (s)	d (ft)
1	16
2	62
3	138
4	242
5	366
6	504
7	652
8	808
9	971
10	1 138
11	1 309
12	1 483
13	1 657
14	1 831
15	2 005
16	2 179
17	2 353
18	2 527
19	2 701
20	2 875

ANSWERS TO QUICK QUIZZES

6.1 No. The tangential acceleration changes just the speed part of the velocity vector. For the car to move in a circle, the *direction* of its velocity vector must change, and the only way this can happen is for there to be a centripetal acceleration.

6.2 (a) The ball travels in a circular path that has a larger radius than the original circular path, and so there must be some external force causing the change in the velocity vector's direction. The external force must not be as strong as the original tension in the string because if it were, the ball would follow the original path. (b) The ball again travels in an arc, implying some kind of external force. As in part (a), the external force is directed toward the center of the new arc and not toward the center of the original circular path. (c) The ball undergoes an abrupt change in velocity—from tangent to the circle to perpendicular to it—and so must have experienced a large force that had one component opposite the ball's velocity (tangent to the circle) and another component radially outward. (d) The ball travels in a straight line tangent to the original path. If there is an external force, it cannot have a component perpendicular to this line because if it did, the path would curve. In fact, if the string breaks and there is no other force acting on the ball, Newton's first law says the ball will travel along such a tangent line at constant speed.

6.3 At Ⓐ the path is along the circumference of the larger circle. Therefore, the wire must be exerting a force on the bead directed toward the center of the circle. Because the speed is constant, there is no tangential force component. At Ⓑ the path is not curved, and so the wire exerts no force on the bead. At Ⓒ the path is again curved, and so the wire is again exerting a force on the bead. This time the force is directed toward the center of the smaller circle. Because the radius of this circle is smaller, the magnitude of the force exerted on the bead is larger here than at Ⓐ.



PUZZLER

Chum salmon "climbing a ladder" in the McNeil River in Alaska. Why are fish ladders like this often built around dams? Do the ladders reduce the amount of work that the fish must do to get past the dam? *(Daniel J. Cox/Tony Stone Images)*

chapter

7 Work and Kinetic Energy

Chapter Outline

The concept of energy is one of the most important topics in science and engineering. In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy. They merely tell us that fuels are needed to do a job and that those fuels provide us with something we call *energy*.

In this chapter, we first introduce the concept of work. Work is done by a force acting on an object when the point of application of that force moves through some distance and the force has a component along the line of motion. Next, we define kinetic energy, which is energy an object possesses because of its motion. In general, we can think of *energy* as the capacity that an object has for performing work. We shall see that the concepts of work and kinetic energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws. In a complex situation, in fact, the "energy approach" can often allow a much simpler analysis than the direct application of Newton's second law. However, it is important to note that the work–energy concepts are based on Newton's laws and therefore allow us to make predictions that are always in agreement with these laws.

This alternative method of describing motion is especially useful when the force acting on a particle varies with the position of the particle. In this case, the acceleration is not constant, and we cannot apply the kinematic equations developed in Chapter 2. Often, a particle in nature is subject to a force that varies with the position of the particle. Such forces include the gravitational force and the force exerted on an object attached to a spring. Although we could analyze situations like these by applying numerical methods such as those discussed in Section 6.5, utilizing the ideas of work and energy is often much simpler. We describe techniques for treating complicated systems with the help of an extremely important theorem called the *work–kinetic energy theorem*, which is the central topic of this chapter.

7.1 WORK DONE BY A CONSTANT FORCE

5.1

Almost all the terms we have used thus far—velocity, acceleration, force, and so on—convey nearly the same meaning in physics as they do in everyday life. Now, however, we encounter a term whose meaning in physics is distinctly different from its everyday meaning. That new term is *work*.

To understand what *work* means to the physicist, consider the situation illustrated in Figure 7.1. A force is applied to a chalkboard eraser, and the eraser slides along the tray. If we are interested in how effective the force is in moving the

(a) (b) (c)

Figure 7.1 An eraser being pushed along a chalkboard tray. *(Charles D. Winters)*

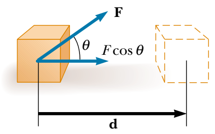


Figure 7.2 If an object undergoes a displacement **d** under the action of a constant force **F**, the work done by the force is $(F \cos \theta)d$.

eraser, we need to consider not only the magnitude of the force but also its direction. If we assume that the magnitude of the applied force is the same in all three photographs, it is clear that the push applied in Figure 7.1b does more to move the eraser than the push in Figure 7.1a. On the other hand, Figure 7.1c shows a situation in which the applied force does not move the eraser at all, regardless of how hard it is pushed. (Unless, of course, we apply a force so great that we break something.) So, in analyzing forces to determine the work they do, we must consider the vector nature of forces. We also need to know how far the eraser moves along the tray if we want to determine the work required to cause that motion. Moving the eraser 3 m requires more work than moving it 2 cm.

Let us examine the situation in Figure 7.2, where an object undergoes a displacement **d** along a straight line while acted on by a constant force **F** that makes an angle θ with **d**.

Work done by a constant force

The **work** W done on an object by an agent exerting a constant force on the object is the product of the component of the force in the direction of the displacement and the magnitude of the displacement:

$$W = Fd \cos \theta \qquad \textbf{(7.1)}$$

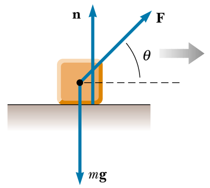


Figure 7.3 When an object is displaced on a frictionless, horizontal, surface, the normal force **n** and the force of gravity $m\mathbf{g}$ do no work on the object. In the situation shown here, **F** is the only force doing work on the object.

As an example of the distinction between this definition of work and our everyday understanding of the word, consider holding a heavy chair at arm's length for 3 min. At the end of this time interval, your tired arms may lead you to think that you have done a considerable amount of work on the chair. According to our definition, however, you have done no work on it whatsoever.[1] You exert a force to support the chair, but you do not move it. A force does no work on an object if the object does not move. This can be seen by noting that if $d = 0$, Equation 7.1 gives $W = 0$—the situation depicted in Figure 7.1c.

Also note from Equation 7.1 that the work done by a force on a moving object is zero when the force applied is perpendicular to the object's displacement. That is, if $\theta = 90°$, then $W = 0$ because $\cos 90° = 0$. For example, in Figure 7.3, the work done by the normal force on the object and the work done by the force of gravity on the object are both zero because both forces are perpendicular to the displacement and have zero components in the direction of **d**.

The sign of the work also depends on the direction of **F** relative to **d**. The work done by the applied force is positive when the vector associated with the component $F \cos \theta$ is in the same direction as the displacement. For example, when an object is lifted, the work done by the applied force is positive because the direction of that force is upward, that is, in the same direction as the displacement. When the vector associated with the component $F \cos \theta$ is in the direction opposite the displacement, W is negative. For example, as an object is lifted, the work done by the gravitational force on the object is negative. The factor $\cos \theta$ in the definition of W (Eq. 7.1) automatically takes care of the sign. It is important to note that **work is an energy transfer;** if energy is transferred *to* the system (object), W is positive; if energy is transferred *from* the system, W is negative.

5.3

[1] Actually, you do work while holding the chair at arm's length because your muscles are continuously contracting and relaxing; this means that they are exerting internal forces on your arm. Thus, work is being done by your body—but internally on itself rather than on the chair.

If an applied force **F** acts along the direction of the displacement, then $\theta = 0$ and $\cos 0 = 1$. In this case, Equation 7.1 gives

$$W = Fd$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the **newton·meter** (N·m). This combination of units is used so frequently that it has been given a name of its own: the **joule** (J).

Quick Quiz 7.1

Can the component of a force that gives an object a centripetal acceleration do any work on the object? (One such force is that exerted by the Sun on the Earth that holds the Earth in a circular orbit around the Sun.)

In general, a particle may be moving with either a constant or a varying velocity under the influence of several forces. In these cases, because work is a scalar quantity, the total work done as the particle undergoes some displacement is the algebraic sum of the amounts of work done by all the forces.

EXAMPLE 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.4a). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

Solution Because they aid us in clarifying which forces are acting on the object being considered, drawings like Figure 7.4b are helpful when we are gathering information and organizing a solution. For our analysis, we use the definition of work (Eq. 7.1):

$$\begin{aligned} W &= (F \cos \theta)d \\ &= (50.0\ \text{N})(\cos 30.0°)(3.00\ \text{m}) = 130\ \text{N·m} \\ &= \boxed{130\ \text{J}} \end{aligned}$$

One thing we should learn from this problem is that the normal force **n**, the force of gravity $\mathbf{F}_g = m\mathbf{g}$, and the upward component of the applied force (50.0 N) (sin 30.0°) do no work on the vacuum cleaner because these forces are perpendicular to its displacement.

Exercise Find the work done by the man on the vacuum cleaner if he pulls it 3.0 m with a horizontal force of 32 N.

Answer 96 J.

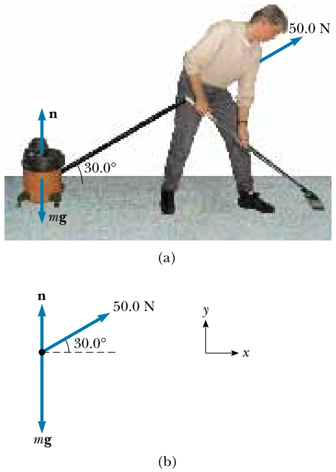


Figure 7.4 (a) A vacuum cleaner being pulled at an angle of 30.0° with the horizontal. (b) Free-body diagram of the forces acting on the vacuum cleaner.

The weightlifter does no work on the weights as he holds them on his shoulders. (If he could rest the bar on his shoulders and lock his knees, he would be able to support the weights for quite some time.) Did he do any work when he raised the weights to this height?

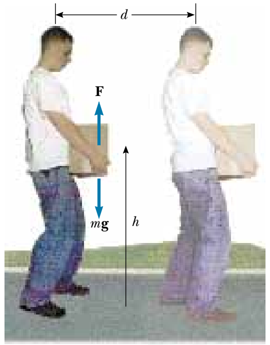


Figure 7.5 A person lifts a box of mass m a vertical distance h and then walks horizontally a distance d.

Quick Quiz 7.2

A person lifts a heavy box of mass m a vertical distance h and then walks horizontally a distance d while holding the box, as shown in Figure 7.5. Determine (a) the work he does on the box and (b) the work done on the box by the force of gravity.

7.2 THE SCALAR PRODUCT OF TWO VECTORS

2.6

Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the **scalar product.** This tool allows us to indicate how **F** and **d** interact in a way that depends on how close to parallel they happen to be. We write this scalar product **F·d.** (Because of the dot symbol, the scalar product is often called the **dot product.**) Thus, we can express Equation 7.1 as a scalar product:

Work expressed as a dot product

$$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos\theta \qquad (7.2)$$

In other words, **F·d** (read "F dot d") is a shorthand notation for $Fd\cos\theta$.

Scalar product of any two vectors **A** and **B**

In general, the scalar product of any two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \qquad (7.3)$$

This relationship is shown in Figure 7.6. Note that **A** and **B** need not have the same units.

In Figure 7.6, $B\cos\theta$ is the projection of **B** onto **A**. Therefore, Equation 7.3 says that **A · B** is the product of the magnitude of **A** and the projection of **B** onto **A**.[2]

From the right-hand side of Equation 7.3 we also see that the scalar product is **commutative.**[3] That is,

The order of the dot product can be reversed

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

Finally, the scalar product obeys the **distributive law of multiplication,** so that

$$\mathbf{A}\cdot(\mathbf{B} + \mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

The dot product is simple to evaluate from Equation 7.3 when **A** is either perpendicular or parallel to **B**. If **A** is perpendicular to **B** ($\theta = 90°$), then $\mathbf{A}\cdot\mathbf{B} = 0$. (The equality $\mathbf{A}\cdot\mathbf{B} = 0$ also holds in the more trivial case when either **A** or **B** is zero.) If vector **A** is parallel to vector **B** and the two point in the same direction ($\theta = 0$), then $\mathbf{A}\cdot\mathbf{B} = AB$. If vector **A** is parallel to vector **B** but the two point in opposite directions ($\theta = 180°$), then $\mathbf{A}\cdot\mathbf{B} = -AB$. The scalar product is negative when $90° < \theta < 180°$.

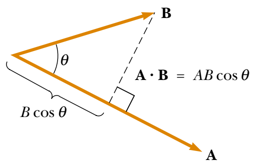


Figure 7.6 The scalar product **A·B** equals the magnitude of **A** multiplied by $B\cos\theta$, which is the projection of **B** onto **A**.

The unit vectors **i**, **j**, and **k**, which were defined in Chapter 3, lie in the positive x, y, and z directions, respectively, of a right-handed coordinate system. Therefore, it follows from the definition of **A · B** that the scalar products of these unit vectors are

Dot products of unit vectors

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1 \qquad (7.4)$$

$$\mathbf{i}\cdot\mathbf{j} = \mathbf{i}\cdot\mathbf{k} = \mathbf{j}\cdot\mathbf{k} = 0 \qquad (7.5)$$

Equations 3.18 and 3.19 state that two vectors **A** and **B** can be expressed in component vector form as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Using the information given in Equations 7.4 and 7.5 shows that the scalar product of **A** and **B** reduces to

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z \qquad (7.6)$$

(Details of the derivation are left for you in Problem 7.10.) In the special case in which **A** = **B**, we see that

$$\mathbf{A}\cdot\mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Quick Quiz 7.3

If the dot product of two vectors is positive, must the vectors have positive rectangular components?

[2] This is equivalent to stating that **A·B** equals the product of the magnitude of **B** and the projection of **A** onto **B**.

[3] This may seem obvious, but in Chapter 11 you will see another way of combining vectors that proves useful in physics and is not commutative.

EXAMPLE 7.2 The Scalar Product

The vectors **A** and **B** are given by $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -\mathbf{i} + 2\mathbf{j}$. (a) Determine the scalar product $\mathbf{A}\cdot\mathbf{B}$.

Solution

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\mathbf{i} + 3\mathbf{j})\cdot(-\mathbf{i} + 2\mathbf{j}) \\ &= -2\mathbf{i}\cdot\mathbf{i} + 2\mathbf{i}\cdot 2\mathbf{j} - 3\mathbf{j}\cdot\mathbf{i} + 3\mathbf{j}\cdot 2\mathbf{j} \\ &= -2(1) + 4(0) - 3(0) + 6(1) \\ &= -2 + 6 = \boxed{4}\end{aligned}$$

where we have used the facts that $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = 1$ and $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{i} = 0$. The same result is obtained when we use Equation 7.6 directly, where $A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

(b) Find the angle θ between **A** and **B**.

Solution The magnitudes of **A** and **B** are

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

Using Equation 7.3 and the result from part (a) we find that

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = \boxed{60.2°}$$

EXAMPLE 7.3 Work Done by a Constant Force

A particle moving in the xy plane undergoes a displacement $\mathbf{d} = (2.0\mathbf{i} + 3.0\mathbf{j})$ m as a constant force $\mathbf{F} = (5.0\mathbf{i} + 2.0\mathbf{j})$ N acts on the particle. (a) Calculate the magnitude of the displacement and that of the force.

Solution

$$d = \sqrt{x^2 + y^2} = \sqrt{(2.0)^2 + (3.0)^2} = \boxed{3.6 \text{ m}}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = \boxed{5.4 \text{ N}}$$

(b) Calculate the work done by **F**.

Solution Substituting the expressions for **F** and **d** into Equations 7.4 and 7.5, we obtain

$$\begin{aligned}W = \mathbf{F}\cdot\mathbf{d} &= (5.0\mathbf{i} + 2.0\mathbf{j})\cdot(2.0\mathbf{i} + 3.0\mathbf{j}) \text{ N}\cdot\text{m} \\ &= 5.0\mathbf{i}\cdot 2.0\mathbf{i} + 5.0\mathbf{i}\cdot 3.0\mathbf{j} + 2.0\mathbf{j}\cdot 2.0\mathbf{i} + 2.0\mathbf{j}\cdot 3.0\mathbf{j} \\ &= 10 + 0 + 0 + 6 = 16 \text{ N}\cdot\text{m} = \boxed{16 \text{ J}}\end{aligned}$$

Exercise Calculate the angle between **F** and **d**.

Answer 35°.

7.3 WORK DONE BY A VARYING FORCE

5.2 Consider a particle being displaced along the x axis under the action of a varying force. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = (F\cos\theta)d$ to calculate the work done by the force because this relationship applies only when **F** is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, shown in Figure 7.7a, then the x component of the force F_x is approximately constant over this interval; for this small displacement, we can express the work done by the force as

$$\Delta W = F_x\,\Delta x$$

This is just the area of the shaded rectangle in Figure 7.7a. If we imagine that the F_x versus x curve is divided into a large number of such intervals, then the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x\,\Delta x$$

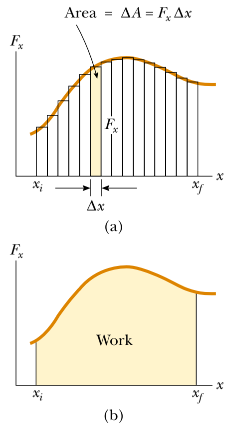


Figure 7.7 (a) The work done by the force component F_x for the small displacement Δx is $F_x\,\Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles. (b) The work done by the component F_x of the varying force as the particle moves from x_i to x_f is exactly equal to the area under this curve.

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area bounded by the F_x curve and the x axis:

$$\lim_{\Delta x\to 0}\sum_{x_i}^{x_f} F_x\,\Delta x = \int_{x_i}^{x_f} F_x\,dx$$

This definite integral is numerically equal to the area under the F_x-versus-x curve between x_i and x_f. Therefore, we can express the work done by F_x as the particle moves from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x\,dx \qquad (7.7)$$

Work done by a varying force

This equation reduces to Equation 7.1 when the component $F_x = F\cos\theta$ is constant.

If more than one force acts on a particle, the total work done is just the work done by the resultant force. If we express the resultant force in the x direction as ΣF_x, then the total work, or *net work*, done as the particle moves from x_i to x_f is

$$\sum W = W_{\text{net}} = \int_{x_i}^{x_f}\left(\sum F_x\right)dx \qquad (7.8)$$

EXAMPLE 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x, as shown in Figure 7.8. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

Solution The work done by the force is equal to the area under the curve from $x_A = 0$ to $x_C = 6.0$ m. This area is equal to the area of the rectangular section from Ⓐ to Ⓑ plus

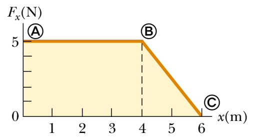


Figure 7.8 The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x_B = 4.0$ m to $x_C = 6.0$ m. The net work done by this force is the area under the curve.

the area of the triangular section from Ⓑ to Ⓒ. The area of the rectangle is (4.0)(5.0) N·m = 20 J, and the area of the triangle is $\frac{1}{2}(2.0)(5.0)$ N·m = 5.0 J. Therefore, the total work done is 25 J.

EXAMPLE 7.5 Work Done by the Sun on a Probe

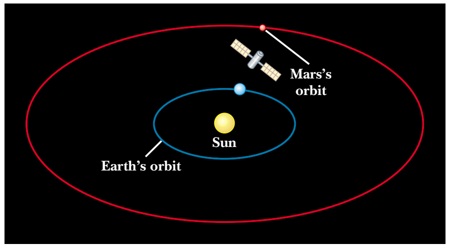
The interplanetary probe shown in Figure 7.9a is attracted to the Sun by a force of magnitude

$$F = -1.3 \times 10^{22}/x^2$$

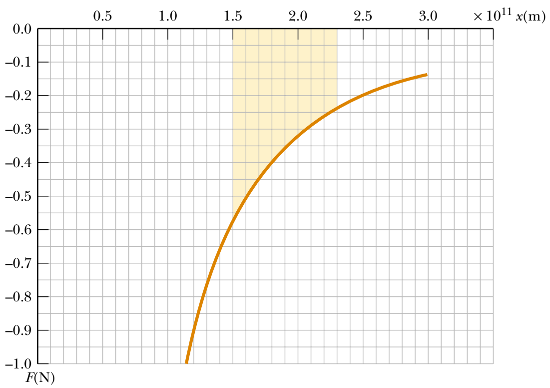
where x is the distance measured outward from the Sun to the probe. Graphically and analytically determine how much work is done by the Sun on the probe as the probe–Sun separation changes from 1.5×10^{11} m to 2.3×10^{11} m.

Graphical Solution The minus sign in the formula for the force indicates that the probe is attracted to the Sun. Because the probe is moving away from the Sun, we expect to calculate a negative value for the work done on it.

A spreadsheet or other numerical means can be used to generate a graph like that in Figure 7.9b. Each small square of the grid corresponds to an area (0.05 N)$(0.1 \times 10^{11}$ m) = 5×10^8 N·m. The work done is equal to the shaded area in Figure 7.9b. Because there are approximately 60 squares shaded, the total area (which is negative because it is below the x axis) is about -3×10^{10} N·m. This is the work done by the Sun on the probe.



(a)



(b)

Figure 7.9 (a) An interplanetary probe moves from a position near the Earth's orbit radially outward from the Sun, ending up near the orbit of Mars. (b) Attractive force versus distance for the interplanetary probe.

Analytical Solution We can use Equation 7.7 to calculate a more precise value for the work done on the probe by the Sun. To solve this integral, we use the first formula of Table B.5 in Appendix B with $n = -2$:

$$W = \int_{1.5\times10^{11}}^{2.3\times10^{11}} \left(\frac{-1.3 \times 10^{22}}{x^2}\right) dx$$

$$= (-1.3 \times 10^{22}) \int_{1.5\times10^{11}}^{2.3\times10^{11}} x^{-2}\, dx$$

$$= (-1.3 \times 10^{22})(-x^{-1}) \Big|_{1.5\times10^{11}}^{2.3\times10^{11}}$$

$$= (-1.3 \times 10^{22})\left(\frac{-1}{2.3 \times 10^{11}} - \frac{-1}{1.5 \times 10^{11}}\right)$$

$$= -3.0 \times 10^{10}\ \text{J}$$

Exercise Does it matter whether the path of the probe is not directed along a radial line away from the Sun?

Answer No; the value of W depends only on the initial and final positions, not on the path taken between these points.

Work Done by a Spring

5.3

A common physical system for which the force varies with position is shown in Figure 7.10. A block on a horizontal, frictionless surface is connected to a spring. If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force of magnitude

$$F_s = -kx \tag{7.9}$$

Spring force

where x is the displacement of the block from its unstretched ($x = 0$) position and k is a positive constant called the **force constant** of the spring. In other words, the force required to stretch or compress a spring is proportional to the amount of stretch or compression x. This force law for springs, known as **Hooke's law,** is valid only in the limiting case of small displacements. The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values.

Quick Quiz 7.4

What are the units for k, the force constant in Hooke's law?

The negative sign in Equation 7.9 signifies that the force exerted by the spring is always directed *opposite* the displacement. When $x > 0$ as in Figure 7.10a, the spring force is directed to the left, in the negative x direction. When $x < 0$ as in Figure 7.10c, the spring force is directed to the right, in the positive x direction. When $x = 0$ as in Figure 7.10b, the spring is unstretched and $F_s = 0$. Because the spring force always acts toward the equilibrium position ($x = 0$), it sometimes is called a *restoring force.* If the spring is compressed until the block is at the point $-x_{max}$ and is then released, the block moves from $-x_{max}$ through zero to $+x_{max}$. If the spring is instead stretched until the block is at the point x_{max} and is then released, the block moves from $+x_{max}$ through zero to $-x_{max}$. It then reverses direction, returns to $+x_{max}$, and continues oscillating back and forth.

Suppose the block has been pushed to the left a distance x_{max} from equilibrium and is then released. Let us calculate the work W_s done by the spring force as the block moves from $x_i = -x_{max}$ to $x_f = 0$. Applying Equation 7.7 and assuming the block may be treated as a particle, we obtain

$$W_s = \int_{x_i}^{x_f} F_s\, dx = \int_{-x_{max}}^{0} (-kx)\, dx = \tfrac{1}{2}kx_{max}^2 \tag{7.10}$$

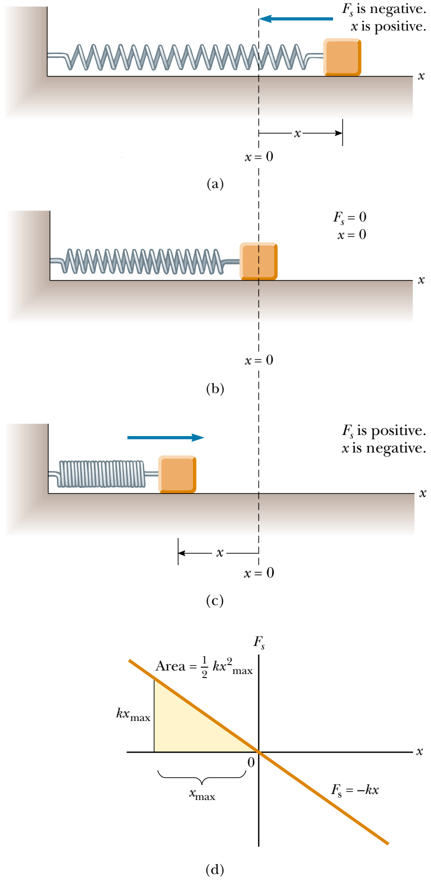


Figure 7.10 The force exerted by a spring on a block varies with the block's displacement x from the equilibrium position $x = 0$. (a) When x is positive (stretched spring), the spring force is directed to the left. (b) When x is zero (natural length of the spring), the spring force is zero. (c) When x is negative (compressed spring), the spring force is directed to the right. (d) Graph of F_s versus x for the block–spring system. The work done by the spring force as the block moves from $-x_{max}$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx^2_{max}$.

where we have used the indefinite integral $\int x^n dx = x^{n+1}/(n+1)$ with $n = 1$. The work done by the spring force is positive because the force is in the same direction as the displacement (both are to the right). When we consider the work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{max}$, we find that

$W_s = -\frac{1}{2}kx^2_{max}$ because for this part of the motion the displacement is to the right and the spring force is to the left. Therefore, the *net* work done by the spring force as the block moves from $x_i = -x_{max}$ to $x_f = x_{max}$ is *zero.*

Figure 7.10d is a plot of F_s versus x. The work calculated in Equation 7.10 is the area of the shaded triangle, corresponding to the displacement from $-x_{max}$ to 0. Because the triangle has base x_{max} and height kx_{max}, its area is $\frac{1}{2}kx^2_{max}$, the work done by the spring as given by Equation 7.10.

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \textbf{(7.11)}$$

Work done by a spring

For example, if the spring has a force constant of 80 N/m and is compressed 3.0 cm from equilibrium, the work done by the spring force as the block moves from $x_i = -3.0$ cm to its unstretched position $x_f = 0$ is 3.6×10^{-2} J. From Equation 7.11 we also see that the work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$). We shall make use of this important result in Chapter 8, in which we describe the motion of this system in greater detail.

Equations 7.10 and 7.11 describe the work done by the spring on the block. Now let us consider the work done on the spring by an *external agent* that stretches the spring very slowly from $x_i = 0$ to $x_f = x_{max}$, as in Figure 7.11. We can calculate this work by noting that at any value of the displacement, the *applied force* $\mathbf{F}_{app}$ is equal to and opposite the spring force $\mathbf{F}_s$, so that $F_{app} = -(-kx) = kx$. Therefore, the work done by this applied force (the external agent) is

$$W_{F_{app}} = \int_0^{x_{max}} F_{app}\,dx = \int_0^{x_{max}} kx\,dx = \tfrac{1}{2}kx^2_{max}$$

This work is equal to the negative of the work done by the spring force for this displacement.

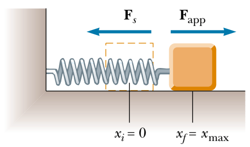


Figure 7.11 A block being pulled from $x_i = 0$ to $x_f = x_{max}$ on a frictionless surface by a force $\mathbf{F}_{app}$. If the process is carried out very slowly, the applied force is equal to and opposite the spring force at all times.

EXAMPLE 7.6 Measuring *k* for a Spring

A common technique used to measure the force constant of a spring is described in Figure 7.12. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position. Because the spring force is upward (opposite the displacement), it must balance the downward force of gravity $m\mathbf{g}$ when the system is at rest. In this case, we can apply Hooke's law to give $|\mathbf{F}_s| = kd = mg$, or

$$k = \frac{mg}{d}$$

For example, if a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, then the force constant is

$$k = \frac{mg}{d} = \frac{(0.55\text{ kg})(9.80\text{ m/s}^2)}{2.0 \times 10^{-2}\text{ m}} = \boxed{2.7 \times 10^2\text{ N/m}}$$

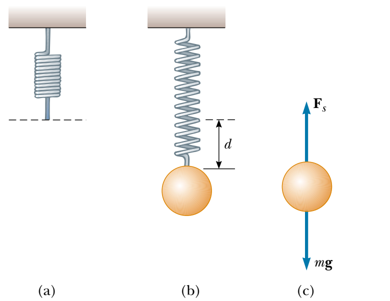


Figure 7.12 Determining the force constant k of a spring. The elongation d is caused by the attached object, which has a weight mg. Because the spring force balances the force of gravity, it follows that $k = mg/d$.

7.4 KINETIC ENERGY AND THE WORK–KINETIC ENERGY THEOREM

5.7 It can be difficult to use Newton's second law to solve motion problems involving complex forces. An alternative approach is to relate the speed of a moving particle to its displacement under the influence of some net force. If the work done by the net force on a particle can be calculated for a given displacement, then the change in the particle's speed can be easily evaluated.

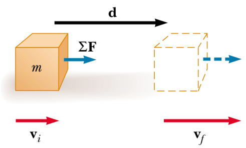


Figure 7.13 A particle undergoing a displacement **d** and a change in velocity under the action of a constant net force Σ**F**.

Figure 7.13 shows a particle of mass m moving to the right under the action of a constant net force $\Sigma\mathbf{F}$. Because the force is constant, we know from Newton's second law that the particle moves with a constant acceleration **a**. If the particle is displaced a distance d, the net work done by the total force $\Sigma\mathbf{F}$ is

$$\sum W = \left(\sum F\right)d = (ma)d \qquad \textbf{(7.12)}$$

In Chapter 2 we found that the following relationships are valid when a particle undergoes constant acceleration:

$$d = \tfrac{1}{2}(v_i + v_f)t \qquad a = \frac{v_f - v_i}{t}$$

where v_i is the speed at $t = 0$ and v_f is the speed at time t. Substituting these expressions into Equation 7.12 gives

$$\sum W = m\left(\frac{v_f - v_i}{t}\right)\tfrac{1}{2}(v_i + v_f)t$$

$$\sum W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \textbf{(7.13)}$$

The quantity $\frac{1}{2}mv^2$ represents the energy associated with the motion of the particle. This quantity is so important that it has been given a special name—**kinetic energy.** The net work done on a particle by a constant net force $\Sigma\mathbf{F}$ acting on it equals the change in kinetic energy of the particle.

In general, the kinetic energy K of a particle of mass m moving with a speed v is defined as

Kinetic energy is energy associated with the motion of a body

$$K \equiv \tfrac{1}{2}mv^2 \qquad \textbf{(7.14)}$$

TABLE 7.1 Kinetic Energies for Various Objects

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^4	2.65×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^3	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^4	3.14×10^{10}
Automobile at 55 mi/h	2 000	25	6.3×10^5
Running athlete	70	10	3.5×10^3
Stone dropped from 10 m	1.0	14	9.8×10^1
Golf ball at terminal speed	0.046	44	4.5×10^1
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

[a] *Escape speed* is the minimum speed an object must attain near the Earth's surface if it is to escape the Earth's gravitational force.

5.4 Kinetic energy is a scalar quantity and has the same units as work. For example, a 2.0-kg object moving with a speed of 4.0 m/s has a kinetic energy of 16 J. Table 7.1 lists the kinetic energies for various objects.

It is often convenient to write Equation 7.13 in the form

$$\sum W = K_f - K_i = \Delta K \qquad \textbf{(7.15)}$$

Work–kinetic energy theorem

That is, $K_i + \Sigma W = K_f$.

Equation 7.15 is an important result known as the **work–kinetic energy theorem.** It is important to note that when we use this theorem, we must include *all* of the forces that do work on the particle in the calculation of the net work done. From this theorem, we see that the speed of a particle increases if the net work done on it is positive because the final kinetic energy is greater than the initial kinetic energy. The particle's speed decreases if the net work done is negative because the final kinetic energy is less than the initial kinetic energy.

The work–kinetic energy theorem as expressed by Equation 7.15 allows us to think of kinetic energy as the work a particle can do in coming to rest, or the amount of energy stored in the particle. For example, suppose a hammer (our particle) is on the verge of striking a nail, as shown in Figure 7.14. The moving hammer has kinetic energy and so can do work on the nail. The work done on the nail is equal to Fd, where F is the average force exerted on the nail by the hammer and d is the distance the nail is driven into the wall.[4]

Figure 7.14 The moving hammer has kinetic energy and thus can do work on the nail, driving it into the wall.

We derived the work–kinetic energy theorem under the assumption of a constant net force, but it also is valid when the force varies. To see this, suppose the net force acting on a particle in the x direction is ΣF_x. We can apply Newton's second law, $\Sigma F_x = ma_x$, and use Equation 7.8 to express the net work done as

$$\sum W = \int_{x_i}^{x_f}\left(\sum F_x\right)dx = \int_{x_i}^{x_f} ma_x\, dx$$

If the resultant force varies with x, the acceleration and speed also depend on x. Because we normally consider acceleration as a function of t, we now use the following chain rule to express a in a slightly different way:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Substituting this expression for a into the above equation for ΣW gives

$$\sum W = \int_{x_i}^{x_f} mv\frac{dv}{dx}\,dx = \int_{v_i}^{v_f} mv\, dv$$

$$\sum W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \textbf{(7.16)}$$

The net work done on a particle equals the change in its kinetic energy

The limits of the integration were changed from x values to v values because the variable was changed from x to v. Thus, we conclude that the net work done on a particle by the net force acting on it is equal to the change in the kinetic energy of the particle. This is true whether or not the net force is constant.

[4] Note that because the nail and the hammer are *systems* of particles rather than single particles, part of the hammer's kinetic energy goes into warming the hammer and the nail upon impact. Also, as the nail moves into the wall in response to the impact, the large frictional force between the nail and the wood results in the continuous transformation of the kinetic energy of the nail into further temperature increases in the nail and the wood, as well as in deformation of the wall. Energy associated with temperature changes is called *internal energy* and will be studied in detail in Chapter 20.

Situations Involving Kinetic Friction

One way to include frictional forces in analyzing the motion of an object sliding on a *horizontal* surface is to describe the kinetic energy lost because of friction. Suppose a book moving on a horizontal surface is given an initial horizontal velocity $\mathbf{v}_i$ and slides a distance d before reaching a final velocity $\mathbf{v}_f$ as shown in Figure 7.15. The external force that causes the book to undergo an acceleration in the negative x direction is the force of kinetic friction $\mathbf{f}_k$ acting to the left, opposite the motion. The initial kinetic energy of the book is $\frac{1}{2}mv_i^2$, and its final kinetic energy is $\frac{1}{2}mv_f^2$. Applying Newton's second law to the book can show this. Because the only force acting on the book in the x direction is the friction force, Newton's second law gives $-f_k = ma_x$. Multiplying both sides of this expression by d and using Equation 2.12 in the form $v_{xf}^2 - v_{xi}^2 = 2a_xd$ for motion under constant acceleration give $-f_kd = (ma_x)d = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$ or

Loss in kinetic energy due to friction

$$\Delta K_{\text{friction}} = -f_kd \qquad \textbf{(7.17a)}$$

This result specifies that the amount by which the force of kinetic friction changes the kinetic energy of the book is equal to $-f_kd$. Part of this lost kinetic energy goes into warming up the book, and the rest goes into warming up the surface over which the book slides. In effect, the quantity $-f_kd$ is equal to the work done by kinetic friction on the book *plus* the work done by kinetic friction on the surface. (We shall study the relationship between temperature and energy in Part III of this text.) When friction—as well as other forces—acts on an object, the work–kinetic energy theorem reads

$$K_i + \sum W_{\text{other}} - f_kd = K_f \qquad \textbf{(7.17b)}$$

Here, ΣW_{other} represents the sum of the amounts of work done on the object by forces other than kinetic friction.

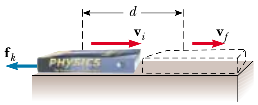


Figure 7.15 A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the book is $\mathbf{v}_i$, and its final velocity is $\mathbf{v}_f$. The normal force and the force of gravity are not included in the diagram because they are perpendicular to the direction of motion and therefore do not influence the book's velocity.

Quick Quiz 7.5

Can frictional forces ever *increase* an object's kinetic energy?

EXAMPLE 7.7 A Block Pulled on a Frictionless Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

Solution We have made a drawing of this situation in Figure 7.16a. We could apply the equations of kinematics to determine the answer, but let us use the energy approach for

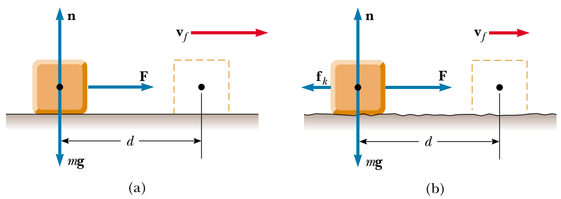


Figure 7.16 A block pulled to the right by a constant horizontal force. (a) Frictionless surface. (b) Rough surface.

practice. The normal force balances the force of gravity on the block, and neither of these vertically acting forces does work on the block because the displacement is horizontal. Because there is no friction, the net external force acting on the block is the 12-N force. The work done by this force is

$$W = Fd = (12\text{ N})(3.0\text{ m}) = 36\text{ N}\cdot\text{m} = 36\text{ J}$$

Using the work–kinetic energy theorem and noting that the initial kinetic energy is zero, we obtain

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f^2 = \frac{2W}{m} = \frac{2(36\text{ J})}{6.0\text{ kg}} = 12\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{3.5\text{ m/s}}$$

Exercise Find the acceleration of the block and determine its final speed, using the kinematics equation $v_{xf}^2 = v_{xi}^2 + 2a_xd$.

Answer $a_x = 2.0\text{ m/s}^2$; $v_f = 3.5\text{ m/s}$.

EXAMPLE 7.8 A Block Pulled on a Rough Surface

Find the final speed of the block described in Example 7.7 if the surface is not frictionless but instead has a coefficient of kinetic friction of 0.15.

Solution The applied force does work just as in Example 7.7:

$$W = Fd = (12\text{ N})(3.0\text{ m}) = 36\text{ J}$$

In this case we must use Equation 7.17a to calculate the kinetic energy lost to friction $\Delta K_{\text{friction}}$. The magnitude of the frictional force is

$$f_k = \mu_kn = \mu_kmg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

The change in kinetic energy due to friction is

$$\Delta K_{\text{friction}} = -f_kd = -(8.82\text{ N})(3.0\text{ m}) = -26.5\text{ J}$$

The final speed of the block follows from Equation 7.17b:

$$\tfrac{1}{2}mv_i^2 + \sum W_{\text{other}} - f_kd = \tfrac{1}{2}mv_f^2$$

$$0 + 36\text{ J} - 26.5\text{ J} = \tfrac{1}{2}(6.0\text{ kg})v_f^2$$

$$v_f^2 = 2(9.5\text{ J})/(6.0\text{ kg}) = 3.18\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{1.8\text{ m/s}}$$

After sliding the 3-m distance on the rough surface, the block is moving at a speed of 1.8 m/s; in contrast, after covering the same distance on a frictionless surface (see Example 7.7), its speed was 3.5 m/s.

Exercise Find the acceleration of the block from Newton's second law and determine its final speed, using equations of kinematics.

Answer $a_x = 0.53\text{ m/s}^2$; $v_f = 1.8\text{ m/s}$.

CONCEPTUAL EXAMPLE 7.9 Does the Ramp Lessen the Work Required?

A man wishes to load a refrigerator onto a truck using a ramp, as shown in Figure 7.17. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his statement valid?

Solution No. Although less force is required with a longer ramp, that force must act over a greater distance if the same amount of work is to be done. Suppose the refrigerator is wheeled on a dolly up the ramp at constant speed. The

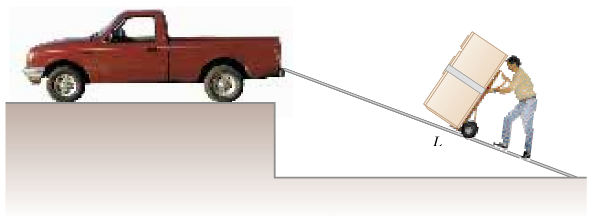


Figure 7.17 A refrigerator attached to a frictionless wheeled dolly is moved up a ramp at constant speed.

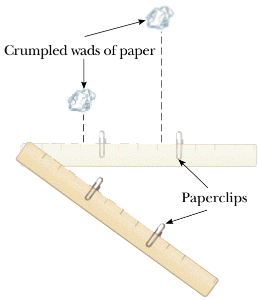
normal force exerted by the ramp on the refrigerator is directed 90° to the motion and so does no work on the refrigerator. Because $\Delta K = 0$, the work–kinetic energy theorem gives

$$\sum W = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the force of gravity equals the weight of the refrigerator mg times the vertical height h through which it is displaced times cos 180°, or $W_{\text{by gravity}} = -mgh$. (The minus sign arises because the downward force of gravity is opposite the displacement.) Thus, the man must do work mgh on the refrigerator, regardless of the length of the ramp.

QuickLab

Attach two paperclips to a ruler so that one of the clips is twice the distance from the end as the other. Place the ruler on a table with two small wads of paper against the clips, which act as stops. Sharply swing the ruler through a small angle, stopping it abruptly with your finger. The outer paper wad will have twice the speed of the inner paper wad as the two slide on the table away from the ruler. Compare how far the two wads slide. How does this relate to the results of Conceptual Example 7.10?



Consider the chum salmon attempting to swim upstream in the photograph at the beginning of this chapter. The "steps" of a fish ladder built around a dam do not change the total amount of work that must be done by the salmon as they leap through some vertical distance. However, the ladder allows the fish to perform that work in a series of smaller jumps, and the net effect is to raise the vertical position of the fish by the height of the dam.

These cyclists are working hard and expending energy as they pedal uphill in Marin County, CA.

CONCEPTUAL EXAMPLE 7.10 Useful Physics for Safer Driving

A certain car traveling at an initial speed v slides a distance d to a halt after its brakes lock. Assuming that the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

Solution Let us assume that the force of kinetic friction between the car and the road surface is constant and the same for both speeds. The net force multiplied by the displacement of the car is equal to the initial kinetic energy of the car (because $K_f = 0$). If the speed is doubled, as it is in this example, the kinetic energy is quadrupled. For a given constant applied force (in this case, the frictional force), the distance traveled is four times as great when the initial speed is doubled, and so the estimated distance that the car slides is $4d$.

EXAMPLE 7.11 A Block–Spring System

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0×10^3 N/m, as shown in Figure 7.10. The spring is compressed 2.0 cm and is then released from rest. (a) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

Solution In this situation, the block starts with $v_i = 0$ at $x_i = -2.0$ cm, and we want to find v_f at $x_f = 0$. We use Equation 7.10 to find the work done by the spring with $x_{\max} = x_i = -2.0\text{ cm} = -2.0 \times 10^{-2}$ m:

$$W_s = \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}(1.0 \times 10^3\text{ N/m})(-2.0 \times 10^{-2}\text{ m})^2 = 0.20\text{ J}$$

Using the work–kinetic energy theorem with $v_i = 0$, we obtain the change in kinetic energy of the block due to the work done on it by the spring:

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$0.20\text{ J} = \tfrac{1}{2}(1.6\text{ kg})v_f^2 - 0$$

$$v_f^2 = \frac{0.40\text{ J}}{1.6\text{ kg}} = 0.25\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{0.50\text{ m/s}}$$

(b) Calculate the speed of the block as it passes through the equilibrium position if a constant frictional force of 4.0 N retards its motion from the moment it is released.

Solution Certainly, the answer has to be less than what we found in part (a) because the frictional force retards the motion. We use Equation 7.17 to calculate the kinetic energy lost because of friction and add this negative value to the kinetic energy found in the absence of friction. The kinetic energy lost due to friction is

$$\Delta K = -f_kd = -(4.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.080\text{ J}$$

In part (a), the final kinetic energy without this loss was found to be 0.20 J. Therefore, the final kinetic energy in the presence of friction is

$$K_f = 0.20\text{ J} - 0.080\text{ J} = 0.12\text{ J} = \tfrac{1}{2}mv_f^2$$

$$\tfrac{1}{2}(1.6\text{ kg})v_f^2 = 0.12\text{ J}$$

$$v_f^2 = \frac{0.24\text{ J}}{1.6\text{ kg}} = 0.15\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{0.39\text{ m/s}}$$

As expected, this value is somewhat less than the 0.50 m/s we found in part (a). If the frictional force were greater, then the value we obtained as our answer would have been even smaller.

7.5 POWER

5.8 Imagine two identical models of an automobile: one with a base-priced four-cylinder engine; and the other with the highest-priced optional engine, a mighty eight-cylinder powerplant. Despite the differences in engines, the two cars have the same mass. Both cars climb a roadway up a hill, but the car with the optional engine takes much less time to reach the top. Both cars have done the same amount of work against gravity, but in different time periods. From a practical viewpoint, it is interesting to know not only the work done by the vehicles but also the *rate* at which it is done. In taking the ratio of the amount of work done to the time taken to do it, we have a way of quantifying this concept. The time rate of doing work is called **power.**

If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the **average power** expended during this interval is defined as

$$\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}$$

Average power

The work done on the object contributes to the increase in the energy of the object. Therefore, a more general definition of power is the *time rate of energy transfer.* In a manner similar to how we approached the definition of velocity and acceleration, we can define the **instantaneous power** $\mathcal{P}$ as the limiting value of the average power as Δt approaches zero:

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

where we have represented the increment of work done by dW. We find from Equation 7.2, letting the displacement be expressed as $d\mathbf{s}$, that $dW = \mathbf{F} \cdot d\mathbf{s}$. Therefore, the instantaneous power can be written

Instantaneous power

$$\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad (7.18)$$

where we use the fact that $\mathbf{v} = d\mathbf{s}/dt$.

The SI unit of power is joules per second (J/s), also called the *watt* (W) (after James Watt, the inventor of the steam engine):

The watt

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^3$$

The symbol W (not italic) for watt should not be confused with the symbol W (italic) for work.

A unit of power in the British engineering system is the horsepower (hp):

$$1 \text{ hp} = 746 \text{ W}$$

A unit of energy (or work) can now be defined in terms of the unit of power. One **kilowatt hour** (kWh) is the energy converted or consumed in 1 h at the constant rate of 1 kW = 1 000 J/s. The numerical value of 1 kWh is

The kilowatt hour is a unit of energy

$$1 \text{ kWh} = (10^3 \text{ W})(3\,600 \text{ s}) = 3.60 \times 10^6 \text{ J}$$

It is important to realize that a kilowatt hour is a unit of energy, not power. When you pay your electric bill, you pay the power company for the total electrical energy you used during the billing period. This energy is the power used multiplied by the time during which it was used. For example, a 300-W lightbulb run for 12 h would convert (0.300 kW)(12 h) = 3.6 kWh of electrical energy.

Quick Quiz 7.6

Suppose that an old truck and a sports car do the same amount of work as they climb a hill but that the truck takes much longer to accomplish this work. How would graphs of $\mathcal{P}$ versus t compare for the two vehicles?

EXAMPLE 7.12 Power Delivered by an Elevator Motor

An elevator car has a mass of 1 000 kg and is carrying passengers having a combined mass of 800 kg. A constant frictional force of 4 000 N retards its motion upward, as shown in Figure 7.18a. (a) What must be the minimum power delivered by the motor to lift the elevator car at a constant speed of 3.00 m/s?

Solution The motor must supply the force of magnitude T that pulls the elevator car upward. Reading that the speed is constant provides the hint that $a = 0$, and therefore we know from Newton's second law that $\Sigma F_y = 0$. We have drawn a free-body diagram in Figure 7.18b and have arbitrarily specified that the upward direction is positive. From Newton's second law we obtain

$$\sum F_y = T - f - Mg = 0$$

where M is the *total* mass of the system (car plus passengers), equal to 1 800 kg. Therefore,

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3 \text{ N} + (1.80 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 2.16 \times 10^4 \text{ N} \end{aligned}$$

Using Equation 7.18 and the fact that $\mathbf{T}$ is in the same direction as $\mathbf{v}$, we find that

$$\begin{aligned} \mathcal{P} &= \mathbf{T}\cdot\mathbf{v} = Tv \\ &= (2.16 \times 10^4 \text{ N})(3.00 \text{ m/s}) = \boxed{6.48 \times 10^4 \text{ W}} \end{aligned}$$

(b) What power must the motor deliver at the instant its speed is v if it is designed to provide an upward acceleration of 1.00 m/s^2?

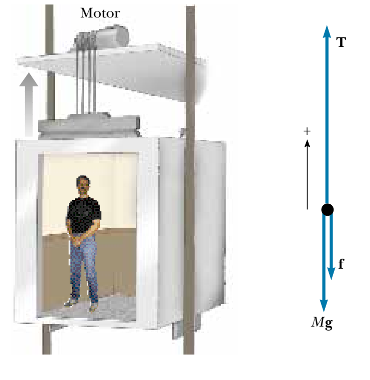
Solution Now we expect to obtain a value greater than we did in part (a), where the speed was constant, because the motor must now perform the additional task of accelerating the car. The only change in the setup of the problem is that now $a > 0$. Applying Newton's second law to the car gives

$$\begin{aligned} \sum F_y &= T - f - Mg = Ma \\ T &= M(a + g) + f \\ &= (1.80 \times 10^3 \text{ kg})(1.00 + 9.80)\text{m/s}^2 + 4.00 \times 10^3 \text{ N} \\ &= 2.34 \times 10^4 \text{ N} \end{aligned}$$

Therefore, using Equation 7.18, we obtain for the required power

$$\mathcal{P} = Tv = \boxed{(2.34 \times 10^4 v) \text{ W}}$$

where v is the instantaneous speed of the car in meters per second. The power is less than that obtained in part (a) as long as the speed is less than $\mathcal{P}/T = 2.77$ m/s, but it is greater when the elevator's speed exceeds this value.



(a) (b)

Figure 7.18 (a) The motor exerts an upward force $\mathbf{T}$ on the elevator car. The magnitude of this force is the tension T in the cable connecting the car and motor. The downward forces acting on the car are a frictional force $\mathbf{f}$ and the force of gravity $\mathbf{F}_g = M\mathbf{g}$. (b) The free-body diagram for the elevator car.

CONCEPTUAL EXAMPLE 7.13

In part (a) of the preceding example, the motor delivers power to lift the car, and yet the car moves at constant speed. A student analyzing this situation notes that the kinetic energy of the car does not change because its speed does not change. This student then reasons that, according to the work–kinetic energy theorem, $W = \Delta K = 0$. Knowing that $\mathcal{P} = W/t$, the student concludes that the power delivered by the motor also must be zero. How would you explain this apparent paradox?

Solution The work–kinetic energy theorem tells us that the *net* force acting on the system multiplied by the displacement is equal to the change in the kinetic energy of the system. In our elevator case, the net force is indeed zero (that is, $T - Mg - f = 0$), and so $W = (\Sigma F_y)d = 0$. However, the power from the motor is calculated not from the *net* force but rather from the force exerted by the motor acting in the direction of motion, which in this case is T and not zero.

Optional Section

7.6 ENERGY AND THE AUTOMOBILE

Automobiles powered by gasoline engines are very inefficient machines. Even under ideal conditions, less than 15% of the chemical energy in the fuel is used to power the vehicle. The situation is much worse under stop-and-go driving conditions in a city. In this section, we use the concepts of energy, power, and friction to analyze automobile fuel consumption.

Many mechanisms contribute to energy loss in an automobile. About 67% of the energy available from the fuel is lost in the engine. This energy ends up in the atmosphere, partly via the exhaust system and partly via the cooling system. (As we shall see in Chapter 22, the great energy loss from the exhaust and cooling systems is required by a fundamental law of thermodynamics.) Approximately 10% of the available energy is lost to friction in the transmission, drive shaft, wheel and axle bearings, and differential. Friction in other moving parts dissipates approximately 6% of the energy, and 4% of the energy is used to operate fuel and oil pumps and such accessories as power steering and air conditioning. This leaves a mere 13% of the available energy to propel the automobile! This energy is used mainly to balance the energy loss due to flexing of the tires and the friction caused by the air, which is more commonly referred to as *air resistance.*

Let us examine the power required to provide a force in the forward direction that balances the combination of the two frictional forces. The coefficient of rolling friction μ between the tires and the road is about 0.016. For a 1 450-kg car, the weight is 14 200 N and the force of rolling friction has a magnitude of $\mu n = \mu mg = 227$ N. As the speed of the car increases, a small reduction in the normal force occurs as a result of a decrease in atmospheric pressure as air flows over the top of the car. (This phenomenon is discussed in Chapter 15.) This reduction in the normal force causes a slight reduction in the force of rolling friction f_r with increasing speed, as the data in Table 7.2 indicate.

Now let us consider the effect of the resistive force that results from the movement of air past the car. For large objects, the resistive force f_a associated with air friction is proportional to the square of the speed (in meters per second; see Section 6.4) and is given by Equation 6.6:

$$f_a = \tfrac{1}{2}D\rho A v^2$$

where D is the drag coefficient, ρ is the density of air, and A is the cross-sectional area of the moving object. We can use this expression to calculate the f_a values in Table 7.2, using $D = 0.50$, $\rho = 1.293\ \text{kg/m}^3$, and $A \approx 2\ \text{m}^2$.

The magnitude of the total frictional force f_t is the sum of the rolling frictional force and the air resistive force:

$$f_t = f_r + f_a$$

At low speeds, road friction is the predominant resistive force, but at high speeds air drag predominates, as shown in Table 7.2. Road friction can be decreased by a reduction in tire flexing (for example, by an increase in the air pressure slightly above recommended values) and by the use of radial tires. Air drag can be reduced through the use of a smaller cross-sectional area and by streamlining the car. Although driving a car with the windows open increases air drag and thus results in a 3% decrease in mileage, driving with the windows closed and the air conditioner running results in a 12% decrease in mileage.

TABLE 7.2 Frictional Forces and Power Requirements for a Typical Car[a]

v (m/s)	n (N)	f_r (N)	f_a (N)	f_t (N)	$\mathcal{P} = f_t v$ (kW)
0	14 200	227	0	227	0
8.9	14 100	226	51	277	2.5
17.8	13 900	222	204	426	7.6
26.8	13 600	218	465	683	18.3
35.9	13 200	211	830	1 041	37.3
44.8	12 600	202	1 293	1 495	67.0

[a] In this table, n is the normal force, f_r is road friction, f_a is air friction, f_t is total friction, and $\mathcal{P}$ is the power delivered to the wheels.

The total power needed to maintain a constant speed v is $f_t v$, and it is this power that must be delivered to the wheels. For example, from Table 7.2 we see that at $v = 26.8$ m/s (60 mi/h) the required power is

$$\mathcal{P} = f_t v = (683\ \text{N})\left(26.8\frac{\text{m}}{\text{s}}\right) = 18.3\ \text{kW}$$

This power can be broken down into two parts: (1) the power $f_r v$ needed to compensate for road friction, and (2) the power $f_a v$ needed to compensate for air drag. At $v = 26.8$ m/s, we obtain the values

$$\mathcal{P}_r = f_r v = (218\ \text{N})\left(26.8\frac{\text{m}}{\text{s}}\right) = 5.84\ \text{kW}$$

$$\mathcal{P}_a = f_a v = (465\ \text{N})\left(26.8\frac{\text{m}}{\text{s}}\right) = 12.5\ \text{kW}$$

Note that $\mathcal{P} = \mathcal{P}_r + \mathcal{P}_a$.

On the other hand, at $v = 44.8$ m/s (100 mi/h), $\mathcal{P}_r = 9.05$ kW, $\mathcal{P}_a = 57.9$ kW, and $\mathcal{P} = 67.0$ kW. This shows the importance of air drag at high speeds.

EXAMPLE 7.14 Gas Consumed by a Compact Car

A compact car has a mass of 800 kg, and its efficiency is rated at 18%. (That is, 18% of the available fuel energy is delivered to the wheels.) Find the amount of gasoline used to accelerate the car from rest to 27 m/s (60 mi/h). Use the fact that the energy equivalent of 1 gal of gasoline is 1.3×10^8 J.

Solution The energy required to accelerate the car from rest to a speed v is its final kinetic energy $\frac{1}{2}mv^2$:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(800\ \text{kg})(27\ \text{m/s})^2 = 2.9 \times 10^5\ \text{J}$$

If the engine were 100% efficient, each gallon of gasoline would supply 1.3×10^8 J of energy. Because the engine is only 18% efficient, each gallon delivers only $(0.18)(1.3 \times 10^8\ \text{J}) = 2.3 \times 10^7$ J. Hence, the number of gallons used to accelerate the car is

$$\text{Number of gallons} = \frac{2.9 \times 10^5\ \text{J}}{2.3 \times 10^7\ \text{J/gal}} = 0.013\ \text{gal}$$

At cruising speed, this much gasoline is sufficient to propel the car nearly 0.5 mi. This demonstrates the extreme energy requirements of stop-and-start driving.

EXAMPLE 7.15 Power Delivered to Wheels

Suppose the compact car in Example 7.14 gets 35 mi/gal at 60 mi/h. How much power is delivered to the wheels?

Solution By simply canceling units, we determine that the car consumes 60 mi/h ÷ 35 mi/gal = 1.7 gal/h. Using the fact that each gallon is equivalent to 1.3×10^8 J, we find that the total power used is

$$\mathcal{P} = \frac{(1.7\ \text{gal/h})(1.3 \times 10^8\ \text{J/gal})}{3.6 \times 10^3\ \text{s/h}} = \frac{2.2 \times 10^8\ \text{J}}{3.6 \times 10^3\ \text{s}} = 62\ \text{kW}$$

Because 18% of the available power is used to propel the car, the power delivered to the wheels is (0.18)(62 kW) = 11 kW. This is 40% less than the 18.3-kW value obtained for the 1 450-kg car discussed in the text. Vehicle mass is clearly an important factor in power-loss mechanisms.

EXAMPLE 7.16 Car Accelerating Up a Hill

Consider a car of mass m that is accelerating up a hill, as shown in Figure 7.19. An automotive engineer has measured the magnitude of the total resistive force to be

$$f_t = (218 + 0.70v^2)\ \text{N}$$

where v is the speed in meters per second. Determine the power the engine must deliver to the wheels as a function of speed.

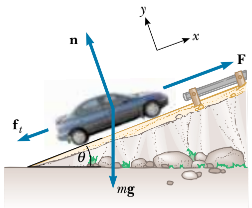


Figure 7.19

Solution The forces on the car are shown in Figure 7.19, in which **F** is the force of friction from the road that propels the car; the remaining forces have their usual meaning. Applying Newton's second law to the motion along the road surface, we find that

$$\sum F_x = F - f_t - mg\sin\theta = ma$$

$$F = ma + mg\sin\theta + f_t$$

$$= ma + mg\sin\theta + (218 + 0.70v^2)$$

Therefore, the power required to move the car forward is

$$\mathcal{P} = Fv = mva + mvg\sin\theta + 218v + 0.70v^3$$

The term mva represents the power that the engine must deliver to accelerate the car. If the car moves at constant speed, this term is zero and the total power requirement is reduced. The term $mvg\sin\theta$ is the power required to provide a force to balance a component of the force of gravity as the car moves up the incline. This term would be zero for motion on a horizontal surface. The term $218v$ is the power required to provide a force to balance road friction, and the term $0.70v^3$ is the power needed to do work on the air.

If we take $m = 1\,450$ kg, $v = 27$ m/s $(=60$ mi/h$)$, $a = 1.0\ \text{m/s}^2$, and $\theta = 10°$, then the various terms in $\mathcal{P}$ are calculated to be

$$mva = (1\,450\ \text{kg})(27\ \text{m/s})(1.0\ \text{m/s}^2)$$
$$= 39\ \text{kW} = 52\ \text{hp}$$

$$mvg\sin\theta = (1\,450\ \text{kg})(27\ \text{m/s})(9.80\ \text{m/s}^2)(\sin 10°)$$
$$= 67\ \text{kW} = 89\ \text{hp}$$

$$218v = 218(27\ \text{m/s}) = 5.9\ \text{kW} = 7.9\ \text{hp}$$

$$0.70v^3 = 0.70(27\ \text{m/s})^3 = 14\ \text{kW} = 19\ \text{hp}$$

Hence, the total power required is 126 kW, or 168 hp.

Note that the power requirements for traveling at constant speed on a horizontal surface are only 20 kW, or 27 hp (the sum of the last two terms). Furthermore, if the mass were halved (as in the case of a compact car), then the power required also is reduced by almost the same factor.

Optional Section

7.7 KINETIC ENERGY AT HIGH SPEEDS

The laws of Newtonian mechanics are valid only for describing the motion of particles moving at speeds that are small compared with the speed of light in a vacuum c $(=3.00 \times 10^8\ \text{m/s})$. When speeds are comparable to c, the equations of Newtonian mechanics must be replaced by the more general equations predicted by the theory of relativity. One consequence of the theory of relativity is that the kinetic energy of a particle of mass m moving with a speed v is no longer given by $K = mv^2/2$. Instead, one must use the relativistic form of the kinetic energy:

Relativistic kinetic energy

$$K = mc^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) \quad \textbf{(7.19)}$$

According to this expression, speeds greater than c are not allowed because, as v approaches c, K approaches ∞. This limitation is consistent with experimental ob-

servations on subatomic particles, which have shown that no particles travel at speeds greater than c. (In other words, c is the ultimate speed.) From this relativistic point of view, the work–kinetic energy theorem says that v can only approach c because it would take an infinite amount of work to attain the speed $v = c$.

All formulas in the theory of relativity must reduce to those in Newtonian mechanics at low particle speeds. It is instructive to show that this is the case for the kinetic energy relationship by analyzing Equation 7.19 when v is small compared with c. In this case, we expect K to reduce to the Newtonian expression. We can check this by using the binomial expansion (Appendix B.5) applied to the quantity $[1 - (v/c)^2]^{-1/2}$, with $v/c \ll 1$. If we let $x = (v/c)^2$, the expansion gives

$$\frac{1}{(1-x)^{1/2}} = 1 + \frac{x}{2} + \frac{3}{8}x^2 + \cdots$$

Making use of this expansion in Equation 7.19 gives

$$K = mc^2\left(1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots - 1\right)$$

$$= \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$

$$= \frac{1}{2}mv^2 \quad \text{for} \quad \frac{v}{c} \ll 1$$

Thus, we see that the relativistic kinetic energy expression does indeed reduce to the Newtonian expression for speeds that are small compared with c. We shall return to the subject of relativity in Chapter 39.

SUMMARY

The work done by a constant force **F** acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. Given a force **F** that makes an angle θ with the displacement vector **d** of a particle acted on by the force, you should be able to determine the work done by **F** using the equation

$$W \equiv Fd\cos\theta \quad \textbf{(7.1)}$$

The **scalar product** (dot product) of two vectors **A** and **B** is defined by the relationship

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \quad \textbf{(7.3)}$$

where the result is a scalar quantity and θ is the angle between the two vectors. The scalar product obeys the commutative and distributive laws.

If a varying force does work on a particle as the particle moves along the x axis from x_i to x_f, you must use the expression

$$W \equiv \int_{x_i}^{x_f} F_x\,dx \quad \textbf{(7.7)}$$

where F_x is the component of force in the x direction. If several forces are acting on the particle, the net work done by all of the forces is the sum of the amounts of work done by all of the forces.

The **kinetic energy** of a particle of mass m moving with a speed v (where v is small compared with the speed of light) is

$$K \equiv \tfrac{1}{2}mv^2 \tag{7.14}$$

The **work–kinetic energy theorem** states that the net work done on a particle by external forces equals the change in kinetic energy of the particle:

$$\sum W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \tag{7.16}$$

If a frictional force acts, then the work–kinetic energy theorem can be modified to give

$$K_i + \sum W_{\text{other}} - f_k d = K_f \tag{7.17b}$$

The **instantaneous power** $\mathcal{P}$ is defined as the time rate of energy transfer. If an agent applies a force **F** to an object moving with a velocity **v**, the power delivered by that agent is

$$\mathcal{P} \equiv \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v} \tag{7.18}$$

QUESTIONS

1. Consider a tug-of-war in which two teams pulling on a rope are evenly matched so that no motion takes place. Assume that the rope does not stretch. Is work done on the rope? On the pullers? On the ground? Is work done on anything?
2. For what values of θ is the scalar product (a) positive and (b) negative?
3. As the load on a spring hung vertically is increased, one would not expect the F_s-versus-x curve to always remain linear, as shown in Figure 7.10d. Explain qualitatively what you would expect for this curve as m is increased.
4. Can the kinetic energy of an object be negative? Explain.
5. (a) If the speed of a particle is doubled, what happens to its kinetic energy? (b) If the net work done on a particle is zero, what can be said about the speed?
6. In Example 7.16, does the required power increase or decrease as the force of friction is reduced?
7. An automobile sales representative claims that a "souped-up" 300-hp engine is a necessary option in a compact car (instead of a conventional 130-hp engine). Suppose you intend to drive the car within speed limits (≤ 55 mi/h) and on flat terrain. How would you counter this sales pitch?
8. One bullet has twice the mass of another bullet. If both bullets are fired so that they have the same speed, which has the greater kinetic energy? What is the ratio of the kinetic energies of the two bullets?
9. When a punter kicks a football, is he doing any work on the ball while his toe is in contact with it? Is he doing any work on the ball after it loses contact with his toe? Are any forces doing work on the ball while it is in flight?
10. Discuss the work done by a pitcher throwing a baseball. What is the approximate distance through which the force acts as the ball is thrown?
11. Two sharpshooters fire 0.30-caliber rifles using identical shells. The barrel of rifle A is 2.00 cm longer than that of rifle B. Which rifle will have the higher muzzle speed? (*Hint:* The force of the expanding gases in the barrel accelerates the bullets.)
12. As a simple pendulum swings back and forth, the forces acting on the suspended mass are the force of gravity, the tension in the supporting cord, and air resistance. (a) Which of these forces, if any, does no work on the pendulum? (b) Which of these forces does negative work at all times during its motion? (c) Describe the work done by the force of gravity while the pendulum is swinging.
13. The kinetic energy of an object depends on the frame of reference in which its motion is measured. Give an example to illustrate this point.
14. An older model car accelerates from 0 to a speed v in 10 s. A newer, more powerful sports car accelerates from 0 to $2v$ in the same time period. What is the ratio of powers expended by the two cars? Consider the energy coming from the engines to appear only as kinetic energy of the cars.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 7.1 Work Done by a Constant Force

1. A tugboat exerts a constant force of 5 000 N on a ship moving at constant speed through a harbor. How much work does the tugboat do on the ship in a distance of 3.00 km?
2. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper as she moves down an aisle 50.0 m in length.
3. A raindrop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at constant speed under the influence of gravity and air resistance. After the drop has fallen 100 m, what is the work done (a) by gravity and (b) by air resistance?
4. A sledge loaded with bricks has a total mass of 18.0 kg and is pulled at constant speed by a rope. The rope is inclined at 20.0° above the horizontal, and the sledge moves a distance of 20.0 m on a horizontal surface. The coefficient of kinetic friction between the sledge and the surface is 0.500. (a) What is the tension of the rope? (b) How much work is done on the sledge by the rope? (c) What is the energy lost due to friction?
5. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done by (a) the applied force, (b) the normal force exerted by the table, and (c) the force of gravity. (d) Determine the total work done on the block.
6. A 15.0-kg block is dragged over a rough, horizontal surface by a 70.0-N force acting at 20.0° above the horizontal. The block is displaced 5.00 m, and the coefficient of kinetic friction is 0.300. Find the work done by (a) the 70-N force, (b) the normal force, and (c) the force of gravity. (d) What is the energy loss due to friction? (e) Find the total change in the block's kinetic energy.

WEB 7. Batman, whose mass is 80.0 kg, is holding onto the free end of a 12.0-m rope, the other end of which is fixed to a tree limb above. He is able to get the rope in motion as only Batman knows how, eventually getting it to swing enough so that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done against the force of gravity in this maneuver?

Section 7.2 The Scalar Product of Two Vectors

In Problems 8 to 14, calculate all numerical answers to three significant figures.

8. Vector **A** has a magnitude of 5.00 units, and vector **B** has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find **A · B**.
9. Vector **A** extends from the origin to a point having polar coordinates (7, 70°), and vector **B** extends from the origin to a point having polar coordinates (4, 130°). Find **A · B**.
10. Given two arbitrary vectors **A** and **B**, show that $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$. (*Hint:* Write **A** and **B** in unit vector form and use Equations 7.4 and 7.5.)

WEB 11. A force $\mathbf{F} = (6\mathbf{i} - 2\mathbf{j})$ N acts on a particle that undergoes a displacement $\mathbf{d} = (3\mathbf{i} + \mathbf{j})$m. Find (a) the work done by the force on the particle and (b) the angle between **F** and **d**.

12. For $\mathbf{A} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{B} = -\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, and $\mathbf{C} = 2\mathbf{j} - 3\mathbf{k}$, find $\mathbf{C}\cdot(\mathbf{A} - \mathbf{B})$.
13. Using the definition of the scalar product, find the angles between (a) $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = 4\mathbf{i} - 4\mathbf{j}$; (b) $\mathbf{A} = -2\mathbf{i} + 4\mathbf{j}$ and $\mathbf{B} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$; (c) $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 3\mathbf{j} + 4\mathbf{k}$.
14. Find the scalar product of the vectors in Figure P7.14.

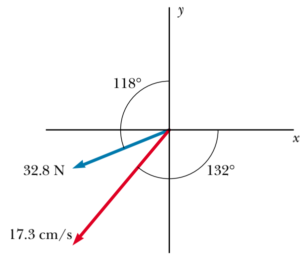


Figure P7.14

Section 7.3 Work Done by a Varying Force

15. The force acting on a particle varies as shown in Figure P7.15. Find the work done by the force as the particle moves (a) from $x = 0$ to $x = 8.00$ m, (b) from $x = 8.00$ m to $x = 10.0$ m, and (c) from $x = 0$ to $x = 10.0$ m.
16. The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3.00$ m. (b) From your graph, find the net work done by this force as the particle moves from $x = 0$ to $x = 3.00$ m.

WEB 17. A particle is subject to a force F_x that varies with position as in Figure P7.17. Find the work done by the force on the body as it moves (a) from $x = 0$ to $x = 5.00$ m,

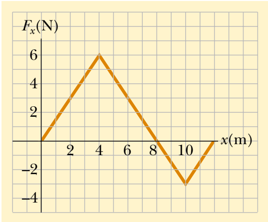


Figure P7.15

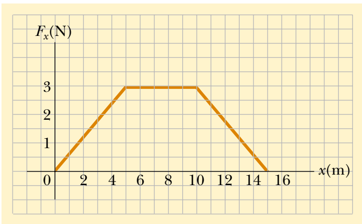


Figure P7.17 Problems 17 and 32.

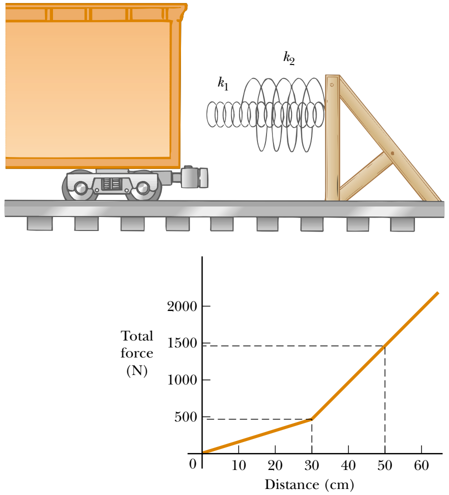


Figure P7.21

(b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

18. A force $\mathbf{F} = (4x\mathbf{i} + 3y\mathbf{j})$ N acts on an object as it moves in the x direction from the origin to $x = 5.00$ m. Find the work $W = \int \mathbf{F} \cdot d\mathbf{r}$ done on the object by the force.

19. When a 4.00-kg mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg mass is removed, (a) how far will the spring stretch if a 1.50-kg mass is hung on it and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

20. An archer pulls her bow string back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work is done by the archer in pulling the bow?

21. A 6 000-kg freight car rolls along rails with negligible friction. The car is brought to rest by a combination of two coiled springs, as illustrated in Figure P7.21. Both springs obey Hooke's law with $k_1 = 1\,600$ N/m and $k_2 = 3\,400$ N/m. After the first spring compresses a distance of 30.0 cm, the second spring (acting with the first) increases the force so that additional compression occurs, as shown in the graph. If the car is brought to rest 50.0 cm after first contacting the two-spring system, find the car's initial speed.

22. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Assuming the origin is placed where the bullet begins to move, the force (in newtons) exerted on the bullet by the expanding gas is $15\,000 + 10\,000x - 25\,000x^2$, where x is in meters. (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) If the barrel is 1.00 m long, how much work is done and how does this value compare with the work calculated in part (a)?

23. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

24. If it takes work W to stretch a Hooke's-law spring a distance d from its unstressed length, determine the extra work required to stretch it an additional distance d.

25. A small mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder, as illustrated in Figure P7.25. (a) If the mass moves at a constant speed, show that $F = mg\cos\theta$. (*Hint:* If the mass moves at a constant speed, the component of its acceleration tangent to the cylinder must be zero at all times.) (b) By directly integrating $W = \int \mathbf{F} \cdot d\mathbf{s}$, find the work done in moving the mass at constant speed from the bottom to the top of the half-cylinder. Here $d\mathbf{s}$ represents an incremental displacement of the small mass.

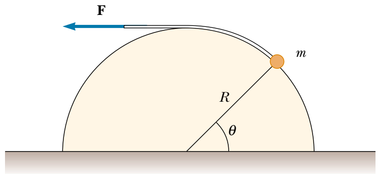


Figure P7.25

26. Express the unit of the force constant of a spring in terms of the basic units meter, kilogram, and second.

Section 7.4 Kinetic Energy and the Work–Kinetic Energy Theorem

27. A 0.600-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B. What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?

28. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) If its speed were doubled, what would be its kinetic energy?

29. A 3.00-kg mass has an initial velocity $\mathbf{v}_i = (6.00\mathbf{i} - 2.00\mathbf{j})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8.00\mathbf{i} + 4.00\mathbf{j})$ m/s. (*Hint:* Remember that $v^2 = \mathbf{v} \cdot \mathbf{v}$.)

30. A mechanic pushes a 2 500-kg car, moving it from rest and making it accelerate from rest to a speed v. He does 5 000 J of work in the process. During this time, the car moves 25.0 m. If friction between the car and the road is negligible, (a) what is the final speed v of the car? (b) What constant horizontal force did he exert on the car?

31. A mechanic pushes a car of mass m, doing work W in making it accelerate from rest. If friction between the car and the road is negligible, (a) what is the final speed of the car? During the time the mechanic pushes the car, the car moves a distance d. (b) What constant horizontal force did the mechanic exert on the car?

32. A 4.00-kg particle is subject to a total force that varies with position, as shown in Figure P7.17. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m, (b) $x = 10.0$ m, (c) $x = 15.0$ m?

33. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between the box and the floor is 0.300, find (a) the work done by the applied force, (b) the energy loss due to friction, (c) the work done by the normal force, (d) the work done by gravity, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

34. You can think of the work–kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem, work out parts (a) and (b) separately from parts (c) and (d) to compare the predictions of the two theories. In a rifle barrel, a 15.0-g bullet is accelerated from rest to a speed of 780 m/s. (a) Find the work that is done on the bullet. (b) If the rifle barrel is 72.0 cm long, find the magnitude of the average total force that acted on it, as $F = W/(d\cos\theta)$. (c) Find the constant acceleration of a bullet that starts from rest and gains a speed of 780 m/s over a distance of 72.0 cm. (d) Find the total force that acted on it as $\Sigma F = ma$.

35. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by gravity? (b) How much energy is lost because of friction? (c) How much work is done by the 100-N force? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after it has been pulled 5.00 m?

36. A block of mass 12.0 kg slides from rest down a frictionless 35.0° incline and is stopped by a strong spring with $k = 3.00 \times 10^4$ N/m. The block slides 3.00 m from the point of release to the point where it comes to rest against the spring. When the block comes to rest, how far has the spring been compressed?

WEB **37.** A sled of mass m is given a kick on a frozen pond. The kick imparts to it an initial speed $v_i = 2.00$ m/s. The coefficient of kinetic friction between the sled and the ice is $\mu_k = 0.100$. Utilizing energy considerations, find the distance the sled moves before it stops.

38. A picture tube in a certain television set is 36.0 cm long. The electrical force accelerates an electron in the tube from rest to 1.00% of the speed of light over this distance. Determine (a) the kinetic energy of the electron as it strikes the screen at the end of the tube, (b) the magnitude of the average electrical force acting on the electron over this distance, (c) the magnitude of the average acceleration of the electron over this distance, and (d) the time of flight.

39. A bullet with a mass of 5.00 g and a speed of 600 m/s penetrates a tree to a depth of 4.00 cm. (a) Use work and energy considerations to find the average frictional force that stops the bullet. (b) Assuming that the frictional force is constant, determine how much time elapsed between the moment the bullet entered the tree and the moment it stopped.

40. An Atwood's machine (see Fig. 5.15) supports masses of 0.200 kg and 0.300 kg. The masses are held at rest beside each other and then released. Neglecting friction, what is the speed of each mass the instant it has moved 0.400 m?

41. A 2.00-kg block is attached to a spring of force constant 500 N/m, as shown in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and is then released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between the block and the surface is 0.350.

Section 7.5 Power

42. Make an order-of-magnitude estimate of the power a car engine contributes to speeding up the car to highway speed. For concreteness, consider your own car (if you use one). In your solution, state the physical quantities you take as data and the values you measure or estimate for them. The mass of the vehicle is given in the owner's manual. If you do not wish to consider a car, think about a bus or truck for which you specify the necessary physical quantities.

WEB 43. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

44. If a certain horse can maintain 1.00 hp of output for 2.00 h, how many 70.0-kg bundles of shingles can the horse hoist (using some pulley arrangement) to the roof of a house 8.00 m tall, assuming 70.0% efficiency?

45. A certain automobile engine delivers 2.24×10^4 W (30.0 hp) to its wheels when moving at a constant speed of 27.0 m/s ($\approx$ 60 mi/h). What is the resistive force acting on the automobile at that speed?

46. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required for him to be pulled a distance of 60.0 m up a 30.0° slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

47. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with its power when it moves at its cruising speed?

48. An energy-efficient lightbulb, taking in 28.0 W of power, can produce the same level of brightness as a conventional bulb operating at 100-W power. The lifetime of the energy-efficient bulb is 10 000 h and its purchase price is $17.0, whereas the conventional bulb has a lifetime of 750 h and costs $0.420 per bulb. Determine the total savings obtained through the use of one energy-efficient bulb over its lifetime as opposed to the use of conventional bulbs over the same time period. Assume an energy cost of $0.080 0 per kilowatt hour.

(Optional)
Section 7.6 Energy and the Automobile

49. A compact car of mass 900 kg has an overall motor efficiency of 15.0%. (That is, 15.0% of the energy supplied by the fuel is delivered to the wheels of the car.) (a) If burning 1 gal of gasoline supplies 1.34×10^8 J of energy, find the amount of gasoline used by the car in accelerating from rest to 55.0 mi/h. Here you may ignore the effects of air resistance and rolling resistance. (b) How many such accelerations will 1 gal provide? (c) The mileage claimed for the car is 38.0 mi/gal at 55 mi/h. What power is delivered to the wheels (to overcome frictional effects) when the car is driven at this speed?

50. Suppose the empty car described in Table 7.2 has a fuel economy of 6.40 km/L (15 mi/gal) when traveling at 26.8 m/s (60 mi/h). Assuming constant efficiency, determine the fuel economy of the car if the total mass of the passengers and the driver is 350 kg.

51. When an air conditioner is added to the car described in Problem 50, the additional output power required to operate the air conditioner is 1.54 kW. If the fuel economy of the car is 6.40 km/L without the air conditioner, what is it when the air conditioner is operating?

(Optional)
Section 7.7 Kinetic Energy at High Speeds

52. An electron moves with a speed of $0.995c$. (a) What is its kinetic energy? (b) If you use the classical expression to calculate its kinetic energy, what percentage error results?

53. A proton in a high-energy accelerator moves with a speed of $c/2$. Using the work–kinetic energy theorem, find the work required to increase its speed to (a) $0.750c$ and (b) $0.995c$.

54. Find the kinetic energy of a 78.0-kg spacecraft launched out of the Solar System with a speed of 106 km/s using (a) the classical equation $K = \frac{1}{2}mv^2$ and (b) the relativistic equation.

ADDITIONAL PROBLEMS

55. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0°. What is the kinetic energy of the baseball at the highest point of the trajectory?

56. While running, a person dissipates about 0.600 J of mechanical energy per step per kilogram of body mass. If a 60.0-kg runner dissipates a power of 70.0 W during a race, how fast is the person running? Assume a running step is 1.50 m in length.

57. A particle of mass m moves with a constant acceleration $\mathbf{a}$. If the initial position vector and velocity of the particle are $\mathbf{r}_i$ and $\mathbf{v}_i$, respectively, use energy arguments to show that its speed v_f at any time satisfies the equation

$$v_f^2 = v_i^2 + 2\mathbf{a} \cdot (\mathbf{r}_f - \mathbf{r}_i)$$

where $\mathbf{r}_f$ is the position vector of the particle at that same time.

58. The direction of an arbitrary vector $\mathbf{A}$ can be completely specified with the angles α, β, and γ that the vector makes with the x, y, and z axes, respectively. If $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$, (a) find expressions for $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ (known as *direction cosines*) and (b) show that these angles satisfy the relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. (*Hint:* Take the scalar product of $\mathbf{A}$ with $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ separately.)

59. A 4.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2.00$ s.

60. A traveler at an airport takes an escalator up one floor (Fig. P7.60). The moving staircase would itself carry him upward with vertical velocity component v between entry and exit points separated by height h. However, while the escalator is moving, the hurried traveler climbs the steps of the escalator at a rate of n steps/s. Assume that the height of each step is h_s. (a) Determine the amount of work done by the traveler during his escalator ride, given that his mass is m. (b) Determine the work the escalator motor does on this person.

Figure P7.60 (©Ron Chapple/FPG)

61. When a certain spring is stretched beyond its proportional limit, the restoring force satisfies the equation $F = -kx + \beta x^3$. If $k = 10.0$ N/m and $\beta = 100$ N/m^3, calculate the work done by this force when the spring is stretched 0.100 m.

62. In a control system, an accelerometer consists of a 4.70-g mass sliding on a low-friction horizontal rail. A low-mass spring attaches the mass to a flange at one end of the rail. When subject to a steady acceleration of $0.800g$, the mass is to assume a location 0.500 cm away from its equilibrium position. Find the stiffness constant required for the spring.

63. A 2 100-kg pile driver is used to drive a steel I-beam into the ground. The pile driver falls 5.00 m before coming into contact with the beam, and it drives the beam 12.0 cm into the ground before coming to rest. Using energy considerations, calculate the average force the beam exerts on the pile driver while the pile driver is brought to rest.

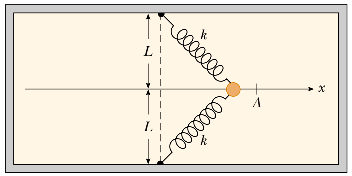
64. A cyclist and her bicycle have a combined mass of 75.0 kg. She coasts down a road inclined at 2.00° with the horizontal at 4.00 m/s and down a road inclined at 4.00° at 8.00 m/s. She then holds on to a moving vehicle and coasts on a level road. What power must the vehicle expend to maintain her speed at 3.00 m/s? Assume that the force of air resistance is proportional to her speed and that other frictional forces remain constant. (*Warning:* You must *not* attempt this dangerous maneuver.)

65. A single constant force $\mathbf{F}$ acts on a particle of mass m. The particle starts at rest at $t = 0$. (a) Show that the instantaneous power delivered by the force at any time t is $(F^2/m)t$. (b) If $F = 20.0$ N and $m = 5.00$ kg, what is the power delivered at $t = 3.00$ s?

66. A particle is attached between two identical springs on a horizontal frictionless table. Both springs have spring constant k and are initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P7.66, show that the force exerted on the particle by the springs is

$$\mathbf{F} = -2kx\left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)\mathbf{i}$$

(b) Determine the amount of work done by this force in moving the particle from $x = A$ to $x = 0$.



Top view

Figure P7.66

67. Review Problem. Two constant forces act on a 5.00-kg object moving in the *xy* plane, as shown in Figure P7.67. Force $\mathbf{F}_1$ is 25.0 N at 35.0°, while $\mathbf{F}_2$ = 42.0 N at 150°. At time $t = 0$, the object is at the origin and has velocity $(4.0\mathbf{i} + 2.5\mathbf{j})$ m/s. (a) Express the two forces in unit–vector notation. Use unit–vector notation for your other answers. (b) Find the total force on the object. (c) Find the object's acceleration. Now, considering the instant $t = 3.00$ s, (d) find the object's velocity, (e) its location, (f) its kinetic energy from $\frac{1}{2}mv_f^2$, and (g) its kinetic energy from $\frac{1}{2}mv_i^2 + \Sigma\mathbf{F}\cdot\mathbf{d}$.

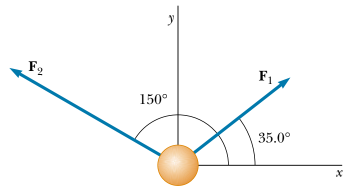


Figure P7.67

68. When different weights are hung on a spring, the spring stretches to different lengths as shown in the following table. (a) Make a graph of the applied force versus the extension of the spring. By least-squares fitting, determine the straight line that best fits the data. (You may not want to use all the data points.) (b) From the slope of the best-fit line, find the spring constant k. (c) If the spring is extended to 105 mm, what force does it exert on the suspended weight?

F (N)	2.0	4.0	6.0	8.0	10	12	14	16	18
L (mm)	15	32	49	64	79	98	112	126	149

69. A 200-g block is pressed against a spring of force constant 1.40 kN/m until the block compresses the spring 10.0 cm. The spring rests at the bottom of a ramp inclined at 60.0° to the horizontal. Using energy considerations, determine how far up the incline the block moves before it stops (a) if there is no friction between the block and the ramp and (b) if the coefficient of kinetic friction is 0.400.

70. A 0.400-kg particle slides around a horizontal track. The track has a smooth, vertical outer wall forming a circle with a radius of 1.50 m. The particle is given an initial speed of 8.00 m/s. After one revolution, its speed has dropped to 6.00 m/s because of friction with the rough floor of the track. (a) Find the energy loss due to friction in one revolution. (b) Calculate the coefficient of kinetic friction. (c) What is the total number of revolutions the particle makes before stopping?

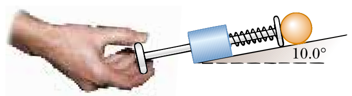


Figure P7.71

WEB **71.** The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm (Fig. P7.71). The surface on which the ball moves is inclined 10.0° with respect to the horizontal. If the spring is initially compressed 5.00 cm, find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.

72. In diatomic molecules, the constituent atoms exert attractive forces on each other at great distances and repulsive forces at short distances. For many molecules, the Lennard–Jones law is a good approximation to the magnitude of these forces:

$$F = F_0\left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7}\right]$$

where r is the center-to-center distance between the atoms in the molecule, σ is a length parameter, and F_0 is the force when $r = \sigma$. For an oxygen molecule, $F_0 = 9.60 \times 10^{-11}$ N and $\sigma = 3.50 \times 10^{-10}$ m. Determine the work done by this force if the atoms are pulled apart from $r = 4.00 \times 10^{-10}$ m to $r = 9.00 \times 10^{-10}$ m.

73. A horizontal string is attached to a 0.250-kg mass lying on a rough, horizontal table. The string passes over a light, frictionless pulley, and a 0.400-kg mass is then attached to its free end. The coefficient of sliding friction between the 0.250-kg mass and the table is 0.200. Using the work–kinetic energy theorem, determine (a) the speed of the masses after each has moved 20.0 m from rest and (b) the mass that must be added to the 0.250-kg mass so that, given an initial velocity, the masses continue to move at a constant speed. (c) What mass must be removed from the 0.400-kg mass so that the same outcome as in part (b) is achieved?

74. Suppose a car is modeled as a cylinder moving with a speed v, as in Figure P7.74. In a time Δt, a column of air of mass Δm must be moved a distance $v\,\Delta t$ and, hence, must be given a kinetic energy $\frac{1}{2}(\Delta m)v^2$. Using this model, show that the power loss due to air resistance is $\frac{1}{2}\rho Av^3$ and that the resistive force is $\frac{1}{2}\rho Av^2$, where ρ is the density of air.

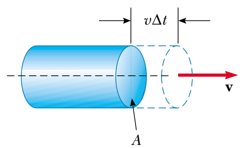


Figure P7.74

75. A particle moves along the x axis from x = 12.8 m to x = 23.7 m under the influence of a force

$$F = \frac{375}{x^3 + 3.75\,x}$$

where F is in newtons and x is in meters. Using numerical integration, determine the total work done by this force during this displacement. Your result should be accurate to within 2%.

76. More than 2 300 years ago the Greek teacher Aristotle wrote the first book called *Physics*. The following passage, rephrased with more precise terminology, is from the end of the book's Section Eta:

> Let $\mathcal{P}$ be the power of an agent causing motion; w, the thing moved; d, the distance covered; and t, the time taken. Then (1) a power equal to $\mathcal{P}$ will in a period of time equal to t move $w/2$ a distance $2d$; or (2) it will move $w/2$ the given distance d in time $t/2$. Also, if (3) the given power $\mathcal{P}$ moves the given object w a distance $d/2$ in time $t/2$, then (4) $\mathcal{P}/2$ will move $w/2$ the given distance d in the given time t.

(a) Show that Aristotle's proportions are included in the equation $\mathcal{P}t = bwd$, where b is a proportionality constant. (b) Show that our theory of motion includes this part of Aristotle's theory as one special case. In particular, describe a situation in which it is true, derive the equation representing Aristotle's proportions, and determine the proportionality constant.

ANSWERS TO QUICK QUIZZES

7.1 No. The force does no work on the object because the force is pointed toward the center of the circle and is therefore perpendicular to the motion.

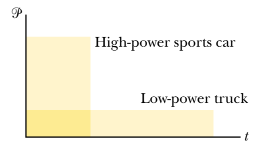
7.2 (a) Assuming the person lifts with a force of magnitude mg, the weight of the box, the work he does during the vertical displacement is mgh because the force is in the direction of the displacement. The work he does during the horizontal displacement is zero because now the force he exerts on the box is perpendicular to the displacement. The net work he does is $mgh + 0 = mgh$. (b) The work done by the gravitational force on the box as the box is displaced vertically is $-mgh$ because the direction of this force is opposite the direction of the displacement. The work done by the gravitational force is zero during the horizontal displacement because now the direction of this force is perpendicular to the direction of the displacement. The net work done by the gravitational force $-mgh + 0 = -mgh$. The total work done on the box is $+mgh - mgh = 0$.

7.3 No. For example, consider the two vectors $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j}$. Their dot product is $\mathbf{A}\cdot\mathbf{B} = 8$, yet both vectors have negative y components.

7.4 Force divided by displacement, which in SI units is newtons per meter (N/m).

7.5 Yes, whenever the frictional force has a component along the direction of motion. Consider a crate sitting on the bed of a truck as the truck accelerates to the east. The static friction force exerted on the crate by the truck acts to the east to give the crate the same acceleration as the truck (assuming that the crate does not slip). Because the crate accelerates, its kinetic energy must increase.

7.6 Because the two vehicles perform the same amount of work, the areas under the two graphs are equal. However, the graph for the low-power truck extends over a longer time interval and does not extend as high on the $\mathcal{P}$ axis as the graph for the sports car does.



PUZZLER

A common scene at a carnival is the Ring-the-Bell attraction, in which the player swings a heavy hammer downward in an attempt to project a mass upward to ring a bell. What is the best strategy to win the game and impress your friends? *(Robert E. Daemmrich/Tony Stone Images)*

chapter

8 Potential Energy and Conservation of Energy

Chapter Outline

In Chapter 7 we introduced the concept of kinetic energy, which is the energy associated with the motion of an object. In this chapter we introduce another form of energy—*potential energy,* which is the energy associated with the arrangement of a system of objects that exert forces on each other. Potential energy can be thought of as stored energy that can either do work or be converted to kinetic energy.

The potential energy concept can be used only when dealing with a special class of forces called *conservative forces.* When only conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy. This balancing of the two forms of energy is known as the *principle of conservation of mechanical energy.*

Energy is present in the Universe in various forms, including mechanical, electromagnetic, chemical, and nuclear. Furthermore, one form of energy can be converted to another. For example, when an electric motor is connected to a battery, the chemical energy in the battery is converted to electrical energy in the motor, which in turn is converted to mechanical energy as the motor turns some device. The transformation of energy from one form to another is an essential part of the study of physics, engineering, chemistry, biology, geology, and astronomy.

When energy is changed from one form to another, the total amount present does not change. Conservation of energy means that although the form of energy may change, if an object (or system) loses energy, that same amount of energy appears in another object or in the object's surroundings.

8.1 POTENTIAL ENERGY

5.3 An object that possesses kinetic energy can do work on another object—for example, a moving hammer driving a nail into a wall. Now we shall introduce another form of energy. This energy, called **potential energy U,** is the energy associated with a system of objects.

Before we describe specific forms of potential energy, we must first define a *system,* which consists of two or more objects that exert forces on one another. **If the arrangement of the system changes, then the potential energy of the system changes.** If the system consists of only two particle-like objects that exert forces on each other, then the work done by the force acting on one of the objects causes a transformation of energy between the object's kinetic energy and other forms of the system's energy.

Gravitational Potential Energy

As an object falls toward the Earth, the Earth exerts a gravitational force $m\mathbf{g}$ on the object, with the direction of the force being the same as the direction of the object's motion. The gravitational force does work on the object and thereby increases the object's kinetic energy. Imagine that a brick is dropped from rest directly above a nail in a board lying on the ground. When the brick is released, it falls toward the ground, gaining speed and therefore gaining kinetic energy. The brick–Earth system has potential energy when the brick is at any distance above the ground (that is, it has the *potential* to do work), and this potential energy is converted to kinetic energy as the brick falls. The conversion from potential energy to kinetic energy occurs continuously over the entire fall. When the brick reaches the nail and the board lying on the ground, it does work on the nail, driving it into the board. What determines how much work the brick is able to do on the nail? It is easy to see that the heavier the brick, the farther in it drives the nail; also the higher the brick is before it is released, the more work it does when it strikes the nail.

The product of the magnitude of the gravitational force mg acting on an object and the height y of the object is so important in physics that we give it a name: the **gravitational potential energy.** The symbol for gravitational potential energy is U_g, and so the defining equation for gravitational potential energy is

Gravitational potential energy

$$U_g \equiv mgy \qquad (8.1)$$

Gravitational potential energy is the potential energy of the object–Earth system. This potential energy is transformed into kinetic energy of the system by the gravitational force. In this type of system, in which one of the members (the Earth) is much more massive than the other (the object), the massive object can be modeled as stationary, and the kinetic energy of the system can be represented entirely by the kinetic energy of the lighter object. Thus, the kinetic energy of the system is represented by that of the object falling toward the Earth. Also note that Equation 8.1 is valid only for objects near the surface of the Earth, where **g** is approximately constant.[1]

Let us now directly relate the work done on an object by the gravitational force to the gravitational potential energy of the object–Earth system. To do this, let us consider a brick of mass m at an initial height y_i above the ground, as shown in Figure 8.1. If we neglect air resistance, then the only force that does work on the brick as it falls is the gravitational force exerted on the brick $m\mathbf{g}$. The work W_g done by the gravitational force as the brick undergoes a downward displacement **d** is

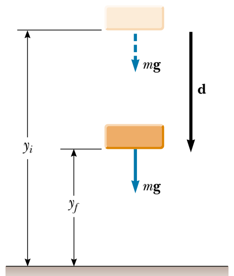


Figure 8.1 The work done on the brick by the gravitational force as the brick falls from a height y_i to a height y_f is equal to $mgy_i - mgy_f$.

$$W_g = (m\mathbf{g}) \cdot \mathbf{d} = (-mg\mathbf{j}) \cdot (y_f - y_i)\,\mathbf{j} = mgy_i - mgy_f$$

where we have used the fact that $\mathbf{j} \cdot \mathbf{j} = 1$ (Eq. 7.4). If an object undergoes both a horizontal and a vertical displacement, so that $\mathbf{d} = (x_f - x_i)\mathbf{i} + (y_f - y_i)\mathbf{j}$, then the work done by the gravitational force is still $mgy_i - mgy_f$ because $-mg\mathbf{j} \cdot (x_f - x_i)\mathbf{i} = 0$. Thus, the work done by the gravitational force depends only on the change in y and not on any change in the horizontal position x.

We just learned that the quantity mgy is the gravitational potential energy of the system U_g, and so we have

$$W_g = U_i - U_f = -(U_f - U_i) = -\Delta U_g \qquad (8.2)$$

From this result, we see that the work done on any object by the gravitational force is equal to the negative of the change in the system's gravitational potential energy. Also, this result demonstrates that it is only the *difference* in the gravitational potential energy at the initial and final locations that matters. This means that we are free to place the origin of coordinates in any convenient location. Finally, the work done by the gravitational force on an object as the object falls to the Earth is the same as the work done were the object to start at the same point and slide down an incline to the Earth. Horizontal motion does not affect the value of W_g.

The unit of gravitational potential energy is the same as that of work—the joule. Potential energy, like work and kinetic energy, is a scalar quantity.

[1] The assumption that the force of gravity is constant is a good one as long as the vertical displacement is small compared with the Earth's radius.

Quick Quiz 8.1

Can the gravitational potential energy of a system ever be negative?

EXAMPLE 8.1 The Bowler and the Sore Toe

A bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the total work done on the ball by the force of gravity as the ball falls. Repeat the calculation, using the top of the bowler's head as the origin of coordinates.

Solution First, we need to estimate a few values. A bowling ball has a mass of approximately 7 kg, and the top of a person's toe is about 0.03 m above the floor. Also, we shall assume the ball falls from a height of 0.5 m. Holding nonsignificant digits until we finish the problem, we calculate the gravitational potential energy of the ball–Earth system just before the ball is released to be $U_i = mgy_i = (7\text{ kg})(9.80\text{ m/s}^2)(0.5\text{ m}) = 34.3\text{ J}$. A similar calculation for when the ball reaches his toe gives $U_f = mgy_f = (7\text{ kg})(9.80\text{ m/s}^2)(0.03\text{ m}) = 2.06\text{ J}$. So, the work done by the gravitational force is $W_g = U_i - U_f = 32.24\text{ J}$. We should probably keep only one digit because of the roughness of our estimates; thus, we estimate that the gravitational force does 30 J of work on the bowling ball as it falls. The system had 30 J of gravitational potential energy relative to the top of the toe before the ball began its fall.

When we use the bowler's head (which we estimate to be 1.50 m above the floor) as our origin of coordinates, we find that $U_i = mgy_i = (7\text{ kg})(9.80\text{ m/s}^2)(-1\text{ m}) = -68.6\text{ J}$ and that $U_f = mgy_f = (7\text{ kg})(9.80\text{ m/s}^2)(-1.47\text{ m}) = -100.8\text{ J}$. The work being done by the gravitational force is still

$$W_g = U_i - U_f = 32.24\text{ J} \approx 30\text{ J}.$$

Elastic Potential Energy

Now consider a system consisting of a block plus a spring, as shown in Figure 8.2. The force that the spring exerts on the block is given by $F_s = -kx$. In the previous chapter, we learned that the work done by the spring force on a block connected to the spring is given by Equation 7.11:

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad (8.3)$$

In this situation, the initial and final x coordinates of the block are measured from its equilibrium position, $x = 0$. Again we see that W_s depends only on the initial and final x coordinates of the object and is zero for any closed path. The **elastic potential energy** function associated with the system is defined by

$$U_s \equiv \tfrac{1}{2}kx^2 \qquad (8.4)$$

Elastic potential energy stored in a spring

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). To visualize this, consider Figure 8.2, which shows a spring on a frictionless, horizontal surface. When a block is pushed against the spring (Fig. 8.2b) and the spring is compressed a distance x, the elastic potential energy stored in the spring is $kx^2/2$. When the block is released from rest, the spring snaps back to its original length and the stored elastic potential energy is transformed into kinetic energy of the block (Fig. 8.2c). The elastic potential energy stored in the spring is zero whenever the spring is undeformed ($x = 0$). Energy is stored in the spring only when the spring is either stretched or compressed. Furthermore, the elastic potential energy is a maximum when the spring has reached its maximum compression or extension (that is, when $|x|$ is a maximum). Finally, because the elastic potential energy is proportional to x^2, we see that U_s is always positive in a deformed spring.

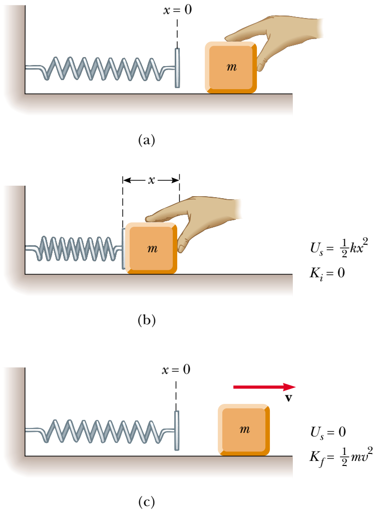


Figure 8.2 (a) An undeformed spring on a frictionless horizontal surface. (b) A block of mass m is pushed against the spring, compressing it a distance x. (c) When the block is released from rest, the elastic potential energy stored in the spring is transferred to the block in the form of kinetic energy.

8.2 CONSERVATIVE AND NONCONSERVATIVE FORCES

The work done by the gravitational force does not depend on whether an object falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. On the other hand, the energy loss due to friction on that incline depends on the distance the object slides. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy loss due to frictional forces. We can use this varying dependence on path to classify forces as either conservative or nonconservative.

Of the two forces just mentioned, the gravitational force is conservative and the frictional force is nonconservative.

Conservative Forces

Properties of a conservative force

Conservative forces have two important properties:

1. A force is conservative if the work it does on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another. As we learned in the preceding section, the work done by the gravitational force on an object moving between any two points near the Earth's surface is $W_g = mgy_i - mgy_f$. From this equation we see that W_g depends only on the initial and final y coordinates of the object and hence is independent of the path. Furthermore, W_g is zero when the object moves over any closed path (where $y_i = y_f$).

For the case of the object–spring system, the work W_s done by the spring force is given by $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$ (Eq. 8.3). Again, we see that the spring force is conservative because W_s depends only on the initial and final x coordinates of the object and is zero for any closed path.

We can associate a potential energy with any conservative force and can do this *only* for conservative forces. In the previous section, the potential energy associated with the gravitational force was defined as $U_g \equiv mgy$. In general, the work W_c done on an object by a conservative force is equal to the initial value of the potential energy associated with the object minus the final value:

Work done by a conservative force

$$W_c = U_i - U_f = -\Delta U \tag{8.5}$$

This equation should look familiar to you. It is the general form of the equation for work done by the gravitational force (Eq. 8.2) and that for the work done by the spring force (Eq. 8.3).

Nonconservative Forces

Properties of a nonconservative force

5.3 **A force is nonconservative if it causes a change in mechanical energy *E*,** which we define as the sum of kinetic and potential energies. For example, if a book is sent sliding on a horizontal surface that is not frictionless, the force of kinetic friction reduces the book's kinetic energy. As the book slows down, its kinetic energy decreases. As a result of the frictional force, the temperatures of the book and surface increase. The type of energy associated with temperature is *internal energy,* which we will study in detail in Chapter 20. Experience tells us that this internal energy cannot be transferred back to the kinetic energy of the book. In other words, the energy transformation is not reversible. Because the force of kinetic friction changes the mechanical energy of a system, it is a nonconservative force.

From the work–kinetic energy theorem, we see that the work done by a conservative force on an object causes a change in the kinetic energy of the object. The change in kinetic energy depends only on the initial and final positions of the object, and not on the path connecting these points. Let us compare this to the sliding book example, in which the nonconservative force of friction is acting between the book and the surface. According to Equation 7.17a, the change in kinetic energy of the book due to friction is $\Delta K_{\text{friction}} = -f_k d$, where d is the length of the path over which the friction force acts. Imagine that the book slides from A to B over the straight-line path of length d in Figure 8.3. The change in kinetic energy is $-f_k d$. Now, suppose the book slides over the semicircular path from A to B. In this case, the path is longer and, as a result, the change in kinetic energy is greater in magnitude than that in the straight-line case. For this particular path, the change in kinetic energy is $-f_k \pi d/2$, since d is the diameter of the semicircle. Thus, we see that for a nonconservative force, the change in kinetic energy depends on the path followed between the initial and final points. If a potential energy is involved, then the change in the total mechanical energy depends on the path followed. We shall return to this point in Section 8.5.

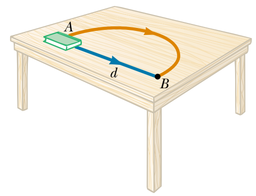


Figure 8.3 The loss in mechanical energy due to the force of kinetic friction depends on the path taken as the book is moved from A to B. The loss in mechanical energy is greater along the red path than along the blue path.

8.3 CONSERVATIVE FORCES AND POTENTIAL ENERGY

In the preceding section we found that the work done on a particle by a conservative force does not depend on the path taken by the particle. The work depends only on the particle's initial and final coordinates. As a consequence, we can define a **potential energy function** U such that the work done by a conservative force equals the decrease in the potential energy of the system. The work done by a conservative force **F** as a particle moves along the x axis is[2]

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U \quad \textbf{(8.6)}$$

where F_x is the component of **F** in the direction of the displacement. That is, **the work done by a conservative force equals the negative of the change in the potential energy associated with that force,** where the change in the potential energy is defined as $\Delta U = U_f - U_i$.

We can also express Equation 8.6 as

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \quad \textbf{(8.7)}$$

Therefore, ΔU is negative when F_x and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

The term *potential energy* implies that the object has the potential, or capability, of either gaining kinetic energy or doing work when it is released from some point under the influence of a conservative force exerted on the object by some other member of the system. It is often convenient to establish some particular location x_i as a reference point and measure all potential energy differences with respect to it. We can then define the potential energy function as

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i \quad \textbf{(8.8)}$$

The value of U_i is often taken to be zero at the reference point. It really does not matter what value we assign to U_i, because any nonzero value merely shifts $U_f(x)$ by a constant amount, and only the *change* in potential energy is physically meaningful.

If the conservative force is known as a function of position, we can use Equation 8.8 to calculate the change in potential energy of a system as an object within the system moves from x_i to x_f. It is interesting to note that in the case of one-dimensional displacement, a force is always conservative if it is a function of position only. This is not necessarily the case for motion involving two- or three-dimensional displacements.

8.4 CONSERVATION OF MECHANICAL ENERGY

5.9

An object held at some height h above the floor has no kinetic energy. However, as we learned earlier, the gravitational potential energy of the object–Earth system is equal to mgh. If the object is dropped, it falls to the floor; as it falls, its speed and thus its kinetic energy increase, while the potential energy of the system decreases. If factors such as air resistance are ignored, whatever potential energy the system loses as the object moves downward appears as kinetic energy of the object. In other words, the sum of the kinetic and potential energies—the total mechanical energy E—remains constant. This is an example of the principle of **conservation of mechanical energy.** For the case of an object in free fall, this principle tells us that any increase (or decrease) in potential energy is accompanied by an equal decrease (or increase) in kinetic energy. Note that **the total mechanical energy of a system remains constant in any isolated system of objects that interact only through conservative forces.**

[2] For a general displacement, the work done in two or three dimensions also equals $U_i - U_f$, where $U = U(x, y, z)$. We write this formally as $W = \int_i^f \mathbf{F} \cdot d\mathbf{s} = U_i - U_f$.

Because the total mechanical energy E of a system is defined as the sum of the kinetic and potential energies, we can write

$$E \equiv K + U \quad \textbf{(8.9)}$$

Total mechanical energy

We can state the principle of conservation of energy as $E_i = E_f$, and so we have

$$K_i + U_i = K_f + U_f \quad \textbf{(8.10)}$$

The mechanical energy of an isolated system remains constant

It is important to note that Equation 8.10 is valid only when no energy is added to or removed from the system. Furthermore, there must be no nonconservative forces doing work within the system.

Consider the carnival Ring-the-Bell event illustrated at the beginning of the chapter. The participant is trying to convert the initial kinetic energy of the hammer into gravitational potential energy associated with a weight that slides on a vertical track. If the hammer has sufficient kinetic energy, the weight is lifted high enough to reach the bell at the top of the track. To maximize the hammer's kinetic energy, the player must swing the heavy hammer as rapidly as possible. The fast-moving hammer does work on the pivoted target, which in turn does work on the weight. Of course, greasing the track (so as to minimize energy loss due to friction) would also help but is probably not allowed!

If more than one conservative force acts on an object within a system, a potential energy function is associated with each force. In such a case, we can apply the principle of conservation of mechanical energy for the system as

$$K_i + \sum U_i = K_f + \sum U_f \quad \textbf{(8.11)}$$

where the number of terms in the sums equals the number of conservative forces present. For example, if an object connected to a spring oscillates vertically, two conservative forces act on the object: the spring force and the gravitational force.

QuickLab

Dangle a shoe from its lace and use it as a pendulum. Hold it to the side, release it, and note how high it swings at the end of its arc. How does this height compare with its initial height? You may want to check Question 8.3 as part of your investigation.

Twin Falls on the Island of Kauai, Hawaii. The gravitational potential energy of the water–Earth system when the water is at the top of the falls is converted to kinetic energy once that water begins falling. How did the water get to the top of the cliff? In other words, what was the original source of the gravitational potential energy when the water was at the top? (*Hint:* This same source powers nearly everything on the planet.)

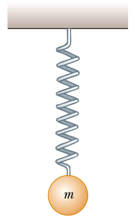


Figure 8.4 A ball connected to a massless spring suspended vertically. What forms of potential energy are associated with the ball–spring–Earth system when the ball is displaced downward?

Quick Quiz 8.2

A ball is connected to a light spring suspended vertically, as shown in Figure 8.4. When displaced downward from its equilibrium position and released, the ball oscillates up and down. If air resistance is neglected, is the total mechanical energy of the system (ball plus spring plus Earth) conserved? How many forms of potential energy are there for this situation?

Quick Quiz 8.3

Three identical balls are thrown from the top of a building, all with the same initial speed. The first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as shown in Figure 8.5. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.

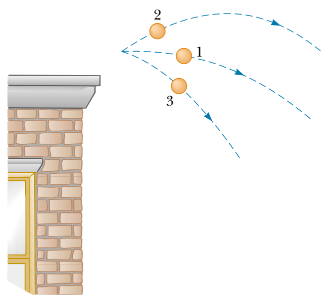


Figure 8.5 Three identical balls are thrown with the same initial speed from the top of a building.

EXAMPLE 8.2 Ball in Free Fall

A ball of mass m is dropped from a height h above the ground, as shown in Figure 8.6. (a) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

Solution Because the ball is in free fall, the only force acting on it is the gravitational force. Therefore, we apply the principle of conservation of mechanical energy to the ball–Earth system. Initially, the system has potential energy but no kinetic energy. As the ball falls, the total mechanical energy remains constant and equal to the initial potential energy of the system.

At the instant the ball is released, its kinetic energy is $K_i = 0$ and the potential energy of the system is $U_i = mgh$. When the ball is at a distance y above the ground, its kinetic energy is $K_f = \frac{1}{2}mv_f^2$ and the potential energy relative to the ground is $U_f = mgy$. Applying Equation 8.10, we obtain

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \tfrac{1}{2}mv_f^2 + mgy$$

$$v_f^2 = 2g(h - y)$$

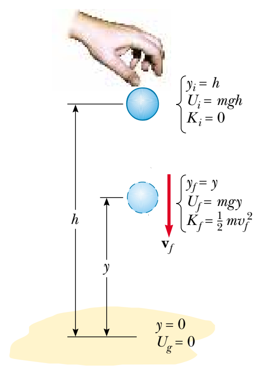


Figure 8.6 A ball is dropped from a height h above the ground. Initially, the total energy of the ball–Earth system is potential energy, equal to mgh relative to the ground. At the elevation y, the total energy is the sum of the kinetic and potential energies.

$$v_f = \sqrt{2g(h - y)}$$

The speed is always positive. If we had been asked to find the ball's velocity, we would use the negative value of the square root as the y component to indicate the downward motion.

(b) Determine the speed of the ball at y if at the instant of release it already has an initial speed v_i at the initial altitude h.

Solution In this case, the initial energy includes kinetic energy equal to $\frac{1}{2}mv_i^2$, and Equation 8.10 gives

$$\tfrac{1}{2}mv_i^2 + mgh = \tfrac{1}{2}mv_f^2 + mgy$$

$$v_f^2 = v_i^2 + 2g(h - y)$$

$$v_f = \sqrt{v_i^2 + 2g(h - y)}$$

This result is consistent with the expression $v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$ from kinematics, where $y_i = h$. Furthermore, this result is valid even if the initial velocity is at an angle to the horizontal (the projectile situation) for two reasons: (1) energy is a scalar, and the kinetic energy depends only on the magnitude of the velocity; and (2) the change in the gravitational potential energy depends only on the change in position in the vertical direction.

EXAMPLE 8.3 The Pendulum

A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure 8.7. The sphere is released from rest when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless. (a) Find the speed of the sphere when it is at the lowest point Ⓑ.

Solution The only force that does work on the sphere is the gravitational force. (The force of tension is always perpendicular to each element of the displacement and hence does no work.) Because the gravitational force is conservative, the total mechanical energy of the pendulum–Earth system is constant. (In other words, we can classify this as an "energy conservation" problem.) As the pendulum swings, continuous transformation between potential and kinetic energy occurs. At the instant the pendulum is released, the energy of the system is entirely potential energy. At point Ⓑ the pendulum has kinetic energy, but the system has lost some potential energy. At Ⓒ the system has regained its initial potential energy, and the kinetic energy of the pendulum is again zero.

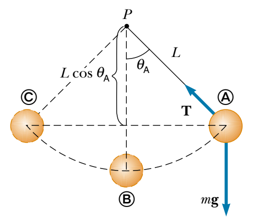


Figure 8.7 If the sphere is released from rest at the angle θ_A it will never swing above this position during its motion. At the start of the motion, position Ⓐ, the energy is entirely potential. This initial potential energy is all transformed into kinetic energy at the lowest elevation Ⓑ. As the sphere continues to move along the arc, the energy again becomes entirely potential energy at Ⓒ.

If we measure the y coordinates of the sphere from the center of rotation, then $y_A = -L\cos\theta_A$ and $y_B = -L$. Therefore, $U_A = -mgL\cos\theta_A$ and $U_B = -mgL$. Applying the principle of conservation of mechanical energy to the system gives

$$K_A + U_A = K_B + U_B$$

$$0 - mgL\cos\theta_A = \tfrac{1}{2}mv_B^2 - mgL$$

$$(1) \qquad v_B = \sqrt{2gL(1 - \cos\theta_A)}$$

(b) What is the tension T_B in the cord at Ⓑ?

Solution Because the force of tension does no work, we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, we obtain

$$(2) \qquad \sum F_r = T_B - mg = ma_r = m\frac{v_B^2}{L}$$

Substituting (1) into (2) gives the tension at point Ⓑ:

$$(3) \qquad T_B = mg + 2mg(1 - \cos\theta_A)$$

$$= mg(3 - 2\cos\theta_A)$$

From (2) we see that the tension at Ⓑ is greater than the weight of the sphere. Furthermore, (3) gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$.

Exercise A pendulum of length 2.00 m and mass 0.500 kg is released from rest when the cord makes an angle of 30.0° with the vertical. Find the speed of the sphere and the tension in the cord when the sphere is at its lowest point.

Answer 2.29 m/s; 6.21 N.

8.5 WORK DONE BY NONCONSERVATIVE FORCES

As we have seen, if the forces acting on objects within a system are conservative, then the mechanical energy of the system remains constant. However, if some of the forces acting on objects within the system are not conservative, then the mechanical energy of the system does not remain constant. Let us examine two types of nonconservative forces: an applied force and the force of kinetic friction.

Work Done by an Applied Force

When you lift a book through some distance by applying a force to it, the force you apply does work W_{app} on the book, while the gravitational force does work W_g on the book. If we treat the book as a particle, then the net work done on the book is related to the change in its kinetic energy as described by the work–kinetic energy theorem given by Equation 7.15:

$$W_{app} + W_g = \Delta K \quad \textbf{(8.12)}$$

Because the gravitational force is conservative, we can use Equation 8.2 to express the work done by the gravitational force in terms of the change in potential energy, or $W_g = -\Delta U$. Substituting this into Equation 8.12 gives

$$W_{app} = \Delta K + \Delta U \quad \textbf{(8.13)}$$

Note that the right side of this equation represents the change in the mechanical energy of the book–Earth system. This result indicates that your applied force transfers energy to the system in the form of kinetic energy of the book and gravitational potential energy of the book–Earth system. Thus, we conclude that if an object is part of a system, then **an applied force can transfer energy into or out of the system.**

Situations Involving Kinetic Friction

Kinetic friction is an example of a nonconservative force. If a book is given some initial velocity on a horizontal surface that is not frictionless, then the force of kinetic friction acting on the book opposes its motion and the book slows down and eventually stops. The force of kinetic friction reduces the kinetic energy of the book by transforming kinetic energy to internal energy of the book and part of the horizontal surface. Only part of the book's kinetic energy is transformed to internal energy in the book. The rest appears as internal energy in the surface. (When you trip and fall while running across a gymnasium floor, not only does the skin on your knees warm up but so does the floor!)

As the book moves through a distance d, the only force that does work is the force of kinetic friction. This force causes a decrease in the kinetic energy of the book. This decrease was calculated in Chapter 7, leading to Equation 7.17a, which we repeat here:

$$\Delta K_{friction} = -f_k d \quad \textbf{(8.14)}$$

If the book moves on an incline that is not frictionless, a change in the gravitational potential energy of the book–Earth system also occurs, and $-f_k d$ is the amount by which the mechanical energy of the system changes because of the force of kinetic friction. In such cases,

$$\Delta E = \Delta K + \Delta U = -f_k d \quad \textbf{(8.15)}$$

where $E_i + \Delta E = E_f$.

QuickLab

Find a friend and play a game of racquetball. After a long volley, feel the ball and note that it is warm. Why is that?

Quick Quiz 8.4

Write down the work–kinetic energy theorem for the general case of two objects that are connected by a spring and acted upon by gravity and some other external applied force. Include the effects of friction as $\Delta E_{friction}$.

Problem-Solving Hints

Conservation of Energy

We can solve many problems in physics using the principle of conservation of energy. You should incorporate the following procedure when you apply this principle:

- Define your system, which may include two or more interacting particles, as well as springs or other systems in which elastic potential energy can be stored. Choose the initial and final points.
- Identify zero points for potential energy (both gravitational and spring). If there is more than one conservative force, write an expression for the potential energy associated with each force.
- Determine whether any nonconservative forces are present. Remember that if friction or air resistance is present, mechanical energy *is not conserved.*
- If mechanical energy is *conserved,* you can write the total initial energy $E_i = K_i + U_i$ at some point. Then, write an expression for the total final energy $E_f = K_f + U_f$ at the final point that is of interest. Because mechanical energy is *conserved,* you can equate the two total energies and solve for the quantity that is unknown.
- If frictional forces are present (and thus mechanical energy is *not conserved*), first write expressions for the total initial and total final energies. In this case, the difference between the total final mechanical energy and the total initial mechanical energy equals the change in mechanical energy in the system due to friction.

EXAMPLE 8.4 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure 8.8. The crate starts from rest at the top, experiences a constant frictional force of magnitude 5.00 N, and continues to move a short distance on the flat floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.

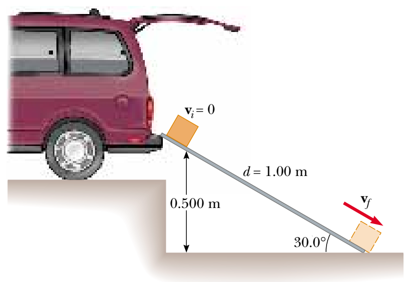


Figure 8.8 A crate slides down a ramp under the influence of gravity. The potential energy decreases while the kinetic energy increases.

Solution Because $v_i = 0$, the initial kinetic energy at the top of the ramp is zero. If the y coordinate is measured from the bottom of the ramp (the final position where the potential energy is zero) with the upward direction being positive, then $y_i = 0.500$ m. Therefore, the total mechanical energy of the crate–Earth system at the top is all potential energy:

$$E_i = K_i + U_i = 0 + U_i = mgy_i$$
$$= (3.00 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m}) = 14.7 \text{ J}$$

When the crate reaches the bottom of the ramp, the potential energy of the system is *zero* because the elevation of the crate is $y_f = 0$. Therefore, the total mechanical energy of the system when the crate reaches the bottom is all kinetic energy:

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

We cannot say that $E_i = E_f$ because a nonconservative force reduces the mechanical energy of the system: the force of kinetic friction acting on the crate. In this case, Equation 8.15 gives $\Delta E = -f_k d$, where d is the displacement along the ramp. (Remember that the forces normal to the ramp do no work on the crate because they are perpendicular to the displacement.) With $f_k = 5.00$ N and $d = 1.00$ m, we have

$$\Delta E = -f_k d = -(5.00\ \text{N})(1.00\ \text{m}) = -5.00\ \text{J}$$

This result indicates that the system loses some mechanical energy because of the presence of the nonconservative frictional force. Applying Equation 8.15 gives

$$E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$

$$\tfrac{1}{2}mv_f^2 = 14.7\ \text{J} - 5.00\ \text{J} = 9.70\ \text{J}$$

$$v_f^2 = \frac{19.4\ \text{J}}{3.00\ \text{kg}} = 6.47\ \text{m}^2/\text{s}^2$$

$$v_f = \boxed{2.54\ \text{m/s}}$$

Exercise Use Newton's second law to find the acceleration of the crate along the ramp, and use the equations of kinematics to determine the final speed of the crate.

Answer 3.23 m/s^2; 2.54 m/s.

Exercise Assuming the ramp to be frictionless, find the final speed of the crate and its acceleration along the ramp.

Answer 3.13 m/s; 4.90 m/s^2.

EXAMPLE 8.5 Motion on a Curved Track

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.9. The child starts from rest at the top. (a) Determine his speed at the bottom, assuming no friction is present.

Solution The normal force **n** does no work on the child because this force is always perpendicular to each element of the displacement. Because there is no friction, the mechanical energy of the child–Earth system is conserved. If we measure the y coordinate in the upward direction from the bottom of the slide, then $y_i = h$, $y_f = 0$, and we obtain

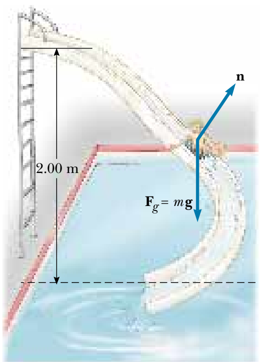


Figure 8.9 If the slide is frictionless, the speed of the child at the bottom depends only on the height of the slide.

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \tfrac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gh}$$

Note that the result is the same as it would be had the child fallen vertically through a distance h! In this example, $h = 2.00$ m, giving

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(2.00\ \text{m})} = \boxed{6.26\ \text{m/s}}$$

(b) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.

Solution In this case, mechanical energy is *not* conserved, and so we must use Equation 8.15 to find the loss of mechanical energy due to friction:

$$\Delta E = E_f - E_i = (K_f + U_f) - (K_i + U_i)$$

$$= (\tfrac{1}{2}mv_f^2 + 0) - (0 + mgh) = \tfrac{1}{2}mv_f^2 - mgh$$

$$= \tfrac{1}{2}(20.0\ \text{kg})(3.00\ \text{m/s})^2 - (20.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})$$

$$= \boxed{-302\ \text{J}}$$

Again, ΔE is negative because friction is reducing mechanical energy of the system (the final mechanical energy is less than the initial mechanical energy). Because the slide is curved, the normal force changes in magnitude and direction during the motion. Therefore, the frictional force, which is proportional to n, also changes during the motion. Given this changing frictional force, do you think it is possible to determine μ_k from these data?

EXAMPLE 8.6 Let's Go Skiing!

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure 8.10. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest?

Solution First, let us calculate her speed at the bottom of the incline, which we choose as our zero point of potential energy. Because the incline is frictionless, the mechanical energy of the skier–Earth system remains constant, and we find, as we did in the previous example, that

$$v_B = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m})} = 19.8\ \text{m/s}$$

Now we apply Equation 8.15 as the skier moves along the rough horizontal surface from Ⓑ to Ⓒ. The change in mechanical energy along the horizontal is $\Delta E = -f_k d$, where d is the horizontal displacement.

To find the distance the skier travels before coming to rest, we take $K_C = 0$. With $v_B = 19.8$ m/s and the frictional force given by $f_k = \mu_k n = \mu_k mg$, we obtain

$$\Delta E = E_C - E_B = -\mu_k mgd$$

$$(K_C + U_C) - (K_B + U_B) = (0 + 0) - (\tfrac{1}{2}mv_B^2 + 0)$$

$$= -\mu_k mgd$$

$$d = \frac{v_B^2}{2\mu_k g} = \frac{(19.8\ \text{m/s})^2}{2(0.210)(9.80\ \text{m/s}^2)}$$

$$= \boxed{95.2\ \text{m}}$$

Exercise Find the horizontal distance the skier travels before coming to rest if the incline also has a coefficient of kinetic friction equal to 0.210.

Answer 40.3 m.

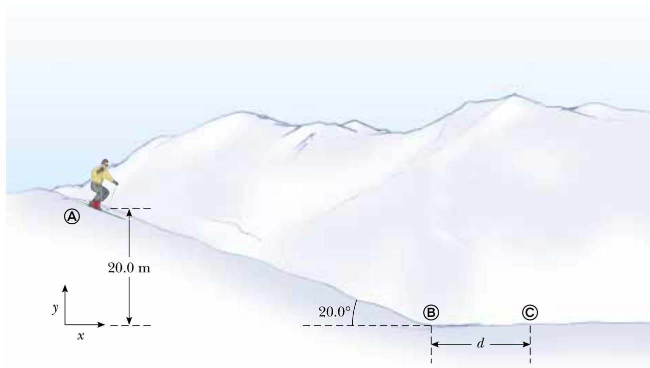


Figure 8.10 The skier slides down the slope and onto a level surface, stopping after a distance d from the bottom of the hill.

EXAMPLE 8.7 The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Fig. 8.11a). When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing. (a) Neglecting all resistive forces, determine the spring constant.

Solution Because the projectile starts from rest, the initial kinetic energy is zero. If we take the zero point for the gravitational potential energy of the projectile–Earth system to be at the lowest position of the projectile x_A, then the initial gravitational potential energy also is zero. The mechanical energy of this system is constant because no nonconservative forces are present.

Initially, the only mechanical energy in the system is the elastic potential energy stored in the spring of the gun, $U_{sA} = kx^2/2$, where the compression of the spring is $x = 0.120$ m. The projectile rises to a maximum height

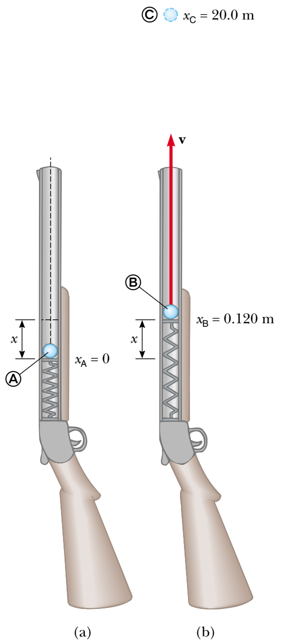


Figure 8.11 A spring-loaded popgun.

$x_C = h = 20.0$ m, and so the final gravitational potential energy when the projectile reaches its peak is mgh. The final kinetic energy of the projectile is zero, and the final elastic potential energy stored in the spring is zero. Because the mechanical energy of the system is constant, we find that

$$E_A = E_C$$
$$K_A + U_{gA} + U_{sA} = K_C + U_{gC} + U_{sC}$$
$$0 + 0 + \tfrac{1}{2}kx^2 = 0 + mgh + 0$$
$$\tfrac{1}{2}k(0.120\ \text{m})^2 = (0.0350\ \text{kg})(9.80\ \text{m/s}^2)(20.0\ \text{m})$$
$$k = \boxed{953\ \text{N/m}}$$

(b) Find the speed of the projectile as it moves through the equilibrium position of the spring (where $x_B = 0.120$ m) as shown in Figure 8.11b.

Solution As already noted, the only mechanical energy in the system at Ⓐ is the elastic potential energy $kx^2/2$. The total energy of the system as the projectile moves through the equilibrium position of the spring comprises the kinetic energy of the projectile $mv_B^2/2$, and the gravitational potential energy mgx_B. Hence, the principle of the conservation of mechanical energy in this case gives

$$E_A = E_B$$
$$K_A + U_{gA} + U_{sA} = K_B + U_{gB} + U_{sB}$$
$$0 + 0 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_B^2 + mgx_B + 0$$

Solving for v_B gives

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B}$$
$$= \sqrt{\frac{(953\ \text{N/m})(0.120\ \text{m})^2}{0.0350\ \text{kg}} - 2(9.80\ \text{m/s}^2)(0.120\ \text{m})}$$
$$= \boxed{19.7\ \text{m/s}}$$

You should compare the different examples we have presented so far in this chapter. Note how breaking the problem into a sequence of labeled events helps in the analysis.

Exercise What is the speed of the projectile when it is at a height of 10.0 m?

Answer 14.0 m/s.

EXAMPLE 8.8 Block–Spring Collision

A block having a mass of 0.80 kg is given an initial velocity $v_A = 1.2$ m/s to the right and collides with a spring of negligible mass and force constant $k = 50$ N/m, as shown in Figure 8.12. (a) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

Solution Our system in this example consists of the block and spring. Before the collision, at Ⓐ, the block has kinetic energy and the spring is uncompressed, so that the elastic potential energy stored in the spring is zero. Thus, the total mechanical energy of the system before the collision is just $\tfrac{1}{2}mv_A^2$. After the collision, at Ⓒ, the spring is fully compressed; now the block is at rest and so has zero kinetic energy, while the energy stored in the spring has its maximum value $\tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2$, where the origin of coordinates $x = 0$ is chosen to be the equilibrium position of the spring and x_m is

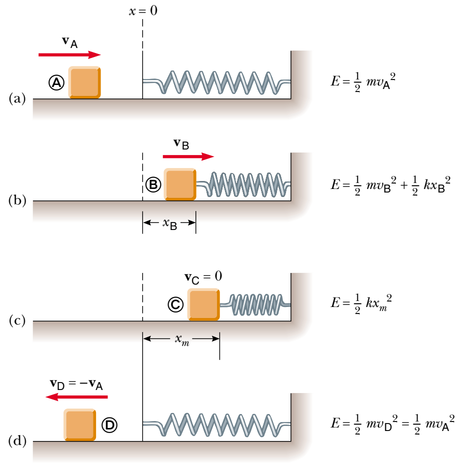


Figure 8.12 A block sliding on a smooth, horizontal surface collides with a light spring. (a) Initially the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy remains constant throughout the motion.

Multiflash photograph of a pole vault event. How many forms of energy can you identify in this picture?

the maximum compression of the spring, which in this case happens to be x_C. The total mechanical energy of the system is conserved because no nonconservative forces act on objects within the system.

Because mechanical energy is conserved, the kinetic energy of the block before the collision must equal the maximum potential energy stored in the fully compressed spring:

$$E_A = E_C$$
$$K_A + U_{sA} = K_C + U_{sC}$$
$$\tfrac{1}{2}mv_A^2 + 0 = 0 + \tfrac{1}{2}kx_m^2$$
$$x_m = \sqrt{\frac{m}{k}}\,v_A = \sqrt{\frac{0.80\ \text{kg}}{50\ \text{N/m}}}\,(1.2\ \text{m/s})$$
$$= \boxed{0.15\ \text{m}}$$

Note that we have not included U_g terms because no change in vertical position occurred.

(b) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_A = 1.2$ m/s, what is the maximum compression in the spring?

Solution In this case, mechanical energy is *not* conserved because a frictional force acts on the block. The magnitude of the frictional force is

$$f_k = \mu_k n = \mu_k mg = 0.50(0.80\ \text{kg})(9.80\ \text{m/s}^2) = 3.92\ \text{N}$$

Therefore, the change in the block's mechanical energy due to friction as the block is displaced from the equilibrium position of the spring (where we have set our origin) to x_B is

$$\Delta E = -f_k x_B = -3.92x_B$$

Substituting this into Equation 8.15 gives

$$\Delta E = E_f - E_i = (0 + \tfrac{1}{2}kx_B^2) - (\tfrac{1}{2}mv_A^2 + 0) = -f_k x_B$$
$$\tfrac{1}{2}(50)x_B^2 - \tfrac{1}{2}(0.80)(1.2)^2 = -3.92x_B$$
$$25x_B^2 + 3.92x_B - 0.576 = 0$$

Solving the quadratic equation for x_B gives $x_B = 0.092$ m and $x_B = -0.25$ m. The physically meaningful root is $x_B = \boxed{0.092\ \text{m.}}$ The negative root does not apply to this situation because the block must be to the right of the origin (positive value of x) when it comes to rest. Note that 0.092 m is less than the distance obtained in the frictionless case of part (a). This result is what we expect because friction retards the motion of the system.

EXAMPLE 8.9 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.13. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

Solution The key word *rest* appears twice in the problem statement, telling us that the initial and final velocities and kinetic energies are zero. (Also note that because we are concerned only with the beginning and ending points of the motion, we do not need to label events with circled letters as we did in the previous two examples. Simply using i and f is sufficient to keep track of the situation.) In this situation, the system consists of the two blocks, the spring, and the Earth. We need to consider two forms of potential energy: gravitational and elastic. Because the initial and final kinetic energies of the system are zero, $\Delta K = 0$, and we can write

$$(1) \qquad \Delta E = \Delta U_g + \Delta U_s$$

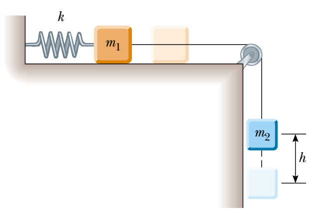


Figure 8.13 As the hanging block moves from its highest elevation to its lowest, the system loses gravitational potential energy but gains elastic potential energy in the spring. Some mechanical energy is lost because of friction between the sliding block and the surface.

where $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. As the hanging block falls a distance h, the horizontally moving block moves the same distance h to the right. Therefore, using Equation 8.15, we find that the loss in energy due to friction between the horizontally sliding block and the surface is

$$(2) \qquad \Delta E = -f_k h = -\mu_k m_1 g h$$

The change in the gravitational potential energy of the system is associated with only the falling block because the vertical coordinate of the horizontally sliding block does not change. Therefore, we obtain

$$(3) \qquad \Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

where the coordinates have been measured from the lowest position of the falling block.

The change in the elastic potential energy stored in the spring is

$$(4) \qquad \Delta U_s = U_{sf} - U_{si} = \tfrac{1}{2}kh^2 - 0$$

Substituting Equations (2), (3), and (4) into Equation (1) gives

$$-\mu_k m_1 g h = -m_2 g h + \tfrac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

This setup represents a way of measuring the coefficient of kinetic friction between an object and some surface. As you can see from the problem, sometimes it is easier to work with the changes in the various types of energy rather than the actual values. For example, if we wanted to calculate the numerical value of the gravitational potential energy associated with the horizontally sliding block, we would need to specify the height of the horizontal surface relative to the lowest position of the falling block. Fortunately, this is not necessary because the gravitational potential energy associated with the first block does not change.

EXAMPLE 8.10 A Grand Entrance

You are designing apparatus to support an actor of mass 65 kg who is to "fly" down to the stage during the performance of a play. You decide to attach the actor's harness to a 130-kg sandbag by means of a lightweight steel cable running smoothly over two frictionless pulleys, as shown in Figure 8.14a. You need 3.0 m of cable between the harness and the nearest pulley so that the pulley can be hidden behind a curtain. For the apparatus to work successfully, the sandbag must never lift above the floor as the actor swings from above the stage to the floor. Let us call the angle that the actor's cable makes with the vertical θ. What is the maximum value θ can have before the sandbag lifts off the floor?

Solution We need to draw on several concepts to solve this problem. First, we use the principle of conservation of mechanical energy to find the actor's speed as he hits the floor as a function of θ and the radius R of the circular path through which he swings. Next, we apply Newton's second

law to the actor at the bottom of his path to find the cable tension as a function of the given parameters. Finally, we note that the sandbag lifts off the floor when the upward force exerted on it by the cable exceeds the gravitational force acting on it; the normal force is zero when this happens.

Applying conservation of energy to the actor–Earth system gives

$$K_i + U_i = K_f + U_f$$

$$(1) \qquad 0 + m_{\text{actor}}\, g y_i = \tfrac{1}{2} m_{\text{actor}} v_f^2 + 0$$

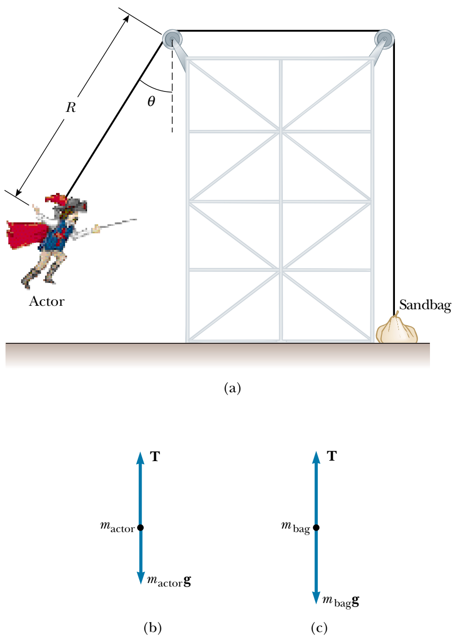


Figure 8.14 (a) An actor uses some clever staging to make his entrance. (b) Free-body diagram for actor at the bottom of the circular path. (c) Free-body diagram for sandbag.

where y_i is the initial height of the actor above the floor and v_f is the speed of the actor at the instant before he lands. (Note that $K_i = 0$ because he starts from rest and that $U_f = 0$ because we set the level of the actor's harness when he is standing on the floor as the zero level of potential energy.) From the geometry in Figure 8.14a, we see that $y_i = R - R\cos\theta = R(1 - \cos\theta)$. Using this relationship in Equation (1), we obtain

$$(2) \qquad v_f^2 = 2gR(1 - \cos\theta)$$

Now we apply Newton's second law to the actor when he is at the bottom of the circular path, using the free-body diagram in Figure 8.14b as a guide:

$$\sum F_y = T - m_{\text{actor}}g = m_{\text{actor}}\frac{v_f^2}{R}$$

$$(3) \qquad T = m_{\text{actor}}g + m_{\text{actor}}\frac{v_f^2}{R}$$

A force of the same magnitude as T is transmitted to the sandbag. If it is to be just lifted off the floor, the normal force on it becomes zero, and we require that $T = m_{\text{bag}}g$, as shown in Figure 8.14c. Using this condition together with Equations (2) and (3), we find that

$$m_{\text{bag}}g = m_{\text{actor}}g + m_{\text{actor}}\frac{2gR(1 - \cos\theta)}{R}$$

Solving for θ and substituting in the given parameters, we obtain

$$\cos\theta = \frac{3m_{\text{actor}} - m_{\text{bag}}}{2m_{\text{actor}}} = \frac{3(65\text{ kg}) - 130\text{ kg}}{2(65\text{ kg})} = \frac{1}{2}$$

$$\theta = 60^\circ$$

Notice that we did not need to be concerned with the length R of the cable from the actor's harness to the leftmost pulley. The important point to be made from this problem is that it is sometimes necessary to combine energy considerations with Newton's laws of motion.

Exercise If the initial angle $\theta = 40^\circ$, find the speed of the actor and the tension in the cable just before he reaches the floor. (*Hint:* You cannot ignore the length $R = 3.0$ m in this calculation.)

Answer 3.7 m/s; 940 N.

8.6 RELATIONSHIP BETWEEN CONSERVATIVE FORCES AND POTENTIAL ENERGY

Once again let us consider a particle that is part of a system. Suppose that the particle moves along the x axis, and assume that a conservative force with an x component F_x acts on the particle. Earlier in this chapter, we showed how to determine the change in potential energy of a system when we are given the conservative force. We now show how to find F_x if the potential energy of the system is known.

In Section 8.2 we learned that the work done by the conservative force as its point of application undergoes a displacement Δx equals the negative of the change in the potential energy associated with that force; that is, $W = F_x \Delta x = -\Delta U$. If the point of application of the force undergoes an infinitesimal displacement dx, we can express the infinitesimal change in the potential energy of the system dU as

$$dU = -F_x\, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship[3]

Relationship between force and potential energy

$$F_x = -\frac{dU}{dx} \qquad \textbf{(8.16)}$$

That is, **any conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to *x*.**

We can easily check this relationship for the two examples already discussed. In the case of the deformed spring, $U_s = \frac{1}{2}kx^2$, and therefore

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

which corresponds to the restoring force in the spring. Because the gravitational potential energy function is $U_g = mgy$, it follows from Equation 8.16 that $F_g = -mg$ when we differentiate U_g with respect to y instead of x.

We now see that U is an important function because a conservative force can be derived from it. Furthermore, Equation 8.16 should clarify the fact that adding a constant to the potential energy is unimportant because the derivative of a constant is zero.

Quick Quiz 8.5

What does the slope of a graph of $U(x)$ versus x represent?

Optional Section

8.7 ENERGY DIAGRAMS AND THE EQUILIBRIUM OF A SYSTEM

The motion of a system can often be understood qualitatively through a graph of its potential energy versus the separation distance between the objects in the system. Consider the potential energy function for a block–spring system, given by $U_s = \frac{1}{2}kx^2$. This function is plotted versus x in Figure 8.15a. (A common mistake is to think that potential energy on the graph represents height. This is clearly not

[3] In three dimensions, the expression is $\mathbf{F} = -\mathbf{i}\frac{\partial U}{\partial x} - \mathbf{j}\frac{\partial U}{\partial y} - \mathbf{k}\frac{\partial U}{\partial z}$, where $\frac{\partial U}{\partial x}$, and so forth, are partial derivatives. In the language of vector calculus, **F** equals the negative of the gradient of the scalar quantity $U(x, y, z)$.

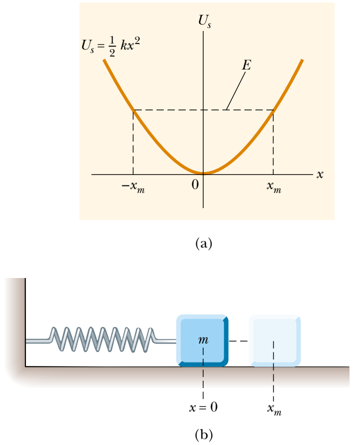


Figure 8.15 (a) Potential energy as a function of x for the block–spring system shown in (b). The block oscillates between the turning points, which have the coordinates $x = \pm x_m$. Note that the restoring force exerted by the spring always acts toward $x = 0$, the position of stable equilibrium.

the case here, where the block is only moving horizontally.) The force F_s exerted by the spring on the block is related to U_s through Equation 8.16:

$$F_s = -\frac{dU_s}{dx} = -kx$$

As we saw in Quick Quiz 8.5, the force is equal to the negative of the slope of the U versus x curve. When the block is placed at rest at the equilibrium position of the spring ($x = 0$), where $F_s = 0$, it will remain there unless some external force F_{ext} acts on it. If this external force stretches the spring from equilibrium, x is positive and the slope dU/dx is positive; therefore, the force F_s exerted by the spring is negative, and the block accelerates back toward $x = 0$ when released. If the external force compresses the spring, then x is negative and the slope is negative; therefore, F_s is positive, and again the mass accelerates toward $x = 0$ upon release.

From this analysis, we conclude that the $x = 0$ position for a block–spring system is one of **stable equilibrium.** That is, any movement away from this position results in a force directed back toward $x = 0$. In general, **positions of stable equilibrium correspond to points for which *U*(*x*) is a minimum.**

From Figure 8.15 we see that if the block is given an initial displacement x_m and is released from rest, its total energy initially is the potential energy stored in the spring $\frac{1}{2}kx_m^2$. As the block starts to move, the system acquires kinetic energy and loses an equal amount of potential energy. Because the total energy must remain constant, the block oscillates (moves back and forth) between the two points $x = -x_m$ and $x = +x_m$, called the *turning points.* In fact, because no energy is lost (no friction), the block will oscillate between $-x_m$ and $+x_m$ forever. (We discuss these oscillations further in Chapter 13.) From an energy viewpoint, the energy of the system cannot exceed $\frac{1}{2}kx_m^2$; therefore, the block must stop at these points and, because of the spring force, must accelerate toward $x = 0$.

Another simple mechanical system that has a position of stable equilibrium is a ball rolling about in the bottom of a bowl. Anytime the ball is displaced from its lowest position, it tends to return to that position when released.

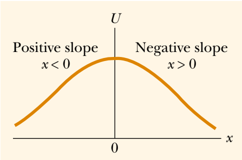


Figure 8.16 A plot of U versus x for a particle that has a position of unstable equilibrium located at $x = 0$. For any finite displacement of the particle, the force on the particle is directed away from $x = 0$.

Now consider a particle moving along the x axis under the influence of a conservative force F_x, where the U versus x curve is as shown in Figure 8.16. Once again, $F_x = 0$ at $x = 0$, and so the particle is in equilibrium at this point. However, this is a position of **unstable equilibrium** for the following reason: Suppose that the particle is displaced to the right ($x > 0$). Because the slope is negative for $x > 0$, $F_x = -dU/dx$ is positive and the particle accelerates *away from* $x = 0$. If instead the particle is at $x = 0$ and is displaced to the left ($x < 0$), the force is negative because the slope is positive for $x < 0$, and the particle again accelerates *away from* the equilibrium position. The position $x = 0$ in this situation is one of unstable equilibrium because for any displacement from this point, the force pushes the particle *farther away from* equilibrium. The force pushes the particle toward a position of lower potential energy. A pencil balanced on its point is in a position of unstable equilibrium. If the pencil is displaced slightly from its absolutely vertical position and is then released, it will surely fall over. In general, **positions of unstable equilibrium correspond to points for which $U(x)$ is a maximum.**

Finally, a situation may arise where U is constant over some region and hence $F_x = 0$. This is called a position of **neutral equilibrium.** Small displacements from this position produce neither restoring nor disrupting forces. A ball lying on a flat horizontal surface is an example of an object in neutral equilibrium.

EXAMPLE 8.11 Force and Energy on an Atomic Scale

The potential energy associated with the force between two neutral atoms in a molecule can be modeled by the Lennard–Jones potential energy function:

$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

where x is the separation of the atoms. The function $U(x)$ contains two parameters σ and ϵ that are determined from experiments. Sample values for the interaction between two atoms in a molecule are $\sigma = 0.263$ nm and $\epsilon = 1.51 \times 10^{-22}$ J. (a) Using a spreadsheet or similar tool, graph this function and find the most likely distance between the two atoms.

Solution We expect to find stable equilibrium when the two atoms are separated by some equilibrium distance and the potential energy of the system of two atoms (the molecule) is a minimum. One can minimize the function $U(x)$ by taking its derivative and setting it equal to zero:

$$\frac{dU(x)}{dx} = 4\epsilon\frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right] = 0$$

$$= 4\epsilon\left[\frac{-12\sigma^{12}}{x^{13}} - \frac{-6\sigma^{6}}{x^{7}}\right] = 0$$

Solving for x—the equilibrium separation of the two atoms in the molecule—and inserting the given information yield

$$x = 2.95 \times 10^{-10}\text{ m.}$$

We graph the Lennard–Jones function on both sides of this critical value to create our energy diagram, as shown in Figure 8.17a. Notice how $U(x)$ is extremely large when the atoms are very close together, is a minimum when the atoms are at their critical separation, and then increases again as the atoms move apart. When $U(x)$ is a minimum, the atoms are in stable equilibrium; this indicates that this is the most likely separation between them.

(b) Determine $F_x(x)$—the force that one atom exerts on the other in the molecule as a function of separation—and argue that the way this force behaves is physically plausible when the atoms are close together and far apart.

Solution Because the atoms combine to form a molecule, we reason that the force must be attractive when the atoms are far apart. On the other hand, the force must be repulsive when the two atoms get very close together. Otherwise, the molecule would collapse in on itself. Thus, the force must change sign at the critical separation, similar to the way spring forces switch sign in the change from extension to compression. Applying Equation 8.16 to the Lennard–Jones potential energy function gives

$$F_x = -\frac{dU(x)}{dx} = -4\epsilon\frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

$$= 4\epsilon\left[\frac{12\sigma^{12}}{x^{13}} - \frac{6\sigma^{6}}{x^{7}}\right]$$

This result is graphed in Figure 8.17b. As expected, the force is positive (repulsive) at small atomic separations, zero when the atoms are at the position of stable equilibrium [recall how we found the minimum of $U(x)$], and negative (attractive) at greater separations. Note that the force approaches zero as the separation between the atoms becomes very great.

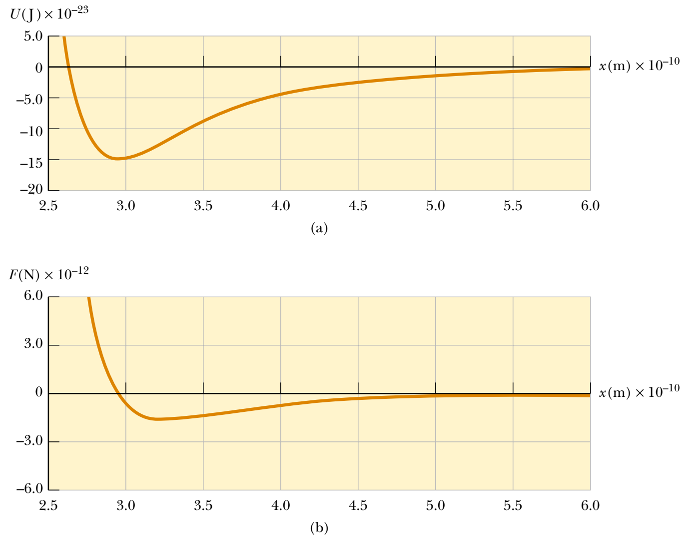


Figure 8.17 (a) Potential energy curve associated with a molecule. The distance x is the separation between the two atoms making up the molecule. (b) Force exerted on one atom by the other.

8.8 CONSERVATION OF ENERGY IN GENERAL

We have seen that the total mechanical energy of a system is constant when only conservative forces act within the system. Furthermore, we can associate a potential energy function with each conservative force. On the other hand, as we saw in Section 8.5, mechanical energy is lost when nonconservative forces such as friction are present.

In our study of thermodynamics later in this course, we shall find that mechanical energy can be transformed into energy stored *inside* the various objects that make up the system. This form of energy is called *internal energy*. For example, when a block slides over a rough surface, the mechanical energy lost because of friction is transformed into internal energy that is stored temporarily inside the block and inside the surface, as evidenced by a measurable increase in the temperature of both block and surface. We shall see that on a submicroscopic scale, this internal energy is associated with the vibration of atoms about their equilibrium positions. Such internal atomic motion involves both kinetic and potential energy. Therefore, if we include in our energy expression this increase in the internal energy of the objects that make up the system, then energy is conserved.

This is just one example of how you can analyze an isolated system and always find that the total amount of energy it contains does not change, as long as you account for all forms of energy. That is, **energy can never be created or destroyed. Energy may be transformed from one form to another, but the**

Total energy is always conserved

total energy of an isolated system is always constant. From a universal point of view, we can say that the **total energy of the Universe is constant.** If one part of the Universe gains energy in some form, then another part must lose an equal amount of energy. No violation of this principle has ever been found.

Optional Section

8.9 MASS–ENERGY EQUIVALENCE

This chapter has been concerned with the important principle of energy conservation and its application to various physical phenomena. Another important principle, **conservation of mass,** states that **in any physical or chemical process, mass is neither created nor destroyed.** That is, the mass before the process equals the mass after the process.

For centuries, scientists believed that energy and mass were two quantities that were separately conserved. However, in 1905 Einstein made the brilliant discovery that the mass of any system is a measure of the energy of that system. Hence, energy and mass are related concepts. The relationship between the two is given by Einstein's most famous formula:

$$E_R = mc^2 \tag{8.17}$$

where c is the speed of light and E_R is the energy equivalent of a mass m. The subscript R on the energy refers to the **rest energy** of an object of mass m—that is, the energy of the object when its speed is $v = 0$.

The rest energy associated with even a small amount of matter is enormous. For example, the rest energy of 1 kg of any substance is

$$E_R = mc^2 = (1\text{ kg})(3 \times 10^8\text{ m/s})^2 = 9 \times 10^{16}\text{ J}$$

This is equivalent to the energy content of about 15 million barrels of crude oil—about one day's consumption in the United States! If this energy could easily be released as useful work, our energy resources would be unlimited.

In reality, only a small fraction of the energy contained in a material sample can be released through chemical or nuclear processes. The effects are greatest in nuclear reactions, in which fractional changes in energy, and hence mass, of approximately 10^{-3} are routinely observed. A good example is the enormous amount of energy released when the uranium-235 nucleus splits into two smaller nuclei. This happens because the sum of the masses of the product nuclei is slightly less than the mass of the original ^{235}U nucleus. The awesome nature of the energy released in such reactions is vividly demonstrated in the explosion of a nuclear weapon.

Equation 8.17 indicates that *energy has mass*. Whenever the energy of an object changes in any way, its mass changes as well. If ΔE is the change in energy of an object, then its change in mass is

$$\Delta m = \frac{\Delta E}{c^2} \tag{8.18}$$

Anytime energy ΔE in any form is supplied to an object, the change in the mass of the object is $\Delta m = \Delta E/c^2$. However, because c^2 is so large, the changes in mass in any ordinary mechanical experiment or chemical reaction are too small to be detected.

EXAMPLE 8.12 Here Comes the Sun

The Sun converts an enormous amount of matter to energy. Each second, 4.19×10^9 kg—approximately the capacity of 400 average-sized cargo ships—is changed to energy. What is the power output of the Sun?

Solution We find the energy liberated per second by means of a straightforward conversion:

$$E_R = (4.19 \times 10^9\text{ kg})(3.00 \times 10^8\text{ m/s})^2 = 3.77 \times 10^{26}\text{ J}$$

We then apply the definition of power:

$$\mathcal{P} = \frac{3.77 \times 10^{26}\text{ J}}{1.00\text{ s}} = 3.77 \times 10^{26}\text{ W}$$

The Sun radiates uniformly in all directions, and so only a very tiny fraction of its total output is collected by the Earth. Nonetheless this amount is sufficient to supply energy to nearly everything on the Earth. (Nuclear and geothermal energy are the only alternatives.) Plants absorb solar energy and convert it to chemical potential energy (energy stored in the plant's molecules). When an animal eats the plant, this chemical potential energy can be turned into kinetic and other forms of energy. You are reading this book with solar-powered eyes!

Optional Section

8.10 QUANTIZATION OF ENERGY

Certain physical quantities such as electric charge are *quantized;* that is, the quantities have discrete values rather than continuous values. The quantized nature of energy is especially important in the atomic and subatomic world. As an example, let us consider the energy levels of the hydrogen atom (which consists of an electron orbiting around a proton). The atom can occupy only certain energy levels, called *quantum states,* as shown in Figure 8.18a. The atom cannot have any energy values lying between these quantum states. The lowest energy level E_1 is called the

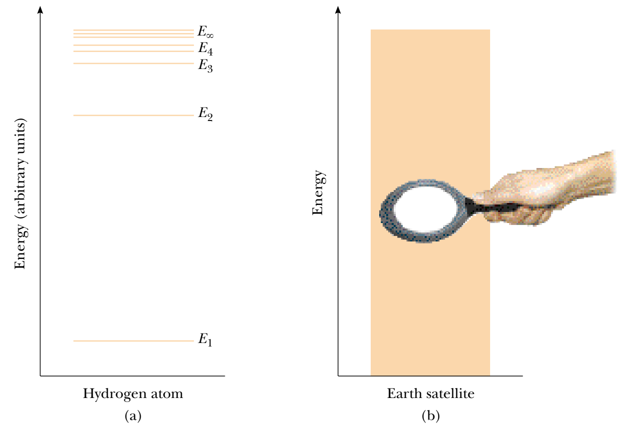


Figure 8.18 Energy-level diagrams: (a) Quantum states of the hydrogen atom. The lowest state E_1 is the ground state. (b) The energy levels of an Earth satellite are also quantized but are so close together that they cannot be distinguished from one another.

ground state of the atom. The ground state corresponds to the state that an isolated atom usually occupies. The atom can move to higher energy states by absorbing energy from some external source or by colliding with other atoms. The highest energy on the scale shown in Figure 8.18a, E_∞, corresponds to the energy of the atom when the electron is completely removed from the proton. The energy difference $E_\infty - E_1$ is called the **ionization energy.** Note that the energy levels get closer together at the high end of the scale.

Next, consider a satellite in orbit about the Earth. If you were asked to describe the possible energies that the satellite could have, it would be reasonable (but incorrect) to say that it could have any arbitrary energy value. Just like that of the hydrogen atom, however, **the energy of the satellite is quantized.** If you were to construct an energy level diagram for the satellite showing its allowed energies, the levels would be so close to one another, as shown in Figure 8.18b, that it would be difficult to discern that they were not continuous. In other words, we have no way of experiencing quantization of energy in the macroscopic world; hence, we can ignore it in describing everyday experiences.

SUMMARY

If a particle of mass m is at a distance y above the Earth's surface, the **gravitational potential energy** of the particle–Earth system is

$$U_g = mgy \quad \textbf{(8.1)}$$

The **elastic potential energy** stored in a spring of force constant k is

$$U_s \equiv \tfrac{1}{2}kx^2 \quad \textbf{(8.4)}$$

You should be able to apply these two equations in a variety of situations to determine the potential an object has to perform work.

A force is **conservative** if the work it does on a particle moving between two points is independent of the path the particle takes between the two points. Furthermore, a force is conservative if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be **nonconservative.**

A **potential energy** function U can be associated only with a conservative force. If a conservative force **F** acts on a particle that moves along the x axis from x_i to x_f, then the change in the potential energy of the system equals the negative of the work done by that force:

$$U_f - U_i = -\int_{x_i}^{x_f} F_x\, dx \quad \textbf{(8.7)}$$

You should be able to use calculus to find the potential energy associated with a conservative force and vice versa.

The **total mechanical energy of a system** is defined as the sum of the kinetic energy and the potential energy:

$$E \equiv K + U \quad \textbf{(8.9)}$$

If no external forces do work on a system and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_i + U_i = K_f + U_f \quad \textbf{(8.10)}$$

If nonconservative forces (such as friction) act on objects inside a system, then mechanical energy is not conserved. In these situations, the difference between the total final mechanical energy and the total initial mechanical energy of the system equals the energy transferred to or from the system by the nonconservative forces.

QUESTIONS

1. Many mountain roads are constructed so that they spiral around a mountain rather than go straight up the slope. Discuss this design from the viewpoint of energy and power.
2. A ball is thrown straight up into the air. At what position is its kinetic energy a maximum? At what position is the gravitational potential energy a maximum?
3. A bowling ball is suspended from the ceiling of a lecture hall by a strong cord. The bowling ball is drawn away from its equilibrium position and released from rest at the tip of the student's nose as in Figure Q8.3. If the student remains stationary, explain why she will not be struck by the ball on its return swing. Would the student be safe if she pushed the ball as she released it?

Figure Q8.3

4. One person drops a ball from the top of a building, while another person at the bottom observes its motion. Will these two people agree on the value of the potential energy of the ball–Earth system? on its change in potential energy? on the kinetic energy of the ball?
5. When a person runs in a track event at constant velocity, is any work done? (*Note:* Although the runner moves with constant velocity, the legs and arms accelerate.) How does air resistance enter into the picture? Does the center of mass of the runner move horizontally?
6. Our body muscles exert forces when we lift, push, run, jump, and so forth. Are these forces conservative?
7. If three conservative forces and one nonconservative force act on a system, how many potential energy terms appear in the equation that describes this system?
8. Consider a ball fixed to one end of a rigid rod whose other end pivots on a horizontal axis so that the rod can rotate in a vertical plane. What are the positions of stable and unstable equilibrium?
9. Is it physically possible to have a situation where $E - U < 0$?
10. What would the curve of U versus x look like if a particle were in a region of neutral equilibrium?
11. Explain the energy transformations that occur during (a) the pole vault, (b) the shot put, (c) the high jump. What is the source of energy in each case?
12. Discuss some of the energy transformations that occur during the operation of an automobile.
13. If only one external force acts on a particle, does it necessarily change the particle's (a) kinetic energy? (b) velocity?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 8.1 Potential Energy
Section 8.2 Conservative and Nonconservative Forces

1. A 1 000-kg roller coaster is initially at the top of a rise, at point A. It then moves 135 ft, at an angle of 40.0° below the horizontal, to a lower point B. (a) Choose point B to be the zero level for gravitational potential energy. Find the potential energy of the roller coaster–Earth system at points A and B and the change in its potential energy as the coaster moves. (b) Repeat part (a), setting the zero reference level at point A.

2. A 40.0-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

3. A 4.00-kg particle moves from the origin to position *C*, which has coordinates $x = 5.00$ m and $y = 5.00$ m (Fig. P8.3). One force on it is the force of gravity acting in the negative *y* direction. Using Equation 7.2, calculate the work done by gravity as the particle moves from *O* to *C* along (a) *OAC*, (b) *OBC*, and (c) *OC*. Your results should all be identical. Why?

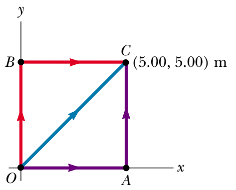


Figure P8.3 Problems 3, 4, and 5.

4. (a) Suppose that a constant force acts on an object. The force does not vary with time, nor with the position or velocity of the object. Start with the general definition for work done by a force

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{s}$$

and show that the force is conservative. (b) As a special case, suppose that the force $\mathbf{F} = (3\mathbf{i} + 4\mathbf{j})$ N acts on a particle that moves from *O* to *C* in Figure P8.3. Calculate the work done by **F** if the particle moves along each one of the three paths *OAC, OBC,* and *OC*. (Your three answers should be identical.)

5. A force acting on a particle moving in the *xy* plane is given by $\mathbf{F} = (2y\mathbf{i} + x^2\mathbf{j})$ N, where *x* and *y* are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00$ m and $y = 5.00$ m, as in Figure P8.3. Calculate the work done by **F** along (a) *OAC*, (b) *OBC*, (c) *OC*. (d) Is **F** conservative or nonconservative? Explain.

Section 8.3 Conservative Forces and Potential Energy

Section 8.4 Conservation of Mechanical Energy

6. At time t_i, the kinetic energy of a particle in a system is 30.0 J and the potential energy of the system is 10.0 J. At some later time t_f, the kinetic energy of the particle is 18.0 J. (a) If only conservative forces act on the particle, what are the potential energy and the total energy at time t_f? (b) If the potential energy of the system at time t_f is 5.00 J, are any nonconservative forces acting on the particle? Explain.

WEB **7.** A single conservative force acts on a 5.00-kg particle. The equation $F_x = (2x + 4)$ N, where *x* is in meters, describes this force. As the particle moves along the *x* axis from $x = 1.00$ m to $x = 5.00$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the system, and (c) the kinetic energy of the particle at $x = 5.00$ m if its speed at $x = 1.00$ m is 3.00 m/s.

8. A single constant force $\mathbf{F} = (3\mathbf{i} + 5\mathbf{j})$ N acts on a 4.00-kg particle. (a) Calculate the work done by this force if the particle moves from the origin to the point having the vector position $\mathbf{r} = (2\mathbf{i} - 3\mathbf{j})$ m. Does this result depend on the path? Explain. (b) What is the speed of the particle at **r** if its speed at the origin is 4.00 m/s? (c) What is the change in the potential energy of the system?

9. A single conservative force acting on a particle varies as $\mathbf{F} = (-Ax + Bx^2)\mathbf{i}$ N, where *A* and *B* are constants and *x* is in meters. (a) Calculate the potential energy function $U(x)$ associated with this force, taking $U = 0$ at $x = 0$. (b) Find the change in potential energy and change in kinetic energy as the particle moves from $x = 2.00$ m to $x = 3.00$ m.

10. A particle of mass 0.500 kg is shot from *P* as shown in Figure P8.10. The particle has an initial velocity $\mathbf{v}_i$ with a horizontal component of 30.0 m/s. The particle rises to a maximum height of 20.0 m above *P*. Using the law of conservation of energy, determine (a) the vertical component of $\mathbf{v}_i$, (b) the work done by the gravitational force on the particle during its motion from *P* to *B*, and (c) the horizontal and the vertical components of the velocity vector when the particle reaches *B*.

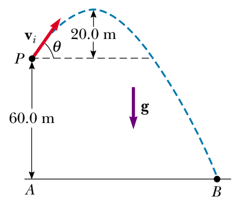


Figure P8.10

11. A 3.00-kg mass starts from rest and slides a distance *d* down a frictionless 30.0° incline. While sliding, it comes into contact with an unstressed spring of negligible mass, as shown in Figure P8.11. The mass slides an additional 0.200 m as it is brought momentarily to rest by compression of the spring ($k = 400$ N/m). Find the initial separation *d* between the mass and the spring.

12. A mass *m* starts from rest and slides a distance *d* down a frictionless incline of angle *θ*. While sliding, it contacts an unstressed spring of negligible mass, as shown in Figure P8.11. The mass slides an additional distance *x* as it is brought momentarily to rest by compression of the spring (of force constant *k*). Find the initial separation *d* between the mass and the spring.

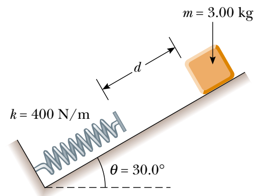


Figure P8.11 Problems 11 and 12.

13. A particle of mass $m = 5.00$ kg is released from point Ⓐ and slides on the frictionless track shown in Figure P8.13. Determine (a) the particle's speed at points Ⓑ and Ⓒ and (b) the net work done by the force of gravity in moving the particle from Ⓐ to Ⓒ.

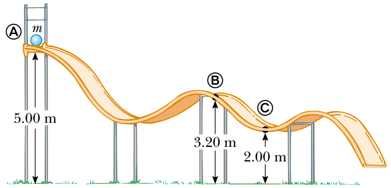


Figure P8.13

14. A simple, 2.00-m-long pendulum is released from rest when the support string is at an angle of 25.0° from the vertical. What is the speed of the suspended mass at the bottom of the swing?

15. A bead slides without friction around a loop-the-loop (Fig. P8.15). If the bead is released from a height $h = 3.50R$, what is its speed at point *A*? How great is the normal force on it if its mass is 5.00 g?

16. A 120-g mass is attached to the bottom end of an unstressed spring. The spring is hanging vertically and has a spring constant of 40.0 N/m. The mass is dropped. (a) What is its maximum speed? (b) How far does it drop before coming to rest momentarily?

17. A block of mass 0.250 kg is placed on top of a light vertical spring of constant $k = 5\,000$ N/m and is pushed downward so that the spring is compressed 0.100 m. After the block is released, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

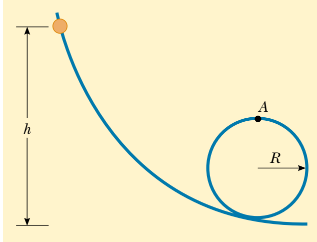


Figure P8.15

18. Dave Johnson, the bronze medalist at the 1992 Olympic decathlon in Barcelona, leaves the ground for his high jump with a vertical velocity component of 6.00 m/s. How far up does his center of gravity move as he makes the jump?

19. A 0.400-kg ball is thrown straight up into the air and reaches a maximum altitude of 20.0 m. Taking its initial position as the point of zero potential energy and using energy methods, find (a) its initial speed, (b) its total mechanical energy, and (c) the ratio of its kinetic energy to the potential energy of the ball–Earth system when the ball is at an altitude of 10.0 m.

20. In the dangerous "sport" of bungee-jumping, a daring student jumps from a balloon with a specially designed

Figure P8.20 Bungee-jumping. *(Gamma)* elastic cord attached to his ankles, as shown in Figure P8.20. The unstretched length of the cord is 25.0 m, the student weighs 700 N, and the balloon is 36.0 m above the surface of a river below. Assuming that Hooke's law describes the cord, calculate the required force constant if the student is to stop safely 4.00 m above the river.

21. Two masses are connected by a light string passing over a light frictionless pulley, as shown in Figure P8.21. The 5.00-kg mass is released from rest. Using the law of conservation of energy, (a) determine the speed of the 3.00-kg mass just as the 5.00-kg mass hits the ground and (b) find the maximum height to which the 3.00-kg mass rises.

22. Two masses are connected by a light string passing over a light frictionless pulley, as shown in Figure P8.21. The mass m_1 (which is greater than m_2) is released from rest. Using the law of conservation of energy, (a) determine the speed of m_2 just as m_1 hits the ground in terms of m_1, m_2, and h, and (b) find the maximum height to which m_2 rises.

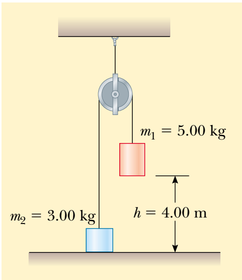


Figure P8.21 Problems 21 and 22.

23. A 20.0-kg cannon ball is fired from a cannon with a muzzle speed of 1 000 m/s at an angle of 37.0° with the horizontal. A second ball is fired at an angle of 90.0°. Use the law of conservation of mechanical energy to find (a) the maximum height reached by each ball and (b) the total mechanical energy at the maximum height for each ball. Let $y = 0$ at the cannon.

24. A 2.00-kg ball is attached to the bottom end of a length of 10-lb (44.5-N) fishing line. The top end of the fishing line is held stationary. The ball is released from rest while the line is taut and horizontal ($\theta = 90.0°$). At what angle θ (measured from the vertical) will the fishing line break?

25. The circus apparatus known as the *trapeze* consists of a bar suspended by two parallel ropes, each of length ℓ. The trapeze allows circus performers to swing in a vertical circular arc (Fig. P8.25). Suppose a performer with mass m and holding the bar steps off an elevated platform, starting from rest with the ropes at an angle of θ_i with respect to the vertical. Suppose the size of the performer's body is small compared with the length ℓ, that she does not pump the trapeze to swing higher, and that air resistance is negligible. (a) Show that when the ropes make an angle of θ with respect to the vertical, the performer must exert a force

$$F = mg(3\cos\theta - 2\cos\theta_i)$$

in order to hang on. (b) Determine the angle θ_i at which the force required to hang on at the bottom of the swing is twice the performer's weight.

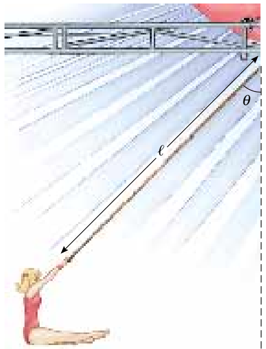


Figure P8.25

26. After its release at the top of the first rise, a roller-coaster car moves freely with negligible friction. The roller coaster shown in Figure P8.26 has a circular loop of radius 20.0 m. The car barely makes it around the loop: At the top of the loop, the riders are upside down and feel weightless. (a) Find the speed of the roller coaster car at the top of the loop (position 3). Find the speed of the roller coaster car (b) at position 1 and (c) at position 2. (d) Find the difference in height between positions 1 and 4 if the speed at position 4 is 10.0 m/s.

27. A light rigid rod is 77.0 cm long. Its top end is pivoted on a low-friction horizontal axle. The rod hangs straight down at rest, with a small massive ball attached to its bottom end. You strike the ball, suddenly giving it a horizontal velocity so that it swings around in a full circle. What minimum speed at the bottom is required to make the ball go over the top of the circle?

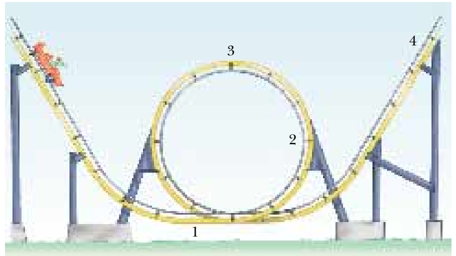


Figure P8.26

Section 8.5 Work Done by Nonconservative Forces

28. A 70.0-kg diver steps off a 10.0-m tower and drops straight down into the water. If he comes to rest 5.00 m beneath the surface of the water, determine the average resistance force that the water exerts on the diver.

29. A force F_x, shown as a function of distance in Figure P8.29, acts on a 5.00-kg mass. If the particle starts from rest at $x = 0$ m, determine the speed of the particle at $x = 2.00$, 4.00, and 6.00 m.

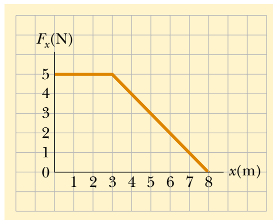


Figure P8.29

30. A softball pitcher swings a ball of mass 0.250 kg around a vertical circular path of radius 60.0 cm before releasing it from her hand. The pitcher maintains a component of force on the ball of constant magnitude 30.0 N in the direction of motion around the complete path. The speed of the ball at the top of the circle is 15.0 m/s. If the ball is released at the bottom of the circle, what is its speed upon release?

WEB **31.** The coefficient of friction between the 3.00-kg block and the surface in Figure P8.31 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

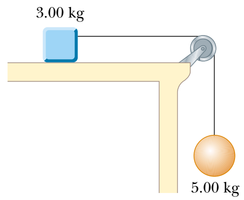


Figure P8.31

32. A 2 000-kg car starts from rest and coasts down from the top of a 5.00-m-long driveway that is sloped at an angle of 20.0° with the horizontal. If an average friction force of 4 000 N impedes the motion of the car, find the speed of the car at the bottom of the driveway.

33. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine (a) the change in the block's kinetic energy, (b) the change in the potential energy, and (c) the frictional force exerted on it (assumed to be constant). (d) What is the coefficient of kinetic friction?

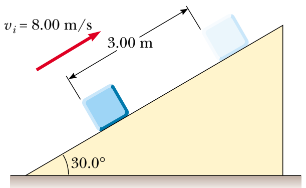


Figure P8.33

34. A boy in a wheelchair (total mass, 47.0 kg) wins a race with a skateboarder. He has a speed of 1.40 m/s at the crest of a slope 2.60 m high and 12.4 m long. At the bottom of the slope, his speed is 6.20 m/s. If air resistance and rolling resistance can be modeled as a constant frictional force of 41.0 N, find the work he did in pushing forward on his wheels during the downhill ride.

35. A parachutist of mass 50.0 kg jumps out of a balloon at a height of 1 000 m and lands on the ground with a speed of 5.00 m/s. How much energy was lost to air friction during this jump?

36. An 80.0-kg sky diver jumps out of a balloon at an altitude of 1 000 m and opens the parachute at an altitude of 200.0 m. (a) Assuming that the total retarding force on the diver is constant at 50.0 N with the parachute closed and constant at 3 600 N with the parachute open, what is the speed of the diver when he lands on the ground? (b) Do you think the sky diver will get hurt? Explain. (c) At what height should the parachute be opened so that the final speed of the sky diver when he hits the ground is 5.00 m/s? (d) How realistic is the assumption that the total retarding force is constant? Explain.

37. A toy cannon uses a spring to project a 5.30-g soft rubber ball. The spring is originally compressed by 5.00 cm and has a stiffness constant of 8.00 N/m. When it is fired, the ball moves 15.0 cm through the barrel of the cannon, and there is a constant frictional force of 0.032 0 N between the barrel and the ball. (a) With what speed does the projectile leave the barrel of the cannon? (b) At what point does the ball have maximum speed? (c) What is this maximum speed?

38. A 1.50-kg mass is held 1.20 m above a relaxed, massless vertical spring with a spring constant of 320 N/m. The mass is dropped onto the spring. (a) How far does it compress the spring? (b) How far would it compress the spring if the same experiment were performed on the Moon, where $g = 1.63\ \text{m/s}^2$? (c) Repeat part (a), but this time assume that a constant air-resistance force of 0.700 N acts on the mass during its motion.

39. A 3.00-kg block starts at a height $h = 60.0$ cm on a plane that has an inclination angle of 30.0°, as shown in Figure P8.39. Upon reaching the bottom, the block slides along a horizontal surface. If the coefficient of friction on both surfaces is $\mu_k = 0.200$, how far does the block slide on the horizontal surface before coming to rest? (*Hint:* Divide the path into two straight-line parts.)

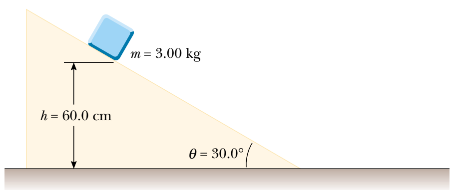


Figure P8.39

40. A 75.0-kg sky diver is falling with a terminal speed of 60.0 m/s. Determine the rate at which he is losing mechanical energy.

Section 8.6 Relationship Between Conservative Forces and Potential Energy

WEB **41.** The potential energy of a two-particle system separated by a distance r is given by $U(r) = A/r$, where A is a constant. Find the radial force $\mathbf{F}_r$ that each particle exerts on the other.

42. A potential energy function for a two-dimensional force is of the form $U = 3x^3y - 7x$. Find the force that acts at the point (x, y).

(Optional) Section 8.7 Energy Diagrams and the Equilibrium of a System

43. A particle moves along a line where the potential energy depends on its position r, as graphed in Figure P8.43. In the limit as r increases without bound, $U(r)$ approaches $+1$ J. (a) Identify each equilibrium position for this particle. Indicate whether each is a point of stable, unstable, or neutral equilibrium. (b) The particle will be bound if its total energy is in what range? Now suppose the particle has energy -3 J. Determine (c) the range of positions where it can be found, (d) its maximum kinetic energy, (e) the location at which it has maximum kinetic energy, and (f) its *binding energy*—that is, the additional energy that it would have to be given in order for it to move out to $r \rightarrow \infty$.

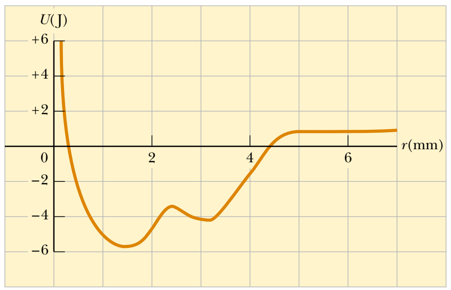


Figure P8.43

44. A right circular cone can be balanced on a horizontal surface in three different ways. Sketch these three equilibrium configurations and identify them as positions of stable, unstable, or neutral equilibrium.

45. For the potential energy curve shown in Figure P8.45, (a) determine whether the force F_x is positive, negative, or zero at the five points indicated. (b) Indicate points of stable, unstable, and neutral equilibrium. (c) Sketch the curve for F_x versus x from $x = 0$ to $x = 9.5$ m.

46. A hollow pipe has one or two weights attached to its inner surface, as shown in Figure P8.46. Characterize each configuration as being stable, unstable, or neutral equilibrium and explain each of your choices ("CM" indicates center of mass).

47. A particle of mass m is attached between two identical springs on a horizontal frictionless tabletop. The

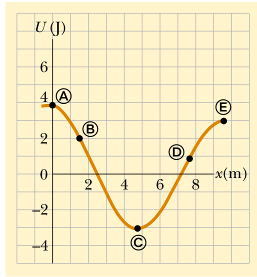


Figure P8.45

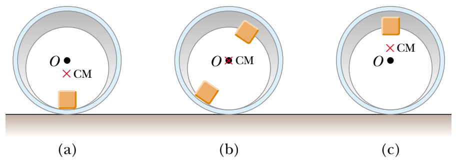


Figure P8.46

springs have spring constant k, and each is initially unstressed. (a) If the mass is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P8.47, show that the potential energy of the system is

$$U(x) = kx^2 + 2kL(L - \sqrt{x^2 + L^2})$$

(*Hint:* See Problem 66 in Chapter 7.) (b) Make a plot of $U(x)$ versus x and identify all equilibrium points. Assume that $L = 1.20$ m and $k = 40.0$ N/m. (c) If the mass is pulled 0.500 m to the right and then released, what is its speed when it reaches the equilibrium point $x = 0$?

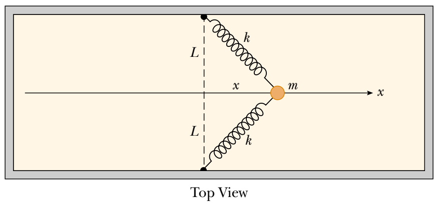


Figure P8.47

(Optional) Section 8.9 Mass–Energy Equivalence

48. Find the energy equivalents of (a) an electron of mass 9.11×10^{-31} kg, (b) a uranium atom with a mass of 4.00×10^{-25} kg, (c) a paper clip of mass 2.00 g, and (d) the Earth (of mass 5.99×10^{24} kg).

49. The expression for the kinetic energy of a particle moving with speed v is given by Equation 7.19, which can be written as $K = \gamma mc^2 - mc^2$, where $\gamma = [1 - (v/c)^2]^{-1/2}$. The term γmc^2 is the total energy of the particle, and the term mc^2 is its rest energy. A proton moves with a speed of $0.990c$, where c is the speed of light. Find (a) its rest energy, (b) its total energy, and (c) its kinetic energy.

ADDITIONAL PROBLEMS

50. A block slides down a curved frictionless track and then up an inclined plane as in Figure P8.50. The coefficient of kinetic friction between the block and the incline is μ_k. Use energy methods to show that the maximum height reached by the block is

$$y_{max} = \frac{h}{1 + \mu_k \cot\theta}$$

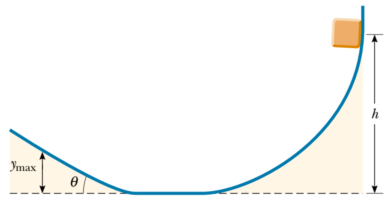


Figure P8.50

51. Close to the center of a campus is a tall silo topped with a hemispherical cap. The cap is frictionless when wet. Someone has somehow balanced a pumpkin at the highest point. The line from the center of curvature of the cap to the pumpkin makes an angle $\theta_i = 0°$ with the vertical. On a rainy night, a breath of wind makes the pumpkin start sliding downward from rest. It loses contact with the cap when the line from the center of the hemisphere to the pumpkin makes a certain angle with the vertical; what is this angle?

52. A 200-g particle is released from rest at point Ⓐ along the horizontal diameter on the inside of a frictionless, hemispherical bowl of radius $R = 30.0$ cm (Fig. P8.52). Calculate (a) the gravitational potential energy when the particle is at point Ⓐ relative to point Ⓑ, (b) the kinetic energy of the particle at point Ⓑ, (c) its speed at point Ⓑ, and (d) its kinetic energy and the potential energy at point Ⓒ.

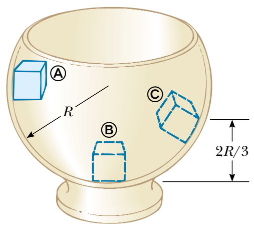


Figure P8.52 Problems 52 and 53.

WEB **53.** The particle described in Problem 52 (Fig. P8.52) is released from rest at Ⓐ, and the surface of the bowl is rough. The speed of the particle at Ⓑ is 1.50 m/s. (a) What is its kinetic energy at Ⓑ? (b) How much energy is lost owing to friction as the particle moves from Ⓐ to Ⓑ? (c) Is it possible to determine μ from these results in any simple manner? Explain.

54. Review Problem. The mass of a car is 1 500 kg. The shape of the body is such that its aerodynamic drag coefficient is $D = 0.330$ and the frontal area is 2.50 m^2. Assuming that the drag force is proportional to v^2 and neglecting other sources of friction, calculate the power the car requires to maintain a speed of 100 km/h as it climbs a long hill sloping at 3.20°.

55. Make an order-of-magnitude estimate of your power output as you climb stairs. In your solution, state the physical quantities you take as data and the values you measure or estimate for them. Do you consider your peak power or your sustainable power?

56. A child's pogo stick (Fig. P8.56) stores energy in a spring ($k = 2.50 \times 10^4$ N/m). At position Ⓐ ($x_A = -0.100$ m), the spring compression is a maximum and the child is momentarily at rest. At position Ⓑ ($x_B = 0$), the spring is relaxed and the child is moving upward. At position Ⓒ, the child is again momentarily at rest at the top of the jump. Assuming that the combined mass of the child and the pogo stick is 25.0 kg, (a) calculate the total energy of the system if both potential energies are zero at $x = 0$, (b) determine x_C, (c) calculate the speed of the child at $x = 0$, (d) determine the value of x for which the kinetic energy of the system is a maximum, and (e) calculate the child's maximum upward speed.

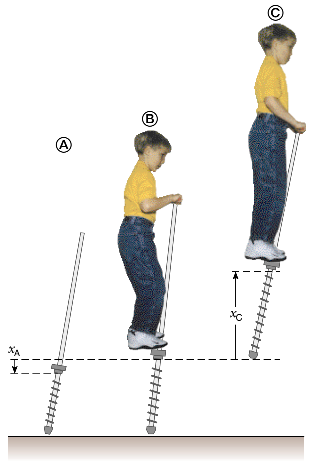


Figure P8.56

57. A 10.0-kg block is released from point Ⓐ in Figure P8.57. The track is frictionless except for the portion between Ⓑ and Ⓒ, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant $k = 2\,250$ N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between Ⓑ and Ⓒ.

58. A 2.00-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m (Fig. P8.58). The pulley is frictionless. The block is released from rest when the spring is unstretched. The block moves 20.0 cm down the incline before coming to rest. Find the coefficient of kinetic friction between block and incline.

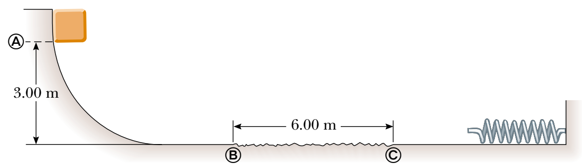


Figure P8.57

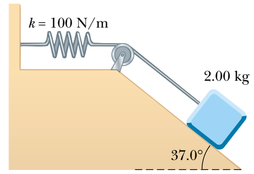


Figure P8.58 Problems 58 and 59.

59. Review Problem. Suppose the incline is frictionless for the system described in Problem 58 (see Fig. P8.58). The block is released from rest with the spring initially unstretched. (a) How far does it move down the incline before coming to rest? (b) What is its acceleration at its lowest point? Is the acceleration constant? (c) Describe the energy transformations that occur during the descent.

60. The potential energy function for a system is given by $U(x) = -x^3 + 2x^2 + 3x$. (a) Determine the force F_x as a function of x. (b) For what values of x is the force equal to zero? (c) Plot $U(x)$ versus x and F_x versus x, and indicate points of stable and unstable equilibrium.

61. A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.61. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and is released from rest. Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).

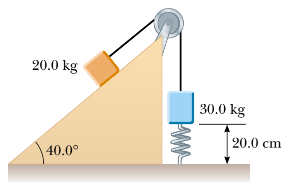


Figure P8.61

62. A 1.00-kg mass slides to the right on a surface having a coefficient of friction $\mu = 0.250$ (Fig. P8.62). The mass has a speed of $v_i = 3.00$ m/s when it makes contact with a light spring that has a spring constant $k = 50.0$ N/m. The mass comes to rest after the spring has been compressed a distance d. The mass is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally, the mass comes to rest at a distance D to the left of the unstretched spring. Find (a) the distance of compression d, (b) the speed v of the mass at the unstretched position when the mass is moving to the left, and (c) the distance D between the unstretched spring and the point at which the mass comes to rest.

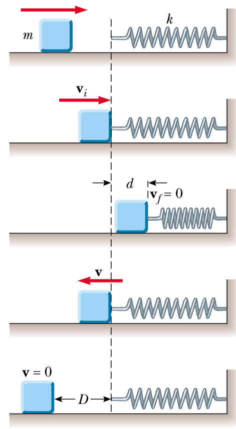


Figure P8.62

WEB **63.** A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance Δx (Fig. P8.63). The spring constant is 450 N/m. When it is released, the block travels along a frictionless, horizontal surface to point B, at the bottom of a vertical circular track of radius $R = 1.00$ m, and continues to move up the track. The speed of the block at the bottom of the track is $v_B = 12.0$ m/s, and the block experiences an average frictional force of 7.00 N while sliding up the track. (a) What is Δx? (b) What speed do you predict for the block at the top of the track? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?

64. A uniform chain of length 8.00 m initially lies stretched out on a horizontal table. (a) If the coefficient of static friction between the chain and the table is 0.600, show that the chain will begin to slide off the table if at least 3.00 m of it hangs over the edge of the table. (b) Determine the speed of the chain as all of it leaves the table, given that the coefficient of kinetic friction between the chain and the table is 0.400.